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CYCLICAL UPGRADING OF LABOR AND EMPLOYMENT DIFFERENCES ACROSS SKILL GROUPS

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Cyclical Upgrading of Labor and Employment Differences across Skill Groups*

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Abstract

This paper examines the cyclical properties of employment rates in a search and matching model that features heterogeneous workers and jobs. I capture heterogeneity by postulating two skill levels: high and low. All low-skill workers can produce in only low-skill jobs, whereas some high-skill workers can produce in both high-and low-skill jobs. My analysis highlights the importance of a vertical type of transitory skill mismatch, in which workers accept jobs below their skill level to escape unemployment and upgrade by on-the-job search, in explaining why employment is typically lower and more procyclical at lower skill levels. The model is also consistent with other important features of the labor market, such as a procyclical rate of job-to-job transitions and evidence on cyclical changes in the composition of job quality. In recessions outflows from unemployment shift the distribution of high-skill workers toward low-skill jobs, while expansions allow them to upgrade to high-skill jobs through job-to-job transitions.

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1 Introduction

Considerable recent research has focused on the question of how well the search and matching model performs in explaining the cyclical properties of central labor market variables, such as the job finding and unemployment rate.[1] However, by focusing only on homogeneous-agent frameworks, no commensurate attention has been paid to the question of how well the matching model performs in explaining the observed salient differences in the levels and cyclical patterns of employment of different skill groups. As shown in Figure 1, the high-school employment rate is lower and more volatile than is the college employment rate. The standard deviation of log deviations from trend of the former is 0.0115 with log deviations ranging from -0.0342 to 0.0164; while for the latter, it is only 0.0046 and its log deviations range from -0.0103 to 0.0068.

In this paper, I adapt an extension of the standard Mortensen-Pissarides model to study the cyclicality of vacancies and employment by skill group. The model allows for worker and firm heterogeneity by postulating two skill levels: high and low. All low-skill workers can produce in only low-skill jobs, whereas some high-skill workers can produce in both high-and low-skill jobs. The latter are skill mismatched for low-skill jobs, because they produce less output in such jobs than they produce in high-skill jobs. However, if they do encounter a low-skill job, they have the option to accept it and to keep searching for a high-skill job while they are employed.

The model is guided by a number of observations that suggest that skilled individuals are less prone to unemployment because they take less-skilled jobs while searching for other jobs. Nickell (1979) was the first to propose this as a possible explanation for the typically lower and less countercyclical skilled unemployment rate, but other studies since then have given empirical support for this view. Pissarides and Wadsworth (1994) show that the propensity to search while on the job is higher among the more educated. Evidence by Polsky (1999) and more recently by Nagypál (2008) shows that the percentage of job separations accounted for by "quits"(resignations) that are followed by a direct transition into a new job is higher when there is a higher education level. Devereux (2002, 2004) finds that when the unemployment rate is high, the educational levels of new hires within occupations are higher and that this effect is more pronounced in less-skilled occupations. Expansions are associated with

[1] See, for example, Shimer (2005b), Hall (2005a), Hagedorn and Manovskii (2008), Costain and Reiter (2008), Pries (2008), and Pissarides (2009).

workers upgrading occupations through job-to-job transitions. The role of on-the-job search in facilitating the reallocation of workers has been emphasized in Barlevy (2002), who cites evidence by Bowlus (1995) and Davis et al. (1996) that jobs created in recessions are of lower quality and pay.

I calibrate the model to the U.S. labor market, with high-skill labor representing college graduates and low-skill labor representing high-school educated workers. I show that the model can explain both the levels and the relative volatility of the two employment rates. As is shown in the data, the standard deviation of log deviations from trend of the college employment rate is more than twice that of the high-school employment rate. However, without allowing for college graduates temporarily taking "high-school" (low-skill) level jobs, the model fails to account for these facts. In particular, it yields a college employment rate that is lower than what is observed and that is excessively volatile relative to the high-school employment rate. The model is also consistent with the evidence on cyclical changes in the composition of job quality mentioned above. When the unemployment rate is high, the share of high-school level jobs occupied by college graduates is higher. In line with well-established evidence by Shimer (2005a) and others, the model generates procyclical job-to-job flows. Consequently, in recessions outflows from unemployment shift the distribution of college graduates toward *high-school* (high-school level) jobs, while expansions allow them to upgrade to *college* (college level) jobs through job-to-job transitions.[2]

In Section 2, I set out the structure of the model. In Section 3, I discuss the impact of skill mismatches on firms' vacancy-posting incentives. There are two channels through which skill mismatches can strengthen incentives for firms to open low-skill vacancies in booms. The first relates to the expected duration of jobs: the shorter a job is expected to last, the greater the benefit from creating that job in a boom than in a recession. Consequently, when firms anticipate that low-skill jobs will be shorter on average due to some workers quitting to take high-skill jobs, they have a stronger incentive to create these jobs in booms. The second relates to the net productivity (i.e., the difference between the worker's output and unemployment income) of skill-mismatched, high-skill workers. An increase in aggregate productivity has a larger impact (in percentage terms) on the value of an employment match if the net productivity of that match is small. Consequently, the impact of a positive

[2]Whenever I refer to "high-school jobs" I mean the jobs that require at least a high-school diploma and whenever I refer to "college jobs" I mean the jobs the require at least a college degree.

productivity shock on firms' expected profits from low-skill vacancy postings is larger when the output of high-skill workers in low-skill jobs is close to their unemployment income, and firms anticipate that it is likely that low-skill vacancies will be occupied by high-skill workers. By contrast, incentives to post high-skill vacancies in booms become weaker when high-skill workers can transitorily take low-skill jobs. The increase in the value of on-the-job search that occurs in a boom is larger than the increase in the value of unemployed search, because on-the-job searchers earn a wage that reflects aggregate economic conditions, while unemployment income is fixed. Consequently, when high-skill workers can search for high-skill jobs while holding low-skill jobs, the improvement in their outside option, and thus in their bargaining power, during cyclical upturns is larger. This makes a boom a less-good time for a firm to post a high-skill vacancy.

In order to quantify the model's implications, in Section 4 I calibrate and simulate the model numerically. I first assume that all unemployed college graduates would take a high-school job and would search on the job for a college job. I show that in this case the model yields a college overeducation rate that ranges from 10% to 17% and that matches the levels and the cyclical behavior of employment rates well. I then examine how the model's predictions change when the proportion of college graduates that are willing to transitorily take high-school jobs is smaller. In line with the intuition mentioned above, the simulations show that allowing for a larger share of college graduates to transitorily take high-school jobs not only lowers the college employment rate, but also contributes to making the posting of high-school vacancies more procyclical than the posting of college vacancies; this helps explain the relative volatility of the employment rates.

The closest precursors of the model analyzed in this paper are the contributions by Albrecht and Vroman (2002), Gautier (2002), and Dolado et al. (2009). These studies use a definition of skills similar to the one used in this paper. However, they focus on explaining uneven developments, in the longrun, in the unemployment rates of different skill groups. Thus, they look only at steady-state equilibria. Note that allowing for two-sided skill heterogeneity and on-the-job search complicates the stochastic equilibrium characterization to a great extent. This is because the distribution of workers across jobs is entered into the model as a state variable, in a complicated nonmonotonic way. This might be one of the reasons existing studies that consider frameworks featuring some sort of heterogeneity and on-the-job search often restrict their analysis to nonstochastic equilibrium, while most of those that allow for aggregate uncertainty consider simplified frameworks where there is a unique matching rate for all labor and job types.[3] Hence, they characterize the cyclical behavior of labor flows only in terms of average values. More fundamentally, they overlook the across-skill congestion effects and externalities that arise when workers of different skill levels compete for the same types of jobs.

The paper by Khalifa (2010), written contemporaneously with my paper, is an exception. He also develops a stochastic model with search frictions and two-sided skill heterogeneity, but has a different objective. He attempts to explain the persistence of unemployment. Moreover, in Khalifa's model, workers can direct their search toward specific vacancy types, whereas in my model search is random. He argues that a possible explanation to the persistence of unemployment is the crowding out of unskilled workers into unemployment caused by intensified competition from skilled workers for unskilled jobs during cyclical downturns. This crowding out effect is a dominant feature in Khalifa's model, because he makes the critical, but questionable, assumption that firms can create unskilled vacancies that are exclusively directed to the skilled workers. During downturns a higher share of unskilled vacancies are directed toward the skilled workers, leading to higher unemployment among the unskilled workers who are suited for only unskilled jobs. His model can explain unemployment persistence, but cannot reproduce the procyclicality of the skilled employment rate, because the increase in the employment of skilled workers in unskilled jobs that occurs in downturns exceeds the reduction in their employment in skilled jobs.

[3] Examples are Barlevy (2002), Krause and Lubik (2006) and Nagypál (2007) who study cyclical dynamics in matching models that feature on-the-job search. Krause and Lubik assume that labor is homogeneous and there are only high-and low-wage jobs, while Barlevy's model accounts for both worker and firm heterogeneity, but focuses only on symmetric equilibria where there is a common job finding/filling rate for all worker/firm types. In Nagypál, both workers and jobs are homogeneous and on-the-job search is motivated by subjective perceptions of job quality by workers. Nagypál restricts the analysis to nonstochastic equilibria. Moreover, Nagypál uses a simple sharing rule for wage setting that makes her model more tractable, but has the considerable disadvantage that wages do not reflect the impact of on-the-job search. The same sharing rule is also used by Barlevy (2002). Pries (2008) also explores cyclical dynamics, but considers a model with only one type of job that can be occupied by either high-or low-productivity workers, and no motivation for on-the-job search.

2 The model

The framework consists of an economy where time is infinite and discrete. The population is of measure 1, and there is a continuum of firms. All agents are risk neutral and discount the future at rate r. Each firm has the choice of opening a vacancy that requires either a high or a low skill level. Jobs and firms are indexed by $j = [h, l]$, where h refers to high-skill jobs and l to low-skill jobs. A fraction δ of workers have a low-skill level and the remaining $1 - \delta$ have a high-skill level. Low-skill workers can produce only in low-skill jobs. A fraction μ of high-skill workers can produce in both high- and low-skill jobs, whereas the remaining $1 - \mu$ can produce only in high-skill jobs. Workers are indexed by $i = [h, \bar{h}, l]$. The subscript h refers to the high-skill workers that can produce in both types of jobs, the subscript $\bar{h}$ to the high-skill workers that can produce only in high-skill jobs and the subscript l to the low-skill workers.

Firms are free to enter the market to create employment matches by posting a vacancy (either high-or low-skill) at flow cost c_j in order to recruit a worker. If the firm succeeds in recruiting a worker, the flow output of the resulting employment match is the product of a stochastic aggregate component, y, and a match specific component, α_{ij}. The aggregate productivity component follows a discrete-state Markov process, with a vector of realizations $\bar{y}$ and a transition matrix Π with elements $\pi_{nm} = prob\{y' = \bar{y}_m \setminus y = \bar{y}_n\}$. Both types of high-skill workers are equally productive in high-skill jobs. That is, $\alpha_{\bar{h}h} = \alpha_{hh}$. A type-$h$ worker matched with a low-skill job is skill mismatched, because he generates lower output than when matched with a high-skill job, i.e., $\alpha_{hh} > \alpha_{hl}$. An employment match can be destroyed either for exogenous reasons or due to the worker quitting to take another job as a consequence of on-the-job search. I allow the exogenous separation rates for the two types of jobs, s_j, to differ, and set $s_l \geq s_h$.[4] Whenever a job is exogenously destroyed, the worker becomes unemployed and begins searching for a new job. While unemployed, the worker receives an unemployment income b_i, which can be interpreted as the flow opportunity cost associated with working. I assume that the unemployment income is the same for both types of high-skill workers, i.e., $b_{\bar{h}} = b_h$.

The firms have no way of signaling the type of vacancies to the workers before the workers search for them. Thus, the job searchers apply randomly to jobs, meaning that they

[4]This assumption is based on the observation that the duration of less-skilled jobs is typically shorter than the duration of more-skilled jobs. See, for example, Abraham and Farber (1987).

sometime apply for jobs they are not (best) suited for. An unemployed type-i worker is willing to accept any job that offers a wage that is higher than b_i. Even if that is not the job he is best suited for, he is better off accepting it, because he can move to a better job through on-the-job search.[5] Below I show that the wage of a type-i worker in a type-j job is always higher than b_i, if $y\alpha_{ij} > b_i$ for all $y \,\epsilon\, \bar{y}$. This condition also ensures that the firm is also better off hiring the worker, instead of keeping the job vacant. Since type-l and type-h workers are not suited for high-and low-skill jobs, respectively, this condition is violated for these two types of matches, but holds for all of the rest of the match types. That is, for all $y \,\epsilon\, \bar{y}$,

$$y\alpha_{hh} > b_h, \quad y\alpha_{hl} > b_h, \quad y\alpha_{ll} > b_l, \quad y\alpha_{lh} \leq b_l \quad y\alpha_{\bar{h}l} \leq b_h \tag{1}$$

The payoff structure is therefore such that if a type-h worker comes across a low-skill job, she accepts it, if unemployed, and searches on the job for a high-skill job. She stays in the low-skill job until either she quits to accept a high-skill job or an exogenous separation occurs that moves her into unemployment. If the job she finds is high-skill, she accepts it, regardless of being employed or unemployed, and stays in it until the job is destroyed for exogenous reasons. A type-$\bar{h}$ (type-l) worker who finds a high-skill (low-skill) job accepts it and stays in it until an exogenous separation occurs, but if the job found is low-skill (high-skill), the worker continues searching until a high-skill (low-skill) job comes along. Type-$\bar{h}$ and type-l workers have no incentive to search on the job, because all jobs of the same type are identical.

2.1 Matching

A single matching market with a matching function determines the number of contacts between searching firms and workers as a function of total number of vacancies, ν, and the total number of job seekers, z. The total number of vacancies is given by $\nu = \nu_l + \nu_h$: the number of low- plus the number of high-skill vacancies. The total number of job seekers is given by $z = u_h + u_{\bar{h}} + u_l + \varepsilon_{hl}$: the number of unemployed high- and low-skill workers, $u_h, u_{\bar{h}}$ and u_l, plus the number of on-the-job searchers, ε_{hl}. The function $m(\nu, z)$ is homogeneous of degree one and increasing and concave in both its arguments. This allows me to write

[5] I assume for convenience that employed search is as efficient as unemployed search, meaning that the arrival rate of future job offers is the same for both employed and unemployed searchers. It follows that the only opportunity cost associated with skill mismatches is the unemployment income, b_h.

the number of contacts per job seeker as $m(\theta)$ and the number of contacts per vacancy as $q(\theta) = m(1, \frac{1}{\theta})$, where $\theta = \frac{\nu}{z}$ measures the tightness of the labor market. A job seeker contacts a low-skill vacancy at rate $m(\theta)\eta$ and a high-skill vacancy at rate $m(\theta)(1 - \eta)$, where $\eta = \frac{\nu_l}{\nu}$. Likewise, a firm finds a type-h unemployed worker at rate $q(\theta)(\frac{u_h}{z})$; a type-$\bar{h}$ unemployed worker at rate $q(\theta)(\frac{u_{\bar{h}}}{z})$; a high-skill worker who is already employed at rate $q(\theta)(\frac{\varepsilon_{hl}}{z})$; and a low-skill (unemployed) worker at rate $q(\theta)(\frac{u_l}{z})$.

Aside from the standard "congestion" effects embedded in the random matching assumption (i.e., the assumption that job seekers are unable to self-select prior to applying for a job), the existence of skill-heterogeneity implies additional across-skill congestion effects. For instance, if job vacancies that come on the market are more high skill, the chances that a low-skill worker will find a job are reduced. Likewise, an increase in the share of low-skill workers in the pool of job seekers lowers the matching rate of high-skill firms. This choice of modeling the matching process is guided by the observation that what is often burdensome for firms is not the task of locating applicants, but the task of sifting through applicants searching for the one that best fits their vacancy. For instance, van Ours and Ridder (1993) find that vacancy durations are mainly selection periods, and that attracting a pool of applicants takes relatively little time. Moreover, if job seekers could effectively direct their searches toward only the jobs they are best suited for, then vacancies attracting higher numbers of applicants should be of shorter duration. But, evidence suggests that this is not always true. Van Ours and Ridder (1992) and Barron, Berger and Black (1997) show that although the number of applicants per job offer is higher for skilled positions, vacancy duration is higher for these positions.

Several other studies also make the random matching assumption in search models that feature skill heterogeneity.[6] Gautier et al. (2010) argue that allowing for skill heterogeneity in a model in which matching is random is relevant; one of the most important reasons for search is to find the right person for the job. They also argue, by citing evidence that job-to-job flows are a salient labor-market feature, for the existence of information frictions that prevent workers from immediately matching with their optimal job type.[7] Acemoglu (1999) argues

[6]Plesca (2010) calibrates a model with a similar setup as the model in this paper in order to investigate the properties of random versus directed search, in the context of the Employment Service.

[7]Nagypál (2008) shows that in the U.S. the rate of job-to-job transitions is twice as large as employment-to-unemployment transition rate. Moreover, Blanchard and Diamond (1989a) show that, on average, hires from other jobs are about 50% of hires from unemployment.

for the plausibility of random matching in an environment with skill heterogeneity - pointing out that it is difficult for firms to target recruiting exclusively to workers with a particular skill level, because skill is imperfectly correlated with observable characteristics. Pries (2008) argues in addition that although firms may be able to identify a worker's productivity once the worker is interviewed by the firm - as assumed in the context of random matching - what matters regarding the feasibility of directed search is whether a worker's productivity can be accurately identified prior to meetings, which is less likely.[8]

2.2 Timing and flow equations

The timing within a period is as follows. At the beginning of the period aggregate productivity is revealed and agents produce. Subsequently, some of the existing matches are destroyed due to exogenous reasons. Firms post vacancies and search takes place. Some of the skill-mismatched workers that survived the exogenous separation will quit their low-skill jobs to take high-skill jobs and some unemployed workers will find jobs.

Let $e = \{e_{ll}, e_{lh}, e_{hh}, e_{\bar{h}h}\}$ denote the distribution of employed workers across types of jobs at the beginning of the period and $x = \{y, e\}$ denote the vector of state variables. The next-period distribution satisfies:

$$
\begin{aligned}
e'_{ll} &= \varepsilon_{ll} + p_l(x)\left[\delta - \varepsilon_{ll}\right] \\
e'_{hh} &= \varepsilon_{hh} + p_h(x)[\mu(1-\delta) - \varepsilon_{hh}] \\
e'_{\bar{h}h} &= \varepsilon_{\bar{h}h} + p_h(x)[(1-\mu)(1-\delta) - \varepsilon_{\bar{h}h}] \\
e'_{hl} &= \varepsilon_{hl} + p_l(x)\left[\mu(1-\delta) - \varepsilon_{hl} - \varepsilon_{hh}\right] - p_h(x)\varepsilon_{hl}
\end{aligned} \tag{2}
$$

where $p_l(x) = m(\theta(x))\eta(x)$ and $p_h(x) = m(\theta(x))(1-\eta(x))$ are the probabilities of finding a high- and low-skill job, respectively, and $\varepsilon_{ij} = e_{ij}(1-s_j)$, gives the number of jobs that survived exogenous separations. Since the share of each type of worker in the labor force is constant and the labor force is normalized to 1, we can write: $u_h = \mu(1-\delta) - \varepsilon_{hh} - \varepsilon_{hl}$, $u_h = (1-\mu)(1-\delta) - \varepsilon_{\bar{h}h}$ and $u_l = \delta - \varepsilon_{ll}$. The total number of high-skill job seekers is given by $u_h + u_{\bar{h}} + \varepsilon_{hl} = 1 - \delta - \varepsilon_{hh} - \varepsilon_{\bar{h}h}$. I label the low-skill employment rate as $\tilde{\varepsilon}_l = \frac{\varepsilon_{ll}}{\delta}$, and the

[8]What also matters is whether firms can effectively prevent being contacted by job seekers who are not well suited. Even if firms target recruiting exclusively to workers with a particular skill level, this may still not prevent unsuited job seekers from applying, especially since the cost of sending one application out is trivial relative to the cost of losing an employment opportunity.

overall high-skill employment rate as $\tilde{\varepsilon}_h = \frac{\varepsilon_{hh}+\varepsilon_{\bar{h}h}+\varepsilon_{hl}}{1-\delta}$. Using the same notation, $\tilde{\varepsilon}_{hl} = \frac{\varepsilon_{hl}}{1-\delta}$, $\tilde{\varepsilon}_{hh} = \frac{\varepsilon_{hh}}{1-\delta}$ and $\tilde{\varepsilon}_{\bar{h}h} = \frac{\varepsilon_{\bar{h}h}}{1-\delta}$.

2.3 Value functions

To derive the conditions for job entry and the optimality of job search strategies, the value functions associated with firms and workers, matched and unmatched, need first to be specified. I denote the value of unemployment by U_i; the value of employment by W_{ij}; the value of a vacant job by V_j; and the value of a filled job by J_{ij}.

The value functions for unemployed workers are given by:

$$\begin{aligned} U_l(x) &= b_l + \beta E_{x'|x}\left[p_l(x)W_{ll}(x') + (1-p_l(x))U_l(x')\right] && (3)\\ U_{\bar{h}}(x) &= b_h + \beta E_{x'|x}\left[p_h(x)W_{\bar{h}h}(x') + (1-p_h(x))U_{\bar{h}}(x')\right] && (4)\\ U_h(x) &= b_h + \beta E_{x'|x}\left[p_l(x)W_{hl}(x') + p_h(x)W_{hh}(x')\right] \\ &\quad + \beta E_{x'|x}\left[(1-p_l(x)-p_h(x))U_h(x')\right] && (5) \end{aligned}$$

where $E_{x'|x}$ is the expectation operator and $\beta = \frac{1}{1+r}$ is the discount factor. The expected payoffs in the brackets depend on the transition matrix of aggregate productivity Π, and the flow equations described in (2).

The value functions for employed workers satisfy:

$$\begin{aligned} W_{ll}(x) &= w_{ll}(x) + \beta E_{x'|x}\left[s_l U_l(x') + (1-s_l)W_{ll}(x')\right] && (6)\\ W_{hh}(x) &= w_{hh}(x) + \beta E_{x'|x}\left[s_h U_h(x') + (1-s_h)W_{hh}(x')\right] && (7)\\ W_{\bar{h}h}(x) &= w_{\bar{h}h}(x) + \beta E_{x'|x}\left[s_h U_h(x') + (1-s_h)W_{\bar{h}h}(x')\right] && (8)\\ W_{hl}(x) &= w_{hl}(x) + \beta E_{x'|x}\left[s_l U_h(x') + (1-s_l)W_{hl}(x')\right] \\ &\quad + \beta E_{x'|x}\left[(1-s_l)p_h(x)\left(W_{hh}(x') - W_{hl}(x')\right)\right] && (9) \end{aligned}$$

The interpretation of the value functions is straightforward. In (3) and (4), the payoff in the current period for an unemployed type-i worker is b_i. With probability $p_j(x)$, search results in a match, yielding an employment value $W_{ij}(x')$ and with probability $1-p_j(x)$ there is no match and the continuation value is $U_i(x')$. In (5), search results in a match with a high-skill job with probability $p_h(x)$ or with a low-skill job with probability $p_l(x)$. The resulting employment values are $W_{hh}(x')$ and $W_{hl}(x')$, respectively. In (6)-(8) the employed worker earns the wage $w_{ij}(x)$, keeps the job in the next period with a probability $1-s_j$ and loses it with probability s_j. For a skill-mismatched worker, the value given by (9), incorporates in addition the expected payoff from on-the-job search. This is given by the last term in the bracket. With probability $(1-s_l)p_h(x)$ the worker survives the exogenous separation and matches with a high-skill job, thereby generating a surplus $W_{hh}(x')-W_{hl}(x')$ from switching jobs.

A similar interpretation applies to the value functions for firms. The values of filled jobs are given by,

$$
\begin{aligned}
J_{ll}(x) &= y\alpha_{ll} - w_{ll}(x) + \beta E_{x'|x}\left[s_l V_l(x') + (1-s_l)J_{ll}(x')\right] && (10)\\
J_{hh}(x) &= y\alpha_{hh} - w_{hh}(x) + \beta E_{x'|x}\left[s_h V_h(x') + (1-s_h)J_{hh}(x')\right] && (11)\\
J_{\bar{h}h}(x) &= y\alpha_{hh} - w_{\bar{h}h}(x) + \beta E_{x'|x}\left[s_h V_h(x') + (1-s_h)J_{\bar{h}h}(x')\right] && (12)\\
J_{hl}(x) &= y\alpha_{hl} - w_{hl}(x) + \beta E_{x'|x}\left[s_l V_l(x') + (1-s_l)J_{hl}(x')\right] \\
&\quad -\beta E_{x'|x}\left[(1-s_l)p_h(x)(J_{hl}(x') - V_l(x'))\right] && (13)
\end{aligned}
$$

and the values of vacancies by:

$$
\begin{aligned}
V_h(x) &= -c_h + \beta E_{x'|x}\left[q_h(x)J_{hh}(x') + q_{\bar{h}}(x)J_{\bar{h}h}(x')\right] \\
&\quad +E_{x'|x}\left[(1-q_h(x)-q_{\bar{h}}(x))V_h(x')\right] && (14)\\
V_l(x) &= -c_l + E_{x'|x}\left[q_l(x)J_{ll}(x') + q_h(x)\varphi(x)J_{hl}(x')\right] \\
&\quad +E_{x'|x}\left[(1-q_l(x)-q_h(x)\varphi(x))V_l(x')\right] && (15)
\end{aligned}
$$

where $q_h(x) = q(\theta(x))\frac{u_h+\varepsilon_{hl}}{z}$, $q_{\bar{h}}(x) = q(\theta(x))\frac{u_{\bar{h}}}{z}$, $q_l(x) = q(\theta(x))\frac{u_l}{z}$, and $\varphi(x) = \frac{u_h}{u_h+\varepsilon_{hl}}$.

2.4 Wage setting

Shimer (2006) shows that the standard Nash bargaining solution in matching models without on-the-job search is not valid in models that feature on-the-job search. In particular, Shimer demonstrates that firms may find it profitable to pay a higher wage in order to reduce the probability of a quit.[9] Allowing for such a feature would complicate the model considerably. For this reason, I follow Pissarides (1994) and Dolado et al. (2009) in adopting simplifying assumptions that allow the use of a wage setting rule that looks identical to the typical Nash bargaining rule.

[9]Nash bargaining is not valid because the bargaining set is no longer convex when there is on-the-job search. See also, Gautier et al. (2010).

Wages are such that a share γ of the flow surplus of the match goes to the worker and the rest goes to the firm. The possibility of long-term contracts is ruled out by assuming that wages can be continuously revised at no cost. A worker could therefore start negotiating with a new employer before quitting the current job, but this would not affect the equilibrium wage, because the new employer would immediately renegotiate the wage once the worker quits the previous job. This assumption also eliminates the scope for equilibria where low-skill firms are matching the offers from high-skill firms in order to prevent high-skill workers from quitting. Since long-term contracts are not possible, the workers realize that once they decline an offer from a high-skill firm in order to accept the matching offer from their current employer, their current employer will immediately renegotiate the wage back to its initial level.

Note also that since long-term wage contracts are not allowed, a worker's only credible alternative to the current job is quitting into unemployment. Consequently, a worker's threat point in wage setting is her unemployment value, irrespective of being unemployed or not. This, in turn, implies that the surplus, and therefore the wage that splits the surplus, is the same for both employed and unemployed applicants, so that firms are indifferent between the two.

Under the assumption that workers cannot write long-term wage contracts, the wage, $w_{ij}(x)$, satisfies the typical Nash bargaining solution:

$$\begin{aligned} W_{ij}(x) - U_j(x) &= \gamma S_{ij}(x) \\ J_{ij}(x) - V_i(x) &= (1-\gamma) S_{ij}(x). \end{aligned} \tag{16}$$

where $S_{ij}(x)$ is the match surplus, defined as:

$$S_{ij}(x) = W_{ij}(x) + J_{ij}(x) - U_i(x) - V_j(x) \tag{17}$$

Using (3)-(16) we can solve for the equilibrium wages;

$$w_{ll}(x) = \gamma(y\alpha_{ll} - b_l) + b_l + \gamma p_l(x)\beta E_{x'|x} J_{ll}(x') \tag{18}$$

$$w_{\bar{h}h}(x) = \gamma(y\alpha_{hh} - b_h) + b_h + \gamma p_h(x)\beta E_{x'|x} J_{\bar{h}h}(x') \tag{19}$$

$$w_{hh}(x) = \gamma(y\alpha_{hh} - b_h) + b_h + \gamma p_l(x)\beta E_{x'|x} J_{hl}(x') + \gamma p_h(x)\beta E_{x'|x} J_{hh}(x') \tag{20}$$

$$\begin{aligned} w_{hl}(x) &= \gamma(y\alpha_{hl} - b_h) + b_h + \gamma p_l(x)\beta E_{x'|x} J_{hl}(x') + \gamma p_h(x)\beta E_{x'|x} J_{hh}(x') \\ &\quad -\gamma(1-s_l) p_h(x)\beta E_{x'|x} J_{hh}(x') \end{aligned} \tag{21}$$

A worker's wage is such that she gets a share γ of the net productivity the job creates, given by $y\alpha_{ij} - b_i(x)$, plus the value her outside option. The latter is given by unemployment income, b_i, plus the surplus the worker expects to generate from unemployed search. The skill-mismatched workers need to compensate their employers for their higher probability of quitting by accepting a wage decrease. This explains why the term $\gamma(1-s_l)p_h(x)\beta E_{x'|x}J_{hh}(x')$ enters negatively in (21).

Comparing (20) with (21) shows that $w_{hh}(x) > w_{hl}(x)$, both because on-the-job searchers suffer a wage reduction, and because $\alpha_{hh} > \alpha_{hl}$, by assumption. It follows that skill-mismatched workers are better off quitting to go to high-skill jobs, which ensures that on-the-job search is optimal for these workers. Inspecting the wage equations shows also that the conditions in (1) are sufficient to ensure that $w_{ll}(x) > b_l$, $w_{hh}(x) > b_h$, $w_{\bar{h}h}(x) > b_h$ and $w_{hl}(x) > b_h$. As noted earlier, these imply that a low-skill worker is better off accepting a low-skill job, a type-$\bar{h}$ is better off accepting a high-skill job, and that a type-h worker is better off accepting either type of job, instead of remaining unemployed.

2.5 Equilibrium

Given free entry, in equilibrium vacancies must yield zero profits: $V_h(x) = V_l(x) = 0$ and $V_h(x') = V_l(x') = 0$. Using these zero profit conditions in (14) and (15), gives the free-entry conditions for low- and high-skill vacancies, respectively,

$$\beta E_{x'|x}\left[\frac{u_l}{z}J_{ll}(x') + \frac{u_h}{z}J_{hl}(x')\right] = \frac{c_l}{q(\theta(x))} \tag{22}$$

$$\beta E_{x'|x}\left[\frac{u_{\bar{h}}}{z}J_{\bar{h}h}(x') + \frac{(u_h + \varepsilon_{hl})}{z}J_{hh}(x')\right] = \frac{c_h}{q(\theta(x))} \tag{23}$$

With the values in (10) to (13) and the wages in (18) to (21) substituted in, these conditions implicitly define $\theta(x)$ and $\eta(x)$. More formally, the equilibrium is given by a vector $\{\theta, \eta\}$ that for each realization of aggregate state, y, and distribution of employment, $e = \{e_{ll}, e_{lh}, e_{hh}, e_{\bar{h}h}\}$, the values of opening low- and high-skill vacancies both equal zero. As in the standard model, when the left-hand sides of these equations are higher than their right-hand sides, then the expected gains from vacancy postings exceed the expected vacancy-posting costs, and firms post a higher number of vacancies per job seeker (increase $\theta(x)$) until all rents are exhausted.

The equilibrium skill-mix of vacancies, $\eta(x)$, depends on the relative profitability of the two types of vacancies. Combining the two free-entry condition gives:

$$c_h E_{x'|x}\left[u_l J_{ll}(x') + u_h J_{hl}(x')\right] = c_l E_{x'|x}\left[(u_h + \varepsilon_{hl}) J_{hh}(x') + u_{\bar{h}} J_{\bar{h}h}(x')\right] \tag{24}$$

If the left-hand side of (24) is higher than its right-hand side, then the expected profits from low-skill vacancy postings are higher than those from high-skill vacancy postings. It can be easily verified from (2) and the value functions in (10)-(13) that the right-hand side of (24) increases as the share of low-skill vacancies increases, while the impact on the left-hand side of (24) is in general ambiguous. The impact on the left-hand side is ambiguous, because an increase in $\eta(x)$ lowers $J_{ll}(x)$, by improving a low-skill worker's outside option and therefore wage, but raises $J_{hl}(x)$, by lowering the probability of a quit.[10] Because of the two countervailing effects on the left-hand side of (24), to establish the intuitive notion that when the expected profits from low-skill vacancy postings increase (decrease) relative to the expected profits from high-skill vacancy postings, firms must shift the vacancy mix toward low-skill (high-skill) vacancies to maintain zero profits, requires additional parameter restrictions.

To establish this intuitive notion, in what follows I assume that the model parameters are such that: $\delta \geq \mu(1-\delta)$ and $\gamma(y\alpha_{ll} - b_l) \geq (1-\gamma)(y\alpha_{hl} - b_h)$ for all $y \,\epsilon\, \bar{y}$. The first condition ensures that $u_l \geq u_h$. The second condition ensures that the fall in $J_{ll}(x)$, due to an increase in $\eta(x)$, dominates the increase in $J_{hl}(x)$.[11] When both of these conditions are satisfied the left-hand side of (24) decreases as $\eta(x)$ increases so that a positive productivity shock that raises the expected profits of both types of vacancies induces firms to open a higher number of vacancies of both types, but also to shift the vacancy mix toward the relatively more profitable type.

3 Skill mismatch and employment volatility

If the share of low-skill vacancies, $\eta(x)$, is procyclical, meaning that a positive productivity shock leads to a larger percentage increase in $\nu_l(x)$ than in $\nu_h(x)$, then the rate at

[10] An increase in $\eta(x)$ has two opposite effects on $J_{hl}(x)$. On the one hand, it raises it by reducing the chance of a quit; on the other hand, it lowers it by improving a high-skill worker's outside option and therefore wage, $w_{hl}(x)$. It can be easily shown that the first effect dominates so that $J_{hl}(x)$ increases as $\eta(x)$ increases.

[11] Intuitively, these two conditions ensure that a higher portion of the profits that firms expect to generate from filling low-skill vacancies comes from filling these vacancies with low- instead of high-skill workers.

which job seekers find low-skill jobs is more procyclical than the rate at which they find high-skill jobs, leading to a more strongly procyclical low-skill employment rate, in line with the evidence. Therefore, it is worthwhile to discuss the channels by which skill mismatches can make $\eta(x)$ procyclical.

Setting $\mu = 0$ implies that there are no skill mismatches in the model, whereas as μ approaches 1 the probability of skill mismatches increases. In particular, a higher value of μ places a larger weight in the free-entry condition for low-skill vacancies on $J_{hl}(x')$, while leaving the weight on $J_{ll}(x')$ intact. That is, at a higher value of μ low-skill firms are more likely to become skill mismatched and less likely to remain vacant. In addition, setting a higher value for μ implies a shift in the weights in the free-entry condition for high-skill vacancies from $J_{\bar{h}h}(x')$ to $J_{hh}(x')$.[12] We can say that $\eta(x)$ is procyclical if a positive productivity shock has a larger impact in percentage terms on the left-hand side of (22) than on the left-hand side of (23). That is, the percentage increase in $\nu_l(x)$ is larger than the percentage increase in $\nu_h(x)$, if the percentage increase in the expected profits from posting low-skill vacancies is larger than that from posting high-skill vacancies. Therefore, in what follows I discuss how the shift in the weights in the free-entry conditions that results from an increase in μ can make the left-hand side of (22) more volatile and the left-hand side of (23) less volatile.

3.1 On-the-job search and match duration

As shown in the Appendix, the value of matches that are of shorter expected duration follows the business cycle more closely, i.e., it is more volatile. To understand why, consider a match that is expected to last, say, for only one period. What matters for the firm's value of that match is its current productivity. But, if the match is expected to last for several periods, then its current productivity becomes less important and what matters instead is its average productivity. Since skill-mismatched workers search on the job, skill-mismatched low-skill jobs are expected to be of shorter duration than correctly matched low-skill jobs. This lead us to expect that $J_{hl}(x')$ is more volatile than $J_{ll}(x')$ so that by placing a larger weight on

[12] Notice that u_l is independent of μ. Moreover, a high-skill worker (of any type) will leave the pool of job seekers only if she finds a high-skill job. Thus, the total number of high-skill job seekers, $u_h + \varepsilon_{hl} + u_{\bar{h}}$, is also independent of μ. In particular, an increase in μ increases $u_h + \varepsilon_{hl}$ by exactly as much as it decreases $u_{\bar{h}}$.

$J_{hl}(x')$, a higher value of μ can make the left-hand side of (22) more volatile. Intuitively, since low-skill jobs are more likely to be temporary, firms have a greater incentive to create these jobs in booms.

3.2 Net productivity and separation rate differentials

The value of a match to the firm is also more sensitive to changes in aggregate productivity if the worker's productivity is small relative to his unemployment income. If $\frac{\alpha_{ij}}{b_i}$ is small, then a small productivity shock has a larger impact, in percentage terms, on the net productivity of the match, $y\alpha_{ij} - b_i$, and thus on the value of the match.[13] Hence, $J_{hl}(x')$ can be more volatile than $J_{ll}(x')$ due to $\frac{\alpha_{hl}}{b_h}$ being smaller than $\frac{\alpha_{ll}}{b_l}$, say because b_h is much larger than b_l.

Evidently, since the expected duration and the volatility of the net productivity of a match matter for the volatility of the value of that match, the size of s_l relative to the size of s_h, and of $\frac{\alpha_{ll}}{b_l}$ relative to $\frac{\alpha_{hh}}{b_h}$ are also important in determining the cyclical behavior of the vacancy mix. If $\frac{\alpha_{ll}}{b_l} < \frac{\alpha_{hh}}{b_h}$, and in addition $s_l > s_h$, then $J_{ll}(x')$ is more volatile than $J_{\bar{h}h}(x')$, meaning that even in the absence of skill mismatches, i.e., when $\mu = 0$, the expected profits from posting low-skill vacancies can be more volatile than those from posting high-skill vacancies. The question that follows is whether these differentials are large enough to explain the magnitude of observed differences in the levels and volatilities of employment rates of the two skill groups, in the absence of skill mismatches. I address this question in subsequent sections where I calibrate the model to the U.S. labor market.

3.3 The worker's outside option

The channel by which allowing for skill mismatches can make the expected profits from posting high-skill vacancies less volatile relates to high-skill workers' outside option. Recall that the worker's outside option reduces the firm's surplus from having a filled job, because it increases the worker's wage. It follows that the more cyclical a worker's outside option is, the more cyclical is her wage, and thus the less cyclical is the payoff to the firm from

[13] To see this, note that by substituting (18) in (10) we obtain: $J_{ll}(x) = (1-\gamma)[y\alpha_{ll} - b_l] + \beta E_{x'|x}[(1 - s_l - \gamma p_l(x))J_{ll}(x')]$. Similar expressions can also be obtained for the rest of the match types. The volatility of the net productivity, $y\alpha_{ij} - b_i$, therefore matters for the volatility of $J_{ij}(x)$. This has also been also pointed out by Shimer (2005b), Hagedorn and Manovskii (2008) and Pries (2008).

hiring that worker. A high-skill worker that searches for a high-skill job while employed in a low-skill job earns a wage, $w_{hl}(x)$, which fluctuates with aggregate productivity, whereas an unemployed high-skill searcher receives an unemployment income b_h that is fixed. The outside option of a type-h worker is therefore more cyclical than that of a type-$\bar{h}$ worker, because the former can search for a high-skill job either while unemployed or unemployed, whereas the latter can search only while unemployed. There is therefore reason to suspect that $w_{hh}(x)$ is more volatile than $w_{\bar{h}h}(x)$, so that $J_{hh}(x')$ is less volatile than $J_{\bar{h}h}(x')$.[14] If this is the case, then the shift in the weights in the free-entry condition for high-skill vacancies that results from an increase in μ makes the left-hand-side of (23) less volatile.

3.4 Firm incentives

If booms are associated with an increase in the expected profits from posting low-skill vacancies relative to those from posting high-skill vacancies, then at higher values for μ, firms have a stronger incentive to shift the vacancy mix toward low-skill vacancies in booms.

When $\eta(x)$ increases $J_{hl}(x)$ also increases, because the probability of quits declines. Allowing for skill mismatches introduces a positive feedback, therefore, from $\eta(x)$ on the profits firms expect to generate from posting low-skill vacancies. Allowing for skill mismatches introduces a also a negative feedback from $\eta(x)$ on the expected profits of high-skill vacancies. An increase in $\eta(x)$ raises the expected gains from high-skill vacancy postings, by worsening high-skill workers' outside option. But, if a high-skill worker can take transitorily a low-skill job, the negative impact on the outside option is smaller. In other words, an increase in $\eta(x)$ has a smaller negative impact on $w_{hh}(x)$ than on $w_{\bar{h}h}(x)$, and therefore a smaller positive impact on $J_{hh}(x)$ than on $J_{\bar{h}h}(x)$. Consequently, the shift in the weights in the free-entry conditions that results from an increase in μ implies a smaller negative feedback from $\eta(x)$ on the left-hand side of (22) and a smaller positive feedback from $\eta(x)$ on the left-hand side of (23). This, implies, in turn, that at a higher value of μ, firms respond to a given increase

[14] Since the term $\beta\gamma p_l(x)E_{x'|x}J_{hl}(x')$ enters positively in (20), but not in (19), then $w_{hh}(x)$ is more cyclical, but also larger than $w_{\bar{h}h}(x)$. It is therefore difficult to establish that $w_{hh}(x)$ is more volatile than $w_{\bar{h}h}(x)$. If $J_{hl}(x')$ is small and very cyclical (i.e., if it is volatile), which implies that the difference in size between $w_{hh}(x)$ and $w_{\bar{h}h}(x)$ is small, while their difference in terms of cyclicality is large, then there is more reason to suspect that $w_{hh}(x)$ is more volatile than $w_{\bar{h}h}(x)$. It follows that this channel is likely to be more important the smaller that $\frac{\alpha_{hl}}{b_h}$ is, and the the larger the break up probability of skill mismatches. For the reasons explained above, both of these factors contribute to making $J_{hl}(x')$ more volatile.

in the expected profits from opening low-skill vacancies relative to the expected profits from opening high-skill vacancies by a larger increase in $\eta(x)$ in order to maintain zero profits.

4 Quantitative analysis

With two-sided skill heterogeneity and on-the-job search, the distribution of workers across jobs, $e = \{e_{ll}, e_{lh}, e_{hh}, e_{\bar{h}h}\}$, enters the model as a state variable in a complicated nonmonotonic way, making it difficult to characterize cyclical dynamics analytically. This section therefore turns to numerical simulations.

The model parameters are calibrated assuming that the low-skill labor type represents workers who have at least a high-school diploma but no college degree, and that the high-skill labor types consist of college graduates. I exclude workers with no high-school diploma from the low-skill group for two reasons. First, the random matching assumption is better suited for narrowly defined skill categories.[15] Second, all available evidence suggest that underemployed college graduates are typically in high-school level jobs. I begin the quantitative analysis with the baseline case calibrated for $\mu = 1$, when all unemployed college workers would take temporarily a high-school job. I then examine how the model's predictions change when we allow lower values for μ.

This section examines whether the model-implied employment rates can mimic the cyclical behavior of empirical employment rates and quantitatively assess the role of skill mismatches in explaining the observed patterns. It is evident, from Figure 1, that both employment rates are procyclical, but the high-school employment rate is lower and more volatile than the college employment rate. The first averages to 95%, while the second averages to 98%. Moreover, the standard deviation of log-deviations from trend of the former is 2.49 times that of the latter. I examine whether the model can match both the levels and the relative volatility of the employment rates for these two educational groups (i.e., relative standard deviation of log-deviations from trend of the high-school to the college employment rate). I also discuss the model's predictions regarding the cyclical behavior of job-to-job transitions and proportion of skill-mismatched college workers.

[15] Ideally the model would be calibrated to narrowly defined skill, instead of educational, categories. However, such calibration is difficult, if not impossible, due to the difficulty associated with measuring skill.

4.1 Calibration

Table 1 summarizes the parameter values used in the baseline case ($\mu = 1$). I choose the model period to be one month and set the discount rate to r=0.004. On average, about 27% of the high-school-educated U.S. labor force has a college degree or higher. I therefore set $\delta = 0.73$.[16] The choice of values of parameters that are common to both labor types is not essential for the purpose of this paper. For the workers' bargaining power, the matching function and the aggregate productivity process I therefore make the standard choices. I set $\gamma = 0.5$ and assume a Cobb Douglas functional form, $m = Mz^a v^{1-a}$, where a is the elasticity of matches with respect to unemployment and M is a matching efficiency parameter. I set the elasticity parameter a equal to 0.4, consistent with the Blanchard and Diamond (1989b) estimate. The matching efficiency parameter $M = 0.567$ is chosen to imply an average value of θ equal to 0.53.[17] The vector of states, $\bar{y}$, and transition matrix, Π, are chosen so that the logarithm of y approximates an AR(1) process with mean zero. The autocorrelation and standard deviation of the process are chosen so that the model-generated output per worker (after aggregating to a quarterly frequency) matches that of the US, which at quarterly frequency has standard deviation 0.02 and autocorrelation 0.9.[18]

The remaining parameters are those that differ across the two skill types and thus are likely to affect the relative responses of employment rates to productivity shocks. These are the exogenous separation rates, s_h and s_l, match productivities, α_{hh}, α_{ll} and α_{hl}, unemployment incomes, b_h and b_l, and vacancy-posting costs, c_h and c_l. In what follows I discuss my choice of values for these parameters.

4.1.1 Separation rates

To compute the monthly separation rate for college jobs I use data from the Job Openings and Labor Turnover Survey (JOLTS), which reports layoff rates by one digit industry.[19] The Bureau of Labor Statistic's (BLS) provides tables on the distribution of educational

[16] The share of college graduates corresponds to the average from 1964 to 2003 using data from the March CPS Annual Demographic Survey Files.

[17] The value of θ is based on estimates reported in Hall (2005b).

[18] I use the Bureau of Labor Statistic's (BLS) measure of nonfarm business output per person, from 1948 to 2009. Statistics are based on log-deviations from trend. Following Shimer (2005a) I use a Hodrick-Prescott filter with smoothing parameter 10^5.

[19] JOLTS is available from December 2000 to October 2010. Total separations in JOLTS are divided into three categories: Layoffs and Dischargers that include involuntary separations initiated by the employer;

attainment within each of approximately 700 detailed occupations.[20] For each occupation, information is also provided on the most significant source of education or training requirements. I use these tables to distinguish which occupations are college occupations. I label as college occupations those whose most significant source of education is a Bachelor's degree and whose proportion of workers with a Bachelor's degree or higher exceeds 50%. The BLS also provides tables on 2008 industry employment by occupation.[21] I use these together with my definition of college occupations to compute an estimate of the fraction of college jobs in each industry. I multiply this estimate by total employment in each industry to estimate the number of college jobs in each industry.[22] I then compute the separation rate for college jobs as the average layoff rate across industries, weighted by the number of college jobs in each industry. My computation yields $s_h = 0.016$. Following Hall and Milgrom (2008) I target an overall separation rate into unemployment of 3 percent, which implies a monthly (exogenous) separation rate for high-school jobs of $s_l = 0.035$. As is common in the literature, my targeted, overall separation rate takes into account workers who exit the labor force, but whose behavior is similar to those counted as unemployed.[23]

4.1.2 Match productivity and unemployment income

I select values for $\alpha_{hh}, \alpha_{ll}, \alpha_{hl}, b_h$ and b_l to match statistics from the simulated data to empirical measures of, i) the college employment rate, ii) the high-school employment rate, iii) the "college-plus" (college education and higher degrees) to high-school wage premium,

Quits that include employees leaving their jobs voluntarily and Other Separations, which report retirements or transfers to other locations. Since the exogenous separation rate captures transitions into unemployment, I choose to use Layoffs and Discharges to compute the exogenous separation rate for college jobs. Evidence reported in Nagypál's (2008), indicate that about 80% of quits are followed by a direct transition into a new job.

[20] The educational attainment distribution for each of the occupations is based on 2006, 2007 and 2008 data from the American Community Survey data, U.S. Department of Commerce, Census Bureau.

[21] These tables are provided by the Employment Projections Programme (EEP) of the BLS.

[22] I use the BLS measure of employment by industry, from December 2000 to October 2010.

[23] Blanchard and Diamond (1990) show that for the U.S., the "want-a-job" pool for those not in the labor force is roughly equal that of the unemployed. Moreover, they document that only half of the average flow into employment comes from unemployment, with the other half coming from people classified as not in the labor force, signifying that "out of the labor force" job seekers also take part in matching. My targeted separation rate is consistent with the average separation rate measured in the CPS, when roughly half of the flows from employment to out of the labor force are job seeking.

iv) the average job finding rate, and v) the wage gap between skill-mismatched and correctly matched college workers (i.e., those that hold college jobs).

I target the U.S. average employment rate of college graduates and high-school educated workers, which as mentioned above, average to 0.98 and 0.95, respectively, and a college-plus to high-school wage premium of 55%.[24] I take the overall monthly job finding rate to be 0.4. In line with my targeted, overall separation rate, this value is lower than 0.45 (Shimer, 2005b estimate), which is a commonly used value in the literature, to account for job seekers who are out of the labor force.[25]

Turning to the wage gap between skill-mismatched and correctly matched college workers, Sicherman (1991) finds that overeducated workers earn more than their coworkers who are not overeducated, but less than similar workers who are correctly matched. In particular, he finds that the wage of overeducated workers is on average 5% lower than that of correctly matched workers. To my knowledge there is no other good empirical counterpart for the U.S. that can guide my choice of wage gap between skill-mismatched and correctly matched college workers.[26] If skill-mismatched college workers earned the counterfactual of what would have been their wage had they stayed with high-school education only, then the wage difference between a college- and a high-school educated worker in the same type of job should only reflect the effect of ability. Empirical evidence show an upward "ability" bias in OLS estimates of returns to education in the order of 10% to 12%. This suggests a wage gap between college- and high-school educated workers in high-school jobs of around 10% and between skill-mismatched and correctly matched college workers of around 40%.[27] Considering

[24] Both the employment rates and the college-plus to high-school premium are computed using data from the March CPS Annual Demographic Survey Files covering the period 1964-2003. The sample is restricted to civilian adults between 22 and 65 years old.

[25] Hall (2005b) takes advantage of the expanded unemployment rate series available from the BLS starting in 1994, which includes people that are classified as out of the labor force but are likely to move into the labor force soon, to calculate a job finding rate that accounts for out of the labor force job seekers. By approximating the expanded unemployment series for earlier years, Hall calculates an average monthly job finding rate for the period 1948-2004 of about 0.3. This is lower than the value I target. However, my calibration excludes workers with less than high-school education who typically have a lower job-finding rate and are more likely be in the group of "out of the labor force" job seekers in the expanded unemployment rate. I therefore consider an average job finding rate 0.4 as a fair target.

[26] Chevalier (2003) shows that college overeducation in the U.K. is associated with a wage penalty of 22%-22%. In Germany, Bauer (2002) shows that it is associated with a wage penalty of 10%-15%.

[27] For evidence on "ability" bias, see, for instance, Behrman and Rosenzweig (1999).

Sicherman's estimate as a lower bound and this back-of-the envelop calculation as an upper bound, in my baseline calibration I target an average wage gap between skill-mismatched and correctly matched college workers of 20%, but I also examine how predictions change when we allow for a 10% and 30% wage gap.

4.1.3 Vacancy costs

Even though vacancy costs can be treated as scale parameters, they should not be out of line with the general implications of the model. A significant part of the cost of filling a vacancy is the opportunity cost of labor effort devoted to hiring activities. Consequently, vacancy costs should be compatible with labor earnings. Based on Hamermesh (1993), recruiting costs should not be higher than two months of labor earnings. Recruiting costs cannot be too large relative to output, either. The standard upper bound in the literature is 5% of output devoted in job creation activities. Finally, since hiring is typically done by supervisors whose wages are at least as high as new hires' wages, recruiting for high-wage jobs should be more costly than recruiting for low-wage jobs. Setting $c_l = 0.205$ and $c_h = 0.324$ results in 5% of output devoted to vacancy-posting costs and obeys the other two criteria. Specifically, $\frac{c_h}{c_l}$ is roughly equal to the relative wage of high- to low-skill jobs (when correctly matched).

4.2 Quantitative results

With all the parameter values assigned, I use the free entry conditions given by equations (22) and (23) to find the state-contingent market tightness $\theta(x)$ and fraction of low-skill vacancies $\eta(x)$. I then simulate the model as follows. First, I generate a sequence of random aggregate state realizations; then, starting with the first realization of aggregate state and an initial distribution of employment $e = \{e_{hh}, e_{hl}, e_{ll}\}$, I use the flow equations in (2) to compute the new distribution of employment at the beginning of the next period; then I repeat. At the end of each period, I record the values of the variables of interest along the sequence of aggregate state realizations.

4.2.1 Average values

Table 2 summarizes the results from simulations of the baseline model. In all cases considered, the majority of posted vacancies are high-school vacancies so that p_l is substantially higher than p_h. Moreover, the higher employment rate of college graduates is due to a significant fraction of them holding high-school jobs. On average, between 10% (when the wage gap is 30%) to 17% (when the wage gap is 10%) of college employment is in high-school jobs. Available empirical estimates for the U.S. show that on average 15% of college graduates have jobs that do not require a college degree.[28]

Evidence for the U.S., reported in Nagypál (2008), show that quits followed by a direct transition into a new job make up around 50% of all separations for college-graduate workers. Nagypál (2008) also shows that employment-to-employment flows as a share of employment for college workers average about 1.8%.[29] In the model, job-to-job flows as a share of college employment average to 0.01 and as share of overall college separations to 0.36-0.37. These are lower than Nagypál's estimates. However, this is not puzzling since the model captures only transitions to higher job levels, while such a distinction is not made in the data. Moreover, the data in Nagypál cover only the period 1996-2003, in which the U.S. economy experienced one expansion. A longer series would cover additional recessions and the severe contraction at the beginning of the 1980s. Therefore, it would probably yield lower averages.

The hiring rate of firms with low-skill vacancies is higher than that of firms with high-skill vacancies. It therefore takes longer to fill a high-skill vacancy than to fill a low-skill one. In all cases considered, the overall hiring rate is such that the average vacancy duration is a bit less than a month. In the absence of information frictions (i.e., if workers could direct their search toward only jobs they are suited for), the model implied vacancy duration would be much shorter. Yet, the predicted vacancy duration is consistent with the data, suggesting that the existence of information frictions is not an alien feature of the labor market. Blanchard and Diamond (1989b) find that for the period 1968-1981 vacancy duration in the U.S. ranges

[28]Hecker (1992, 1995) measures the proportion of overeducated college graduates using data from the Current Population Survey (CPS) over the period 1967-1990. He labels as "over-educated" college graduates working in occupations within retail sales; administrative support; service precision production, craft and repair; operator, fabricator and laborer; and farm jobs. He considers jobs in managerial, professional specialty, sales representative, and many technician occupations as jobs that require a degree. Based on his findings, the proportion of college graduates having jobs that do not require a degree in overall college employment ranges from 10% to 18%, yielding an average over the whole period of 15%. Graduate over-education measures in the same range can also be found for many European countries. For instance, Green et al. (1999) find that just over 20% of graduates in the UK are genuinely over-educated for their jobs.

[29]Nagypál (2008) uses panel data from the Survey of Income and Program Dynamics, covering the period 1996-2003 and referring to individuals between 25 and 60 years old.

from 2 to 4 weeks. Similar estimates can be found in Davis, Faberman and Haltiwanger (2010) for the period 2000-2005. As expected, the model's prediction lies at the upper end of the empirical estimates, because it excludes vacancies suited for individuals with less than high-school education. These vacancies have typically shorter durations.

4.2.2 Cyclical properties

The cyclical properties of the key variables are summarized in Tables 3 to 5 that report cross correlations with output and standard deviations of log-deviations from trend. The simulation results confirm the insights obtained in Section 3. The proportion of high-school vacancies is procyclical, meaning that ν_l is more strongly procyclical than ν_h. As can be seen, in all cases considered, both the standard deviation and the correlation of ν_l with output are higher than those of ν_h.[30] For this reason, p_l is both more volatile and more strongly correlated with output than p_h, which explains why $\tilde{\varepsilon}_{hh}$ is less volatile and less strongly correlated with output than $\tilde{\varepsilon}_l$.

The net flow of college graduates into low-skill jobs is countercyclical. Both flows in and out become larger in a boom, because both p_h and p_l increase, but eventually outflows become larger than inflows, because the number of unemployed college graduates falls. This explains why output is more negatively correlated with future values of $\tilde{\varepsilon}_{hl}$. The negative correlation between $\tilde{\varepsilon}_{hl}$ and output peaks at a lead of six to seven months. Both the fact that $\tilde{\varepsilon}_{hh}$ is less volatile than $\tilde{\varepsilon}_l$, and the fact that $\tilde{\varepsilon}_{hl}$ is countercyclical, make the overall college employment rate, $\tilde{\varepsilon}_h$, less volatile than the high-school employment rate $\tilde{\varepsilon}_l$, in line with the evidence. As mentioned above, the U.S. high-school employment rate is 2.49 times more volatile than the U.S. college employment rate. The model is consistent with the observed relative volatility of employment rates. The ratio of the standard deviation of log-deviations from trend of the simulated high-school employment rate to that of the simulated college employment is 2.15, at the 10% wage gap, 2.39, at the 20% wage gap and 2.42 at the 30% wage gap. At a yearly frequency, the corresponding measures are 2.24, 2.50, and 2.56, respectively, not far from what is empirically observed.[31]

[30] By "standard deviation" I always refer to the standard deviation of log-deviations from trend.

[31] To compute the yearly measures I first aggregate the simulated series to a yearly frequency. I then use a Hodrick-Prescott filter with smoothing parameter 100 to compute the log-deviations from trend. The same smoothing parameter is used to compute the standard deviation of log-deviations from trend of the empirical employment rates.

To allow for a more meaningful comparison between the model and the data, I also simulate the model under the baseline calibration along a series of aggregate productivity realizations that mimics the U.S. GDP quarterly log deviations from trend for the period from 1964 to 2003. I then aggregate to a yearly frequency to produce annual employment rates, and compare them to the empirical employment rates over the same period. The replication is crude, because I only allow for nine productivity states. Still, as can be seen in Figure 2, in terms of relative deviations the model performs quite well.[32] The magnitude of the simulated employment rate deviations is clearly smaller than in the data. This caveat of the model corresponds to the general failure of the matching model to match the empirical volatility of the vacancy to unemployment ratio.[33] The present model performs considerably better in this dimension, but the model-implied volatility is still about three to four times smaller than in the data. It should be emphasized, however, that the relative volatility of employment rates is not sensitive to this caveat of the model. Higher volatility in the vacancy to unemployment ratio implies a proportionately higher employment volatility for both skill groups, leaving the relative volatility of the two employment rates almost intact.

Figure 3 traces the behavior of the college unemployment rate and three alternative measures of skill mismatch, the proportion of college workers holding high-school jobs, the proportion of employed college graduates that hold high-school jobs and the proportion of high-school jobs that are occupied by college workers, along the same series of aggregate productivity realizations. All three measures of skill mismatch are countercyclical and trail the college unemployment rate. Hence, the model is consistent with the evidence on cyclical changes in the composition of job quality discussed in the introduction. When the unemployment rate is high, the share of skill-mismatched college workers increases. Expansions allow skill-mismatched college workers to upgrade to college jobs through job-to-job transitions. The share of high-school jobs that are occupied by college graduates is also higher in downturns, consisted with evidence by Deveraux (2002, 2004) that the educational levels of new hires within occupations are higher when the unemployment rate is high.

[32] I simulate the model under the baseline calibration assumed above, where the wage gap between skill-mismatched and correctly matched college graduates is 20%. It should be noted, however, that the picture does not change much when we allow for a 10% and 30% wage gap.

[33] See, e.g., Shimer (2005b).

4.2.3 Responses to a negative productivity shock

To better illustrate the various effects that lie beneath the cyclical behavior of the employment rates, this section demonstrates the consequences of a negative productivity shock. I set a high value for y and simulate the model until the endogenous variables converge to a stable value. I then set a lower value for y and simulate the effects. This switch in aggregate productivity results in a reduction in output of approximately one standard deviation.

The evolution of the variables of interest in response to this shock are depicted in Figure 4.[34] The numbers of both types of vacancies and, therefore, the job finding rates, p_l and p_h, decline on impact, leading to higher unemployment and thus higher arrival rates of job seekers to firms in subsequent periods, which encourages firms to post more vacancies. Hence, the number of vacancies and the job finding rates subsequently partially recover from their initial decline, but never reach their original level. The initial percentage decline in ν_l is larger than in ν_h so that η also decreases in impact. The proportion of high-school workers in the pool of job searchers increases. This shift in the composition of job seekers raises the effective matching rate of high-school vacancies and lowers that of the college vacancies, but it is not sufficient to induce firms to shift the vacancy mix toward high-school vacancies. The proportion of high-school vacancies continues to decline in subsequent periods until it settles to a lower level.

At the onset of the recession the proportion of college workers holding high-school jobs, $\tilde{\varepsilon}_{hl}$, declines, reflecting the fall in p_l, but in about two quarters it reaches its initial level and continues to rise in subsequent periods, reflecting the rise in college unemployment. Both employment rates gradually decline and converge to lower levels due to the fall in job finding rates. The evolution of the percentage deviations of the employment rates is depicted in Figure 5. As shown in the second panel, the overall college employment rate declines less in percentage terms, and converges faster than the high-school employment rate. The first panel shows that this is due to both a smaller percentage decline in $\tilde{\varepsilon}_{hh}$ than in $\tilde{\varepsilon}_l$ and the subsequent percentage increase in $\tilde{\varepsilon}_{hl}$.

[34] To derive the responses I use the baseline calibration values. The responses do not change significantly when the model is calibrated to a 10% and 30% wage gap.

4.2.4 Inspecting the role of skill mismatch

In this section I examine how the model's predictions change when we allow lower values for μ. In Table 6, I report simulation results for $\mu = 1$ (as above), $\mu = 0.5$ and $\mu = 0$, under the three alternative calibrations. A lower value of μ makes ν_h more volatile and ν_l less volatile, confirming the intuition discussed in Section 3. For this reason, the correlation of η with output becomes smaller and at $\mu = 0$ it even turns negative. Hence, the volatility of p_h and thus of $\tilde{\varepsilon}_{hh}$ increases, while that of p_l and as a consequence of $\tilde{\varepsilon}_l$ falls.[35] A lower value of μ implies that a higher number of college workers rely on only the supply of high-school vacancies to exit unemployment. This, coupled with the fact that as μ falls, the volatility of p_h increases, explains why the overall college employment rate becomes excessively volatile relative to the high-school employment rate, compared to what we observe in the data.

At lower values for μ, the college employment becomes not only more volatile, but also smaller than in the data, because the number of college workers in high-school jobs falls. Matching the data in this case requires either increasing the net productivity of college jobs, $y\alpha_{hh} - b_h$, or lowering the exogenous separation rate for college jobs, s_h. For the reasons explained above, these changes can moderate the volatility and increase the size of ν_h, and thus improve the model's ability to match both the levels and the relative volatility of employment rates at lower values for μ. However, at the same time, these changes will cause the college-plus to high-school wage premium to rise. With a smaller number of college workers employed in high-school jobs that offer lower wages, the predicted college-plus to high-school wage premium is larger than in the data. Setting, in addition, a smaller value for s_h and a larger value for $y\alpha_{hh} - b_h$ makes the surplus of college jobs even larger, leading to an even larger wage premium.

Nevertheless, to investigate this possibility I first simulate the model with $\mu = 0.5$ and the values for s_h and s_l as calibrated above, searching for the dispersion in net productivity between high-school and college jobs that matches the observed employment rates. To simplify things, I consider the best-case scenario for the profitability of college jobs. Although it seems more reasonable to set $c_h > c_l$ and $b_h > b_l$, I keep the values for c_l and b_l as

[35] The standard deviations of ν_l and p_l decline only moderately, suggesting that the dominant channel through which skill mismatches affect the volatility of vacancy postings is the bargaining position of college workers. This is somewhat expected, because the college unemployment rate is much smaller than the high-school unemployment rate, which means that the presence of college searchers in the high-school sector has a relatively small impact on incentives to post high-school vacancies.

calibrated above, and set $c_h = c_l$ and $b_h = b_l$.[36] Despite the parameter choices that clearly favor the posting of college vacancies, I find that with $\mu = 0.5$, the value for a_{hh} must be about 3 times that for a_{ll} in order for the supply of college vacancies to be sufficiently large to match the college employment rate. This implies an average college-plus to high-school wage premium of about 180%, which is unrealistic. More fundamentally, with the required productivity dispersion, the expected profit from posting a college vacancy is much less volatile than that from posting a high-school vacancy, which makes ν_h excessively less volatile than ν_l, and as a result, makes the college employment rate much less volatile than the high-school employment rate. Specifically, with the required productivity dispersion the standard deviation of the latter is more than 6 times that of the former.

I then experiment with an even lower value for s_h and a higher one for s_l, while setting $c_h = c_l$ and $b_h = b_l$ as above. This helps in matching the dispersion in employment rates at a smaller productivity gap, and thus a smaller wage premium.[37] However, even in this case the resulting wage premium is unrealistically large. More importantly, increasing the dispersion between s_h and s_l, makes the expected profits from posting college vacancies even less volatile compared to the expected profits from posting high-school vacancies, leading to an even less volatile college employment rate compared to the high-school employment rate. For instance, with $s_h = 0.007$ and $s_l = 0.039$, the model matches the employment rate levels at a wage premium of around 85%, but the standard deviation of the high-school employment rate is 13 to 16 times that of the college employment rate.

Note the role of the bargaining parameter γ. If we lower γ for college graduates, their wage declines, meaning that the profits of college jobs increase, leading to a larger supply of college vacancies, at an empirically reasonable college-plus to high-school wage premium. However, there are two problems with this remedy. First, there is little empirical background for such an argument. In fact, it seems more reasonable to assume that the bargaining share of college graduates, who have more skills and capture only a small share of the labor force,

[36] I keep the values of the parameters that are common to both labor types as assumed above and choose a value for a_{hl} that matches the wage gap between skill-mismatched and correctly matched college workers. I simulate the model under the three alternative parameterizations (10%, 20% and 30% wage gap) and find similar results.

[37] Evidently, lowering the dispersion between s_l and s_h does not help in matching the levels and relative volatility of employment rates at lower values of μ. The required productivity dispersion in this case would be even larger, leading to an even less volatile college employment rate and a much higher wage premium.

is larger. Second, even if this helps in matching the dispersion in employment rates at a reasonable wage premium, it does not improve the model's ability to match the relative volatility of employment rates at lower values for μ. If we lower γ for college graduates, their wage becomes less cyclical, but also smaller, meaning that the profits of college jobs become more cyclical, but also larger. As Hagedorn and Manovskii (2008) argue, these two opposite effects cancel each other out, leaving the volatility of vacancies almost intact. Consequently, if we assume that college graduates have a lower bargaining share than do high-school workers, the model can potentially match both the dispersion in employment rates and the wage premium at lower values for μ, but the resulting college employment rate will still be much less volatile relative to the high-school employment rate, because ν_h will still be much less volatile than ν_l.

To recap, this exercise suggests that the model can match both the levels and the relative volatility of employment rates only if we allow for a sufficiently large share of college workers to transitorily take high-school jobs. This feature makes the college employment rate larger and less volatile, while keeping the college-plus to high-school wage premium reasonable. On the one hand, without this feature, matching the observed dispersion in employment rates yields an unrealistically high college-plus to high-school wage premium and an excessively stable college employment rate. On the other hand, matching the wage premium yields a college employment rate that is smaller and much volatile relative to the high-school employment rate.

5 Concluding remarks

Employment is typically lower and less procyclical at lower skill levels. This paper shows that a standard search and matching model modified to allow for two-sided skill heterogeneity and on-the-job search can go a long way in explaining these facts.

The model accounts for the fact that more-skilled workers are qualified for a wider range of job types and thus have a better outside option than do less-skilled workers. They are less likely to remain unemployed until their optimal job type comes across, and are more likely to search for jobs while employed in less-skilled jobs. I show that this feature makes the posting of vacancies suited for skilled individuals less cyclical, because the impact of cyclical shocks on the profits firms expect to generate from posting these vacancies is absorbed more by the wages of these individuals. This feature also implies greater variability in net productivity and match duration in the pool of potential hires for firms that seek to fill less-skilled vacancies, because these vacancies can be occupied by a wider range of worker types and thus are more likely to be skill mismatched. I show that this makes the posting of less-skilled vacancies more cyclical, because firms anticipate that these jobs will generate lower net productivity and be of shorter duration on average. These aspects that arise due to the presence of heterogeneity in the model have been overlooked in previous analyses that consider simplified frameworks where there is a unique matching rate for all worker/job types.

A calibration of the model accurately predicts the observed differences in the levels and cyclical volatility of employment rates between college and high-school educated workers. The model explains the relative volatility of employment rates for the two skill groups, because it allows for a sufficiently large number of college graduates to transitorily take jobs for which they are overeducated. The model accounts for roughly 10% to 17% of college graduates employed in high-school jobs, in line with available empirical measures. The model is also consistent with other important features of the labor market such as a procyclical rate of job-to-job transitions and cyclical changes in job quality with workers occupying jobs that would be normally occupied by less-skilled individuals in downturns and upgrading to higher-skill jobs in booms.

An intriguing property of the model is that it explains important dimensions of the data and provides insights on across-skill, labor-market interactions without introducing complex features relative to the standard search and matching model. A key element of the model is that, conditional on the skills they possess, workers can perform only a limited number of job types, which is a natural consequence of skill heterogeneity in the labor market. Some other aspects may have important implications for how the employment rates of different skill groups respond to changes in aggregate economic conditions. For instance, different matching technologies or wage setting mechanisms across labor market sectors may also induce differential employment responses to business cycle shocks. However, there is little empirical background for such arguments.

APPENDIX

Break up probability and match-value volatility

The purpose of this appendix is to demonstrate that the volatility of the value of a match increases as its break up probability increases. To this end, I consider a simplified framework were matches break up at an exogenous rate, and the worker's wage is such that he gets a share γ of the prevailing match productivity and a share $(1-\gamma)$ of his opportunity cost of working. In this simpler form, the value of a match is given by the following recursive formula:

$$J_y = (1-\gamma)[y\alpha - b] + \beta(1-s)\sum_{y'} \pi_{yy'} J_{y'} \tag{25}$$

where the subscript y distinguishes different states of aggregate productivity, s is the the probability of a break up, α is the productivity of the match, b is the worker's opportunity cost of working, and β is the discount factor. The terms $\pi_{yy'}$ are transition probabilities from current state y to future state y'.

In matrix notation the value of a match can be written as:

$$\mathbf{J} = (1-\gamma)\left[\mathbf{I} + \beta(1-s)\Pi + \beta^2(1-s)^2\Pi^2 + \cdots\right][\alpha\mathbf{Y} - b\mathbf{I}] \tag{26}$$

where $\mathbf{Y}$ is a vector of possible aggregate productivity states, and $[\alpha\mathbf{Y} - b\mathbf{I}]$ is a vector consisting of the net productivity associated with each state. Π is a transition matrix with elements within each row summing up to unity. All powers of Π are again transition matrices. The elements of a vector with a higher power of Π are the weighted averages of the elements of a vector with a lower power of Π. For example, the elements of $\Pi^2[\alpha\mathbf{Y} - b\mathbf{I}]$ are the weighted averages of the elements of $\Pi[\alpha\mathbf{Y} - b\mathbf{I}]$. This means that the elements of vectors with lower powers are less similar than the elements of vectors with higher powers. Therefore, we can say that the elements of $\mathbf{J}$ are less similar the higher the weight associated with lower powers of Π. Note that the weight of lower powers of Π is higher the larger s is. Hence, the elements of $\mathbf{J}$ are less similar the higher s is, meaning that the the value of a match varies more significantly across different realizations of the aggregate state, when the break up probability of that match is high.

References

Abraham, K. and Farber H. (1987), "Job Duration, Seniority, and Earnings," *The American Economic Review*, 77(3), 278-297.

Acemoglu, D. (1999), "Changes in Unemployment and Wage Inequality: an Alternative Theory and some Evidence," *American Economic Review*, 89(5), 1259-1278.

Albrecht, J. and Vroman S. (2002), "A Matching Model with Endogenous Skill Requirements," *International Economic Review*, 43, 283-305.

Barlevy, G. (2002), "The Sullying Effect of Recessions," *The Review of Economic Studies*, 69, 65-96.

Barron, M., Berger, B. and Black, D. (1997), "Employer Search, Training, and Vacancy Duration," *Economic Inquiry*, 35, 167-192.

Bauer, T. (2002), "Educational Mismatch and Wages: a Panel Analysis," *Economics of Education Review*, 21, 221-229.

Behrman, M. and Rosenzweig, M. (1999), ""Ability" Biases in Schooling Returns and Twins: a Test and New Estimates," *Economics of Education Review*, 18, 159-167.

Blanchard, D. and Diamond, P. (1989a), "The Beveridge Curve," *Brookings Papers on Economic Activity*, 1989(1), 1-76.

Blanchard, D. and Diamond, P. (1989b), "The Aggregate Matching Function," (National Bureau of Economic Research, Working Paper 3175).

Blanchard, D. and Diamond, P. (1990), "The Cyclical Behavior of the Gross Flows of U.S. Workers," *Brookings Papers on Economic Activity*, 1990(2), 85-155.

Bowlus, A. (1995), "Matching Workers and Jobs: Cyclical Fluctuations in Match Quality," *Journal of Labor Economics*, 13(2), 335-350.

Chevalier, A. (2003), "Measuring Over-Education," *Economica*, 70, 509-531.

Costain, J. and Reiter, M. (2008), "Business Cycles, Unemployment Insurance, and the Calibration of Matching Models," *Journal of Economic Dynamics and Control*, 32, 1120-1155.

Davis, S., Faberman, J. and Haltiwanger, J. (2010), "The Establishment-Level Behavior of Vacancies and Hiring," (National Bureau of Economic Research, Working Paper 16265).

Davis, S., Haltiwanger, J. and Schuh, S. (1996), *Job Creation and Destruction* (The MIT Press, Cambridge, MA).

Devereux, P. (2002), "Occupational Upgrading and the Business Cycle," *Labour*, 16(3), 423-452.

Devereux, P. (2004), "Cyclical Quality Adjustment in the Labor Market," *Southern Economic Journal*, 70(3), 600-615.

Dolado, J., Jansen, M. and Jimeno, J. (2009), "On-the-job Search in a Matching Model with Heterogeneous Jobs and Workers," *The Economic Journal*, 119(534), 200-228.

Gautier, P. (2002), "Unemployment and Search externalities in a model with Heterogeneous Jobs and Heterogeneous Workers," *Economica*, 69, 21-40.

Gautier, P., Teulings, C. and van Vuuren A. (2010), "On the Job Search Mismatch and Efficiency," *Review of Economic Studies*, 77(1), 245-272.

Green, F., McInstosh, S. and Vignoles, A. (1999), "Overeducation and Skills-Clarifying the Concepts" (CEP discussion paper 435).

Hagedorn, M. and Manovskii, I. (2008), "The Cyclical Behavior of Equilibrium Unemployment and Vacnacies Revisited," *American Economic Review*, 98(4), 1692-1706.

Hall, R. (2005a), "Employment Fluctuations with Equilibrium Wage Stickiness, " *American Economic Review*, 95(1), 50-65.

Hall, R. (2005b), "Job Loss, Job Finding, and Unemployment in the U.S. Economy Over the Past Fifty Years," *NBER Macroeconomics Annual*, 101-137.

Hall, R. and Milgrom, P. (2008), "The Limited Influence of Unemployment on the Wage Bargain," *American Economic Review*, 98(4), 1653-1674.

Hamermesh, D. (1993) *Labor Demand* (Princeton Academic Press, Princeton, NJ).

Hecker, D. (1992), "Reconciling Conflicting Data on Jobs for College Graduates," *Monthly Labor Review*, July.

Hecker, D. (1995), "College Graduates in "High School" Jobs," *Monthly Labor Review*, December.

Khalifa, S. (2010), "Job Competition, Crowding Out and Unemployment Fluctuations," *Macroeconomic Dynamics*, available on CJO 2002 doi: 10.1017/S1365100510000325.

Krause, M. and Lubik, T. (2006), "The Cyclical Upgrading of Labor and On-the-job Search," *Labour Economics*, 13, 459-477.

Mortensen, D. and Pissarides, C. (1994), "Job Creation and Job Destruction in the Theory of Unemployment," *The Review of Economic Studies*, 61(3), 397-415.

Nagypál, E. (2007), "Labor-Market Fluctuations and On-the-Job Search," (mimeo, Northwestern University).

Nagypál, E. (2008), "Worker Reallocation over the Business Cycle: the Importance of Employment-to-Employment Transitions," (mimeo, Northwestern University).

Nickell, S. (1979), "Education and Lifitime Patterns of Unemployment," *Journal of Political Economy*, 87(5), 117-131.

Pissarides, C. (1994), "Search Unemployment with On-the-job Search," *The Review of Economic Studies*, 61(3), 457-475.

Pissarides, C. (2009), "The Unemployment Volatility Puzzle: is Wage Stickiness the Answer?," *Econometrica*, 77(5), 1339-1369.

Pissarides, C. and Wadsworth, J. (1994), "On-the-job Search: some Empirical Evidence from Britain," *European Economic Review*, 38(2), 385-401.

Plesca, M. (2010), "A General Equilibrium Evaluation of the Employment Service," *Journal of Human Capital*, forthcoming.

Polsky, D. (1999), "Changing Consequences of Job Separation in the United States," *Industrial and Labor Relations Review*, 52(4), 564-580.

Pries, M. (2008), "Worker Heterogeneity and Labor Market Volatility in Matching Models," *Review of Economic Dynamics*, 11(3), 664-678.

Royalty, A. (1998), "Job-to-job and Job-to-nonemployment Turnover by Gender and Education Level," *Journal of Labor Economics*, 16(2), 392-443.

Shimer, R. (2005a), "The Cyclicality of Hires, Separations, and Job-to-job Transitions," *Federal Reserve Bank of St. Louis Review*, 87(4), 493-507.

Shimer, R. (2005b), "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review*, 95(1), 25-49.

Shimer, R. (2006), "On-the-Job Search and Strategic Bargaining," *European Economic Review*, 50(4), 811-830.

Sicherman, N. (1991), "Overeducation in the Labor Market," *Journal of Labor Economics*, 9, 101-112.

Van Ours, J. and Ridder, G. (1992), "Vacancies and the Recruitment of New Employees," *Journal of Labor Economics*, 10(2), 138-155.

Van Ours, J. and Ridder, G. (1993), "Vacancy Durations: Search or Selection?," *Oxford Bulletin of Economics and Statistics*, 55(2), 187-198.

Van Ours, J. and Ridder, G. (1995), "Job Matching and Job Competition: are Lower Educated Workers at the Back of Job Queues?," *European Economic Review*, 39, 1717-1731.

Table 1: Calibrated Parameter Values for the U.S. economy

Parameter	Value	Source
Match elasticity, a	0.4	Blanchard and Diamond (1989b)
Match efficiency parameter, M	0.567	Calibrated
Discount rate, r	0.004	Shimer (2005), consistent with a quarterly interest rate of 0.012
A worker's bargaining share, γ	0.5	Den Haan et al. (2000)
Proportion of high-school workers, δ	0.73	March CPS Annual Demographic Survey
Output of a high-school worker in a high-school job, α_{ll}	0.210	Calibrated
Output of a college worker in a college job, α_{hh}	0.341	Calibrated
Output of a college worker in a high-school job, α_{hl}	0.260	Calibrated
Unemployment income of high-school workers, b_l	0.110	Calibrated
Unemployment income of college workers, b_h	0.230	Calibrated
A college job's exogenous separation rate, s_h	0.016	Job Openings and Labor Turnover Survey
A high-school job's exogenous separation rate, s_l	0.035	Calibrated
Cost of maintaining a high-school vacancy, c_l	0.205	Calibrated
Cost of maintaining a college vacancy, c_h	0.324	Calibrated

Note: The table contains the calibrated parameter values in the baseline calibration where $\mu = 1$ and the wage gap between skill-mismatched and correctly matched college workers is 20%.

Table 2: Quantitative Results

Description	Model			U.S. data
	30%	20%	10%	
Proportion of high-school vacancies, η	0.79	0.81	0.85	
Probability of finding a:				
high-school vacancy, p_l	0.38	0.39	0.39	
college vacancy, p_h	0.10	0.09	0.07	
Share of skill-mismatched college graduates, $\frac{\varepsilon_{hl}}{\varepsilon_h}$	0.10	0.12	0.17	0.15
Job-to-job flows/college employment, $\frac{p_h \varepsilon_{hl}}{\varepsilon_h}$	0.01	0.01	0.01	0.02
Job-to-job flows/overall college separations, $\frac{p_h(1-s_l)e_{hl}}{e_{hh}s_h+e_{hl}s_l+p_h(1-s_l)e_{hl}}$	0.37	0.37	0.36	0.50
Average vacancy duration (in weeks)				
high-school	3.39	3.52	3.72	
college	6.73	6.18	5.13	
overall	3.79	3.83	3.87	2-4

Notes: The table reports results from model simulations for the baseline case ($\mu = 1$) when parameters are such that the wage gap between skill-mismatched and correctly matched college workers is 30%, 20% and 10%. The last column reports available results for the U.S.

Table 3: Business Cycle Statistics - 30% Wage Gap

Variable	Cross correlation of output(t) with variable($t+i$)								St. dev.
	t	$t+1$	$t+2$	$t+3$	$t+4$	$t+5$	$t+6$	$t+7$	
$m(\theta)$	0.99	0.87	0.77	0.69	0.62	0.56	0.50	0.45	0.0464
η	0.94	0.90	0.87	0.84	0.80	0.76	0.71	0.67	0.0036
p_l	0.99	0.88	0.79	0.71	0.64	0.58	0.52	0.47	0.0496
p_h	0.97	0.81	0.68	0.58	0.49	0.42	0.36	0.31	0.0352
ν_l	0.93	0.70	0.56	0.46	0.39	0.34	0.29	0.25	0.0668
ν_h	0.79	0.51	0.33	0.22	0.14	0.09	0.04	0.01	0.0555
$\tilde{\varepsilon}_{hh}$	0.85	0.89	0.91	0.91	0.89	0.86	0.82	0.78	0.0020
$\tilde{\varepsilon}_{hl}$	-0.49	-0.60	-0.71	-0.80	-0.85	-0.88	-0.89	-0.88	0.0094
$\tilde{\varepsilon}_h$	0.98	0.96	0.90	0.83	0.75	0.68	0.61	0.55	0.0014
$\tilde{\varepsilon}_l$	0.96	0.96	0.92	0.86	0.80	0.73	0.67	0.60	0.0035
net flow	-0.66	-0.60	-0.55	-0.42	-0.30	-0.20	-0.11	-0.05	0.0932

Relative employment-rate volatility (st. dev. $\tilde{\varepsilon}_l$/st. dev. $\tilde{\varepsilon}_h$): 2.42

Notes: The table reports cross correlations with output and the standard deviations of the key labor market variables for the baseline case ($\mu = 1$). The standard deviations are computed after taking logs and removing a Hodrick-Prescott trend with a smoothing parameter 14440. The "net flow" refers to the flow of college graduates into high-school jobs, minus the flow out of them.

Table 4: Business Cycle Statistics - 20% Wage Gap

Variable	Cross correlation of output(t) with variable($t+i$)								St. dev.
	t	$t+1$	$t+2$	$t+3$	$t+4$	$t+5$	$t+6$	$t+7$	
$m(\theta)$	0.99	0.86	0.75	0.67	0.60	0.54	0.48	0.43	0.0459
η	0.97	0.92	0.86	0.80	0.75	0.69	0.64	0.59	0.0044
p_l	0.99	0.87	0.77	0.69	0.62	0.55	0.50	0.45	0.0500
p_h	0.95	0.75	0.62	0.52	0.43	0.37	0.31	0.26	0.0292
ν_l	0.93	0.70	0.56	0.47	0.40	0.34	0.29	0.26	0.0678
ν_h	0.74	0.43	0.26	0.16	0.09	0.04	0.01	-0.01	0.0507
$\tilde{\varepsilon}_{hh}$	0.84	0.89	0.90	0.90	0.88	0.85	0.81	0.77	0.0017
$\tilde{\varepsilon}_{hl}$	-0.23	-0.37	-0.51	-0.63	-0.72	-0.78	-0.82	-0.83	0.0068
$\tilde{\varepsilon}_h$	0.98	0.96	0.89	0.81	0.74	0.66	0.60	0.54	0.0014
$\tilde{\varepsilon}_l$	0.97	0.96	0.91	0.85	0.78	0.71	0.64	0.58	0.0034
net flow	-0.52	-0.56	-0.49	-0.39	-0.30	-0.22	-0.15	-0.10	0.0844

Relative employment-rate volatility (st. dev. $\tilde{\varepsilon}_l$/st. dev. $\tilde{\varepsilon}_h$): 2.39

Notes: The table reports cross correlations with output and the standard deviations of the key labor market variables for the baseline case ($\mu = 1$). The standard deviations are computed after taking logs and removing a Hodrick-Prescott trend with a smoothing parameter 14440. The "net flow" refers to the flow of college graduates into high-school jobs, minus the flow out of them.

Table 5: Business Cycle Statistics - 10% Wage Gap

Variable	Cross correlation of output(t) with variable($t+i$)								St. dev.
	t	$t+1$	$t+2$	$t+3$	$t+4$	$t+5$	$t+6$	$t+7$	
$m(\theta)$	0.99	0.83	0.72	0.64	0.57	0.51	0.45	0.40	0.0430
η	0.95	0.92	0.87	0.82	0.76	0.71	0.66	0.61	0.0031
p_l	0.99	0.84	0.73	0.66	0.59	0.53	0.47	0.42	0.0457
p_h	0.88	0.62	0.48	0.40	0.31	0.25	0.20	0.16	0.0289
ν_l	0.93	0.68	0.54	0.46	0.39	0.34	0.30	0.26	0.0648
ν_h	0.72	0.39	0.22	0.13	0.07	0.03	0.02	-0.01	0.0510
$\tilde{\varepsilon}_{hh}$	0.84	0.88	0.89	0.88	0.86	0.84	0.80	0.76	0.0016
$\tilde{\varepsilon}_{hl}$	-0.07	-0.22	-0.37	-0.51	-0.61	-0.68	-0.73	-0.76	0.0047
$\tilde{\varepsilon}_h$	0.98	0.95	0.88	0.80	0.72	0.65	0.59	0.53	0.0014
$\tilde{\varepsilon}_l$	0.97	0.96	0.90	0.83	0.76	0.69	0.63	0.56	0.0031
net flow	-0.45	-0.51	-0.45	-0.37	-0.29	-0.22	-0.16	-0.12	0.0628

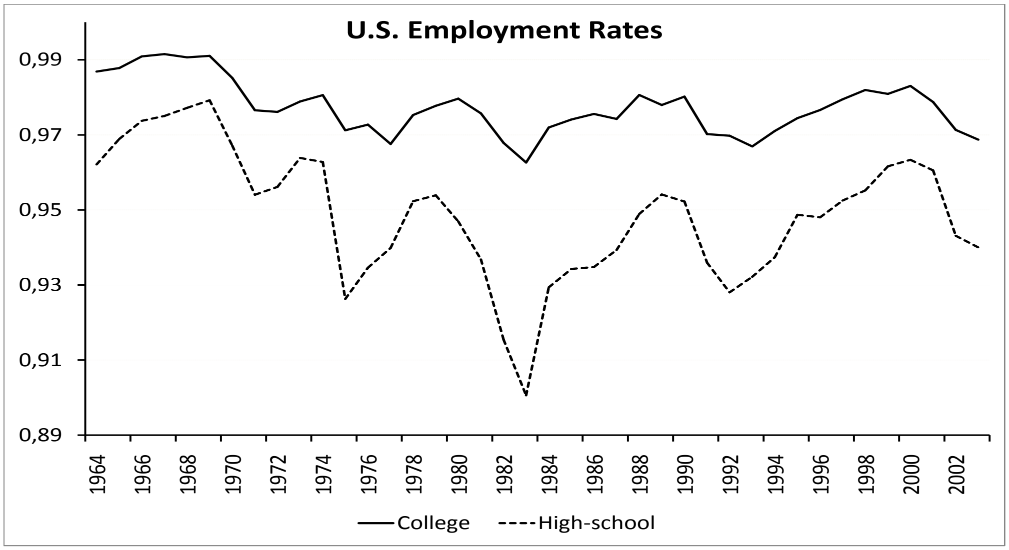
Relative employment-rate volatility (st. dev. $\tilde{\varepsilon}_l$/st. dev. $\tilde{\varepsilon}_h$): 2.15

Notes: The table reports cross correlations with output and the standard deviations of the key labor market variables for the baseline case ($\mu = 1$). The standard deviations are computed after taking logs and removing a Hodrick-Prescott trend with a smoothing parameter 14440. The "net flow" refers to the flow of college graduates into high-school jobs, minus the flow out of them.

Table 6: The Role of Skill Mismatches

	30%			20%			10%		
variable	$\mu = 1$	$\mu = 0.5$	$\mu = 0$	$\mu = 1$	$\mu = 0.5$	$\mu = 0$	$\mu = 1$	$\mu = 0.5$	$\mu = 0$
					Standard Deviations				
$\tilde{\varepsilon}_{hh}$	0.0020	0.0041	0.0075	0.0017	0.0030	0.0038	0.0016	0.0026	0.0028
$\tilde{\varepsilon}_{hl}$	0.0094	0.0213	-	0.0068	0.0149	-	0.0047	0.0125	-
$\tilde{\varepsilon}_{h}$	0.0014	0.0026	0.0075	0.0014	0.0021	0.0038	0.0014	0.0019	0.0028
$\tilde{\varepsilon}_{l}$	0.0035	0.0029	0.0024	0.0034	0.0031	0.0027	0.0031	0.0030	0.0027
ν_l	0.0668	0.0607	0.0524	0.0678	0.0624	0.0574	0.0648	0.0613	0.0577
ν_h	0.0555	0.0879	0.1349	0.0507	0.0688	0.0810	0.0510	0.0639	0.0661
p_l	0.0496	0.0415	0.0341	0.0500	0.0454	0.0410	0.0457	0.0446	0.0423
p_h	0.0352	0.0674	0.1151	0.0292	0.0495	0.638	0.0289	0.0447	0.0498
					Levels				
$\tilde{\varepsilon}_{l}$	0.95	0.95	0.92	0.95	0.95	0.92	0.95	0.95	0.92
$\tilde{\varepsilon}_{h}$	0.98	0.92	0.83	0.98	0.93	0.88	0.98	0.93	0.89
$\tilde{\varepsilon}_{hl}$	0.10	0.06	-	0.12	0.05	-	0.16	0.05	-
					Cross correlation of η with output				
	0.94	-0.73	-0.92	0.97	0.30	-0.82	0.95	0.55	-0.36
					Relative employment-rate volatility				
	2.42	1.11	0.32	2.39	1.52	0.72	2.15	1.60	0.97

Notes: The table reports results for different values of the proportion of college graduates that would transitorily take a high-school job, μ. The standard deviations are computed after taking logs and removing a Hodrick-Prescott trend with a smoothing parameter 14440.



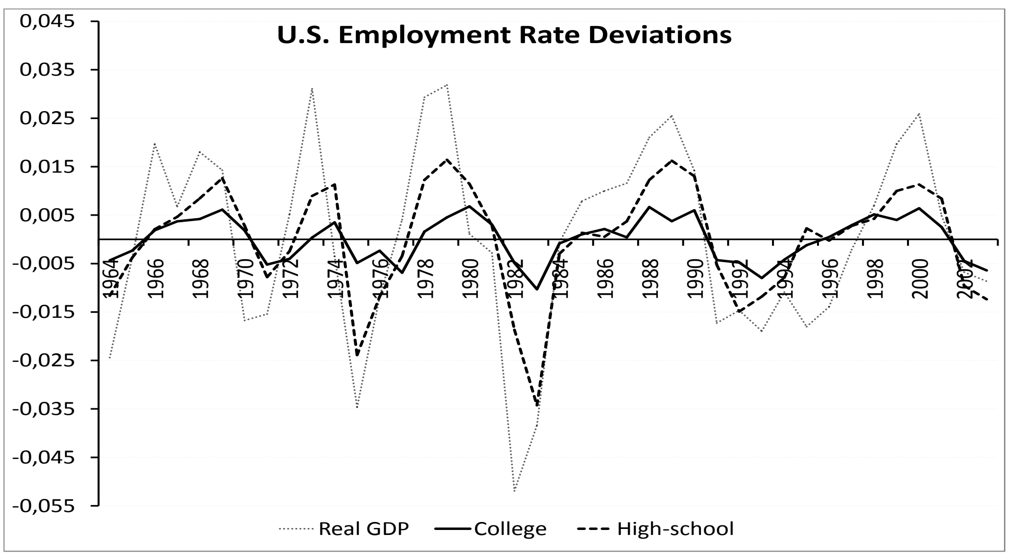


Figure 1: The top figure traces the yearly U.S. employment rates of college and high-school graduates. The bottom figure traces their log-deviations along with the log-deviations of the U.S. real GDP. The log-deviations are computed using a Hodrick Prescott filter with a smoothing parameter 100. The data on employment rates come from the March CPS, Annual Demographic Survey files. The data on real GDP come from the Department of Commerce: Bureau of Economic Analysis.

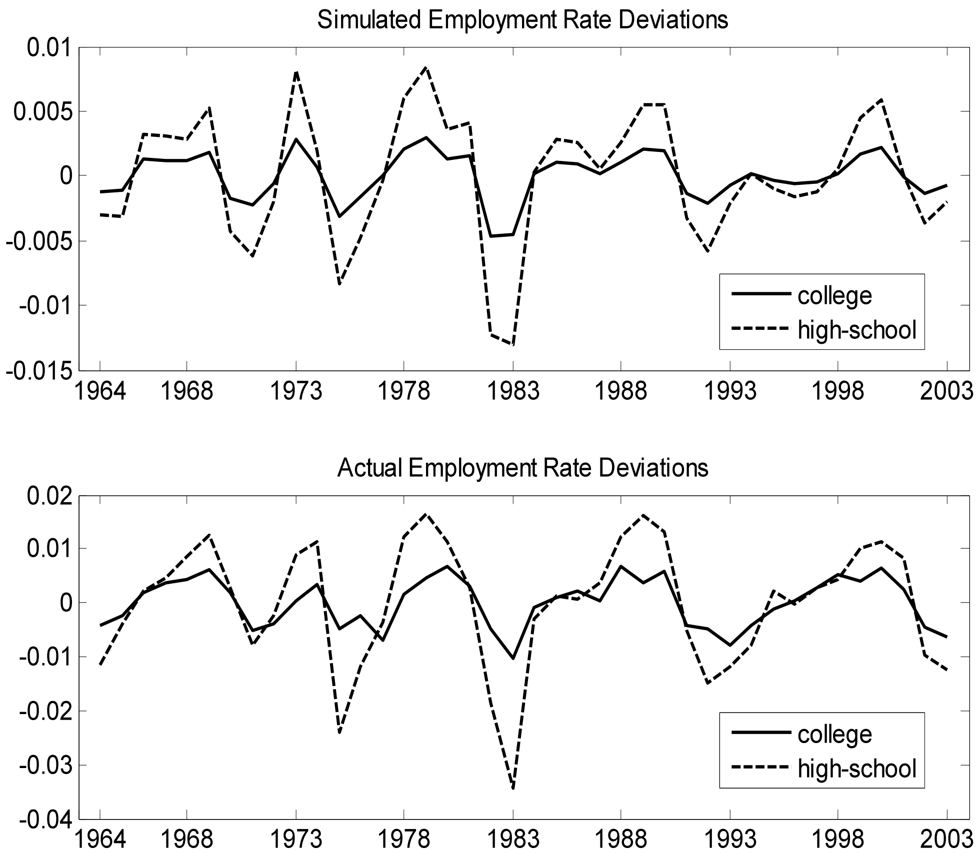


Figure 2: Simulated and actual log-deviations of the employment rates of college and high-school workers. The actual log-deviations come from the March CPS, Annual Demographic Survey, 1964-2003. The simulated log-deviations are along a series of aggregate productivity realizations that replicates the U.S. GDP deviations over the same period. Both the simulated and the actual log-deviations are computed using a Hodrick-Prescott filter with smoothing parameter 100.

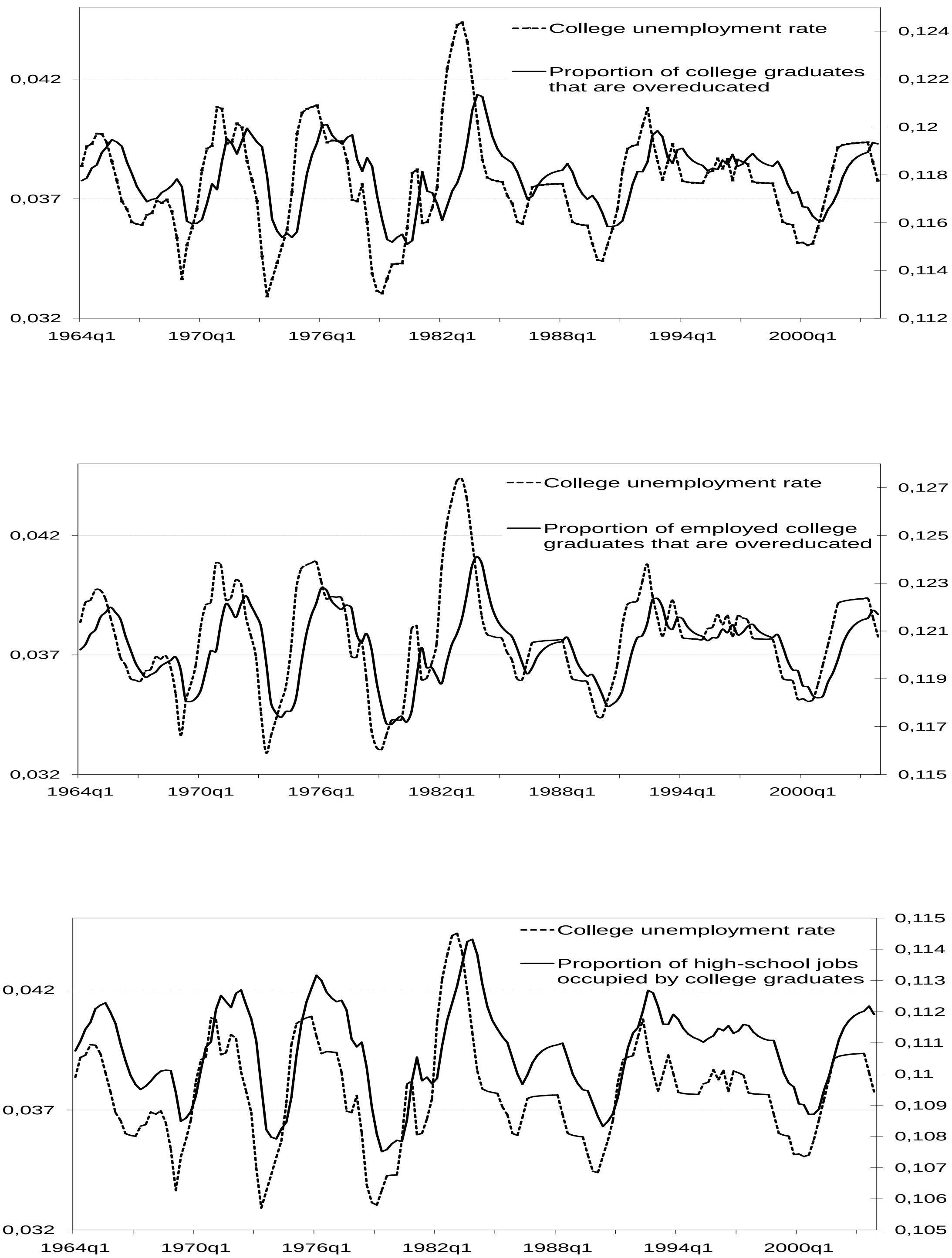
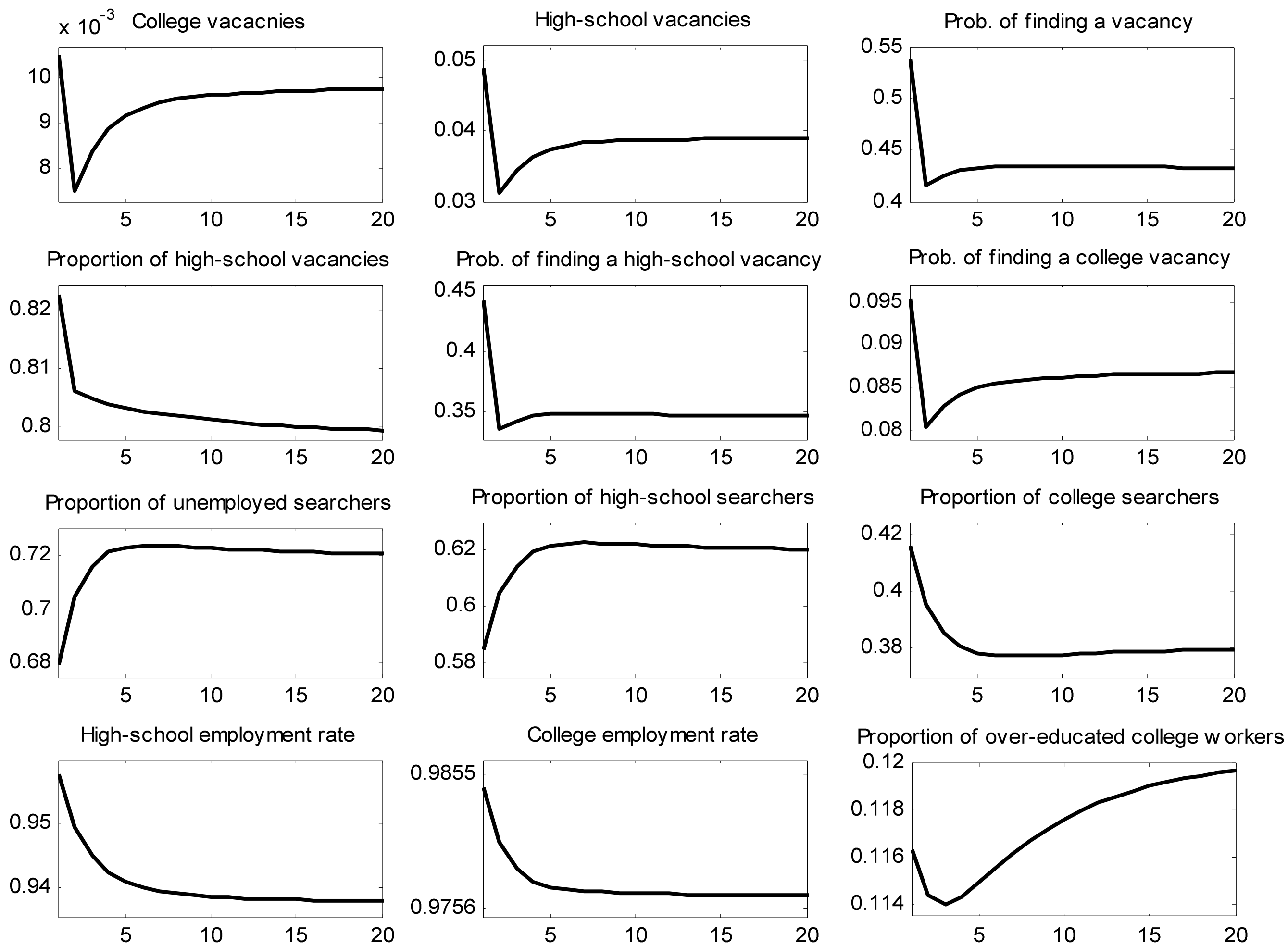


Figure 3: Three alternative measures of skill mismatch along a series of aggregate productivity realizations that replicates the U.S. GDP quarterly deviations from 1964 to 2003.

Figure 4: Responses to a negative productivity shock.



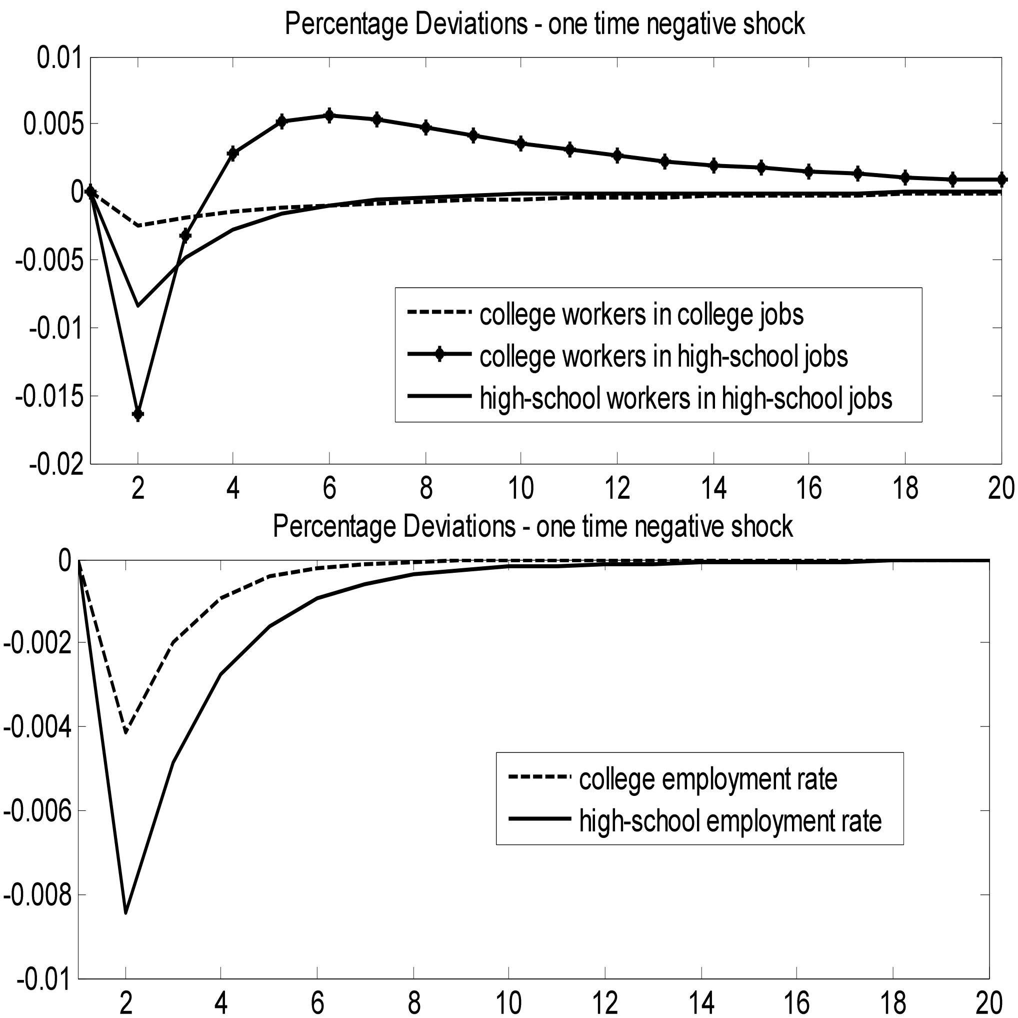


Figure 5: Percentage employment-rate responses to a negative productivity shock.