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**TESTING THE RANK OF ENGEL CURVES  
WITH ENDOGENOUS EXPENDITURE**

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# Testing the Rank of Engel Curves with Endogenous Expenditure

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## Abstract

We find that the endogeneity of consumer expenditure tends to increase the estimated rank of Engel curves. This result, based on nonparametric procedure, is in line with previous results obtained in the context of parametric specifications...

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# 1 Introduction<sup>1</sup>

In empirical demand analysis based on individual household data total consumer expenditure is generally found to be endogenous typically due to measurement error arising from infrequency of purchase (Keen 1986 and Meghir and Robin 1992). Blundell and Duncan (1997) report differences in the shape of Engel curves estimated with and without accounting for the endogeneity of total consumer expenditure. Furthermore, testing the quadratic logarithmic specification for the budget share equations against semiparametric alternatives, report acceptance of this specification at lower significance level when the endogeneity of total consumer expenditure and preference heterogeneity are accounted for. Hausman, Newey and Powell (1995), testing parametrically for the rank of Engel curves, show that when their IV estimates are used no evidence for more than a quadratic (log) expenditure term is present in the budget share equations; whereas the OLS estimates are ambiguous as to whether a cubic (log) expenditure term in the budget share equations is redundant. Banks, Blundell and Lewbel (1997) also do rank tests that control for endogeneity or measurement error and find evidence in support of rank=3.

In this note we test the effect of consumer expenditure endogeneity on the rank of a system of Engel curves (as defined by Lewbel 1989) by applying non-parametric techniques to a large sample of heterogeneous households, after semiparametrically controlling for preference heterogeneity as described in Lyssiotou, Pashardes and Stengos (1997). The empirical analysis is performed with Engel curves estimated from individual household data drawn from UK Family Expenditure Survey (FES).

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## 2 A nonparametric test of rank with endogenous expenditure

Let the budget share equations shares  $w_{ih}$  for goods  $i = 1; \dots; N$  and households  $h = 1; \dots; H$  be written as,

$$w_{ih} = c_i^T z_h + f_i(x_h) + v_{ih}; \quad (1)$$

where  $c_i^T$  is a (transposed) parameter vector for commodity  $i$ ;  $z_h$  a vector of household characteristics and  $x_h$  the logarithm of total consumer expenditure.<sup>2</sup> Here  $x_h$  is considered to be endogenous,  $E(v_{ih}|x_h) \neq 0$ ; and it is possible to express it in terms of a set of instruments  $y_h$

$$x_h = g(y_h) + u_h; \quad (2)$$

where, for simplicity,  $g(y_h)$  is assumed to be parametric, say  $g(y_h) = \beta^T y_h$ . In the context of empirical demand analysis the identifying instruments  $y_h$  are generally taken to be the level of income and its powers on the assumption that  $E(u_h|y_h) = 0$ ; - see Blundell et al (1993) and Lewbel (1996).

We assume that  $E(v_{ih}|y_h; u_h) = E(v_{ih}|u_h)$ : It then follows, since  $E(v_{ih}|u_h) \neq 0$ ; that  $E(v_{ih}|x_h) \neq 0$ : Hence one can decompose  $v_{ih}$  into  $v_{ih} = \mu_i(u_h) + \epsilon_{ih}$ ; where  $\mu_i(u_h) = E(v_{ih}|u_h)$  and  $\epsilon_{ih} = v_{ih} - \mu_i(u_h)$ : Equation (1) then becomes

$$w_{ih} = c_i^T z_h + f_i(x_h) + \mu_i(u_h) + \epsilon_{ih} \quad (3)$$

We replace the unobservable  $u_h$  by the observable  $b_h = x_h - \beta^T y_h$ : Then equation (3) becomes

$$w_{ih} = c_i^T z_h + f_i(x_h) + \mu_i(b_h) + \epsilon_{ih}^a \quad (4)$$

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<sup>2</sup>It should be noted that conditioning on the vector of household characteristics  $z_h$  can reduce the rank of (1). For example Lyssiotou et. al. (1997) show that while the unconditional rank of a demand system is greater than three, the rank=3 hypothesis cannot be rejected after semiparametrically controlling for household characteristics, including ownership of durable goods. Therefore, the inclusion of  $z_h$  in (1) tends to reduce the rank that would have been found if the demand system had been estimated without  $z_h$  and/or with (flows from) durables goods as another good.

The error  $v_{ih}^a$  in (4) is composed of  $v_{ih}$  and  $\gg(u_h)_i \gg(\mathbf{b}_h)$ : In general,  $z_h$  are not uncorrelated with  $y_h$  and  $\mathbf{b}_h$ : For instance it is well known that family size is positively correlated with the level of income and total expenditure of the household. Therefore, rank tests based on the residuals from the regressions above (e.g. Donald's 1997 used here) would be inconsistent due to the inconsistency of the  $c_i$  estimates: To obtain root-n-consistent estimates of  $c_i$  we use a procedure based on the nearest neighbor estimator proposed by Estes and Honore (1995) and Yatchew (1997) described as follows.

Using the residuals  $\mathbf{b}_h$  from regressing  $x_h$  on  $y_h$  we take the data  $(w_{ih}; z_h; \mathbf{b}_{hi}; x_h)$ ;  $i = 1; \dots; N$  and sort them in ascending order according to the pair  $(x_h; \mathbf{b}_h)$ : Relying on the fact that  $x_h$  and  $\mathbf{b}_h$  are continuous variables,  $c_i$  can be consistently estimated from the regression

$$\Phi w_{ih} = c_i^T \Phi z_h + e_{ih} \quad (5)$$

where  $\Phi w_{ih} = w_{ih} - w_{ih-1}$ ,  $\Phi z_h = z_h - z_{h-1}$  and  $e_{ih}$  is a random error term. Alternatively, one can use Robinson's (1988) estimator to obtain root-n-consistent estimates of  $c_i$ :

Once we have obtained a consistent estimate of  $c_i$ ; we can rewrite (4) as

$$w_{ih} - \mathbf{b}_i^T z_h = f_i(x_h) + \gg(\mathbf{b}_h) + v_{ih}^a \quad (6)$$

One can use the marginal integration method proposed by Linton and Nielsen (1995) and Fan et al (1995) to obtain consistent estimates of  $f_i(x_h)$  and  $\gg(\mathbf{b}_h)$ ; say  $\hat{f}_i(x_h)$  and  $\hat{\gg}(\mathbf{b}_h)$ : The latter will provide the basis for applying the rank test of Donald (1997). Alternatively, the rank test can be conducted on the residuals  $\mathbf{b}_{ih} = w_{ih} - \mathbf{b}_i^T z_h - \hat{\gg}(\mathbf{b}_h)$  and  $x_h$ : The test is based on the normalized sum of the smallest eigenvalues of the system and is asymptotically distributed as a standard normal variate.

### 3 Empirical Results

We investigate the effect of the endogeneity of consumer expenditure on rank using a system of Engel curves consisting of six categories of non-durable commodities: food, alcohol, fuel, clothing, other goods and services. The assumption concerning durable goods is that they act as conditioning variables and this is modelled by including dummies for housing tenure, the size and value of the house, car ownership, the presence of smokers in the household etc. in the vector of household characteristics. The data are drawn from the 1970, 1980 and 1985 UK Family Expenditure Surveys and include all two-adult households whose head is under retirement age and not self-employed. The size of each sample is 1305 observations which exceeds substantially the typical sample sizes normally encountered in empirical work elsewhere. The vector  $z_h$  is defined to include a large number of household characteristics reflecting family composition, age, sex, employment and economic position of members, housing tenure etc.

Since the test of rank is carried out using the residuals  $\mathbf{b}_i = w_{ih} \mathbf{b}_i^T z_h$   $\mathbf{b}(\mathbf{b}_h)$ , as described above, its limiting distribution will be affected by the additional noise from the estimation of  $c_i$ . Also, there is an additional source of noise from the replacement of the unobservable  $u_h$  by  $\mathbf{b}_h$  in equation (4). To tackle this problem we bootstrap the null distribution of the test statistic.<sup>3</sup> We find that using residuals introduces a sizable shift to the right of the null distribution when compared with the standard normal variate.

In Table 1 we present the rank test statistics obtained with and without controlling for the endogeneity of total consumer expenditure. The results (reported for two different smoothing factors  $K$ ) show that taking account of consumer ex-

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<sup>3</sup>The bootstrap distribution is obtained using 1000 replications.

Table 1: Non-Parametric RankTest Results

NOT ACCOUNTING FOR ENDOGENEITY

Year	Rank up to 1	Rank up to 2	Rank up to 3	Rank up to 4
Smoothing Factor K=5				
70	10285.29	91.33	3.37	0.01
80	6445.77	110.18	11.43	3.35
85	261.45	35.36	-0.54	-1.90
Smoothing Factor K=7				
70	8710.67	77.30	2.88	0.19
80	5462.81	93.04	9.26	2.47
85	226.28	32.06	1.13	-1.97

ACCOUNTING FOR ENDOGENEITY

Year	Rank up to 1	Rank up to 2	Rank up to 3	Rank up to 4
Smoothing Factor K=5				
70	284.02 (255)	17.76 (14.36)	1.71 (2.59)	-1.69 (2.05)
80	297.90 (248)	40.57 (30.55)	2.42 (2.80)	-1.67 (2.20)
85	390.16 (201)	37.31 (27.10)	1.38 (2.20)	-1.01 (1.50)
Smoothing Factor K=7				
70	240.08 (235)	14.79 (11.51)	1.50 (2.35)	-1.76 (2.01)
80	252.05 (214)	34.87 (23.07)	1.79 (2.36)	-1.59 (1.98)
85	333.68 (193)	34.50 (28.75)	1.96 (2.11)	-1.26 (1.42)

Notes: For the upper panel the critical values are given by the standard normal variate. In the lower panel the entries in parentheses correspond to 95% bootstrap critical values using 1000 bootstrap replications.



penditure endogeneity produces test statistics that are generally more supportive of the rank 3 hypothesis, than the tests statistics produced without controlling for expenditure endogeneity. We have also computed results for other  $K$ 's and the outcome of the tests are similar to the ones reported in Table 1.

## 4 Conclusion

We find that the endogeneity of total consumer expenditure tends to increase the estimated rank of a system of Engel curves. This result, based on a non-parametric procedure, is in line with previous empirical findings reported in the literature where the rank of Engel curves is defined in the context of a parametric specification and tested against parametric or nonparametric alternatives. Our empirical findings are based on a large sample of households consistently controlled for preference heterogeneity.

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