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Working Paper 10-2019

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Stress-Testing the Runoff Rule in the Laboratory

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August 2019

Abstract

When a majority of voters has common values, but private information, then the runoff rule always admits an equilibrium that aggregates information strictly better than the best equilibrium of the plurality rule. But there are cases in which the plurality rule supports equilibria that are strictly better compared to certain undominated equilibria of the runoff rule. Is there any risk with applying the runoff rule in these situations? We conduct a laboratory experiment and we show that the runoff rule consistently delivers better outcomes than the plurality rule even in such unfavorable scenarios. This establishes that the superiority of the runoff rule over the plurality rule in empirical settings outperforms its theoretical advantages.

Keywords: runoff voting; plurality rule; information aggregation; Condorcet jury theorem; experiment.

JEL classification: D72

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1 Introduction

Consider a majority of voters who have common interests but who also have heterogeneous views regarding which of the two majority candidates is the best one due to private information, and a substantial minority (i.e. larger than one third of the population) that supports its own minority candidate. Under plurality rule the prospects of successful coordination of the majority around the best candidate are not great in general. If every voter votes sincerely for the candidate she privately thinks is the most suitable, then this majority will most likely split and the winner will be the candidate least preferred by the majority (i.e. the minority candidate who is also the Condorcet loser). Interestingly, this behavior –let us call it sincere voting— is quite often an equilibrium, and hence coordination failures should not be a surprise in plurality rule elections. So, is coordination of the majority impossible under plurality rule? The answer is no. Coordination might be achieved by the means of a Duvergerian equilibrium: majority voters discard their information and decide to vote for the same majority candidate (Duverger, 1951). These equilibria always elect a majority candidate, but not always the correct one.

To sum up, plurality rule may broadly support two types of pure equilibria: informative equilibria –which reveal information, but lead to the election of the worst candidate with high probability— and Duvergerian ones –which always lead to the election of a majority candidate, but do not exploit the voters' private information and lead often to the election of the inferior majority candidate. Hence, there is an obvious trade-off between voting informatively and the likelihood of electing a majority candidate.

Runoff voting, on the other hand, should be able to free the majority from such coordination concerns. If this society votes according to a runoff rule instead, then the majority could vote sincerely in the first round and then, after the information is revealed, the majority candidate who is considered best by most majority voters will advance in the second round against the minority candidate (who is also the Condorcet loser) and will win with certainty. Hence, the runoff rule should be viewed as unambiguously better compared to the plurality rule –at least in the current divided majority framework— and there should be no reasons to worry that it might lead to worse outcomes and coordination dilemmas.

Or is there? Actually, for the above line of reasoning to be conclusive, it is necessary that this kind of informative play is the only reasonable equilibrium behavior under runoff voting. We start this paper by noticing that in many –actually, in most— situations sincere voting is not an equilibrium in runoff elections. Moreover, Duvergerian equilibria, in which all majority voters vote for a specific majority candidate discarding their information, always exist under the runoff rule –and some of them are in undominated strategies. That is, in most cases every symmetric pure strategy equilibrium of the runoff rule is characterized by uninformative Duvergerian coordination. Perhaps more importantly, we argue that among these generic situations there are instances in which the plurality rule admits a welfare superior sincere equilibrium (i.e. it ensures higher payoffs to majority voters than undominated Duvergerian equilibria, which provide the same payoffs under both rules). That is, there are cases in which the plurality rule admits a symmetric pure strategy equilibrium that gives to majority voters strictly larger payoffs than any symmetric pure strategy equilibrium of the runoff rule!

Fortunately, as we show, in these situations the runoff rule admits other equilibria in mixed and/or asymmetric strategies that lead to strictly larger payoffs compared to the best equilibrium of the plurality rule. That is, the runoff rule is guaranteed to have at least one equilibrium that is better than the best equilibrium of the plurality rule (from the point of view of the majority voters). But it cannot be argued that runoff voting exhibits no coordination concerns, nor that it is unambiguously better than the plurality rule: the best equilibrium of the runoff rule is strictly better than the best equilibrium of the plurality rule, but the best symmetric pure strategy equilibrium of the plurality rule can be strictly better compared to all symmetric pure strategy equilibria of the runoff rule (hence, if majority voters coordinate to such equilibria they will fare better under plurality rule).

These theoretical observations call for empirical analysis of such situations. Given that data from real world elections are hard to produce any interesting insight (mainly, because we do not have accurate access to voters' preferences and information) we turn to the laboratory and specifically test situations as described above. We follow contemporary experimental approaches (e.g. Bouton et al., 2016) in order for our study to be fully comparable with existing results in similar frameworks, and we show that the runoff rule substantially outperforms the plurality rule even in situations in which it is most vulnerable. Therefore, our work raises awareness regarding the fact that runoff rule is not unambiguously better to plurality rule, but at the same time reassures us that rational voters will tend to coordinate close to the payoff superior equilibria of each rule and this will always lead majority voters to enjoy higher payoffs under the runoff rule.

Our experimental results, apart from providing a convincing answer to our main motivating question, also provide insights with respect to other differences between the two voting rules. Namely, we find strong differences in terms of the effective number of parties and the effect of instrumental play on payoffs. Under plurality rule the effective number of parties is substantially smaller than under the runoff rule, a finding that is perfectly in line with Duverger's law and hypothesis (plurality leads to a smaller number of parties compared to runoff). To our knowledge this is the first paper that addresses this question in a common value setting: the only other experimental paper (Bouton et al., 2019) that studies a similar issue focuses on environments with private values.

Moreover, it seems that runoff voting allows instrumental majority voters to enjoy a premium compared to the counterfactual scenario in which they are constrained to vote sincerely, while instrumental play under plurality rule is shown to lead to inferior outcomes compared to sincere behavior. This is, presumably due to the fact that plurality rule might admit multiple equilibria that are similar in terms of welfare but diverse in terms of behavior, and hence, certain subjects might try to coordinate to one of them while others attempt to coordinate to another, leading to profound coordination failures and low payoffs.

Overall, our work highlights the empirical advantages of using the runoff rule in divided majority setups, in a number of dimensions. Runoff seems to aggregate information substantially better than plurality even when, in theory, it is not evident that it should; it guarantees greater pluralism (i.e. a larger number of parties); and seems to interact well with the agents' strategic behavior.

In what follows we briefly discuss the most relevant studies (Section 2), we provide a theoretical analysis (Section 3), we describe our experimental design and state our main hypotheses (Section 4), we present our main experimental results (Section 5), and, finally, we conclude (Section 6).

2 Literature Review

The formal analysis of information aggregation through voting goes back to the infamous Condorcet jury theorem, but the literature regarding its strategic underpinnings is much more recent. Indeed, it is only after Austen-Smith and Banks (1996) first argued that sincere voting might not be an individually rational behavior in two-alternative first-past-the-post elections with common value voters, that the literature started to devote increasing efforts in understanding which electoral rules can help a society aggregate information and which less so.

When the society is composed solely of common value voters, McLennan (1998), Hummel (2011) and Barelli et al. (2018) establish that in large elections plurality rule is sufficient to reach arbitrarily close to perfect information aggregation, providing an equilibrium-analysis support to the intuition of the Condorcet jury theorem. Ahn and Oliveiros (2016) consider the broad family of scoring rules (which includes plurality rule), and show that in such purely common value environments, approval voting performs better than any other scoring rule for any society size. Goertz and Maniquet (2012), establish that when partisan voters also exist in the electorate, then plurality rule –and most scoring rules— cannot always lead to good outcomes, even in large societies.

Especially, in divided majority frameworks –initially proposed by Myerson and Weber (1993) to study coordination issues in private values settings— we know that approval voting outperforms plurality rule in small and large elections (Bouton et al., 2016; Bouton and Castanheira, 2012), under quite general assumptions regarding the information environment; and also that runoff voting does better than plurality rule in large elections. Martinelli (2000) shows that when sincere voting is enough to aggregate information well in large societies, then information will be well aggregated under runoff voting also in equilibrium. Tsakas and Xefteris (2019) establish that the runoff rule admits equilibria that aggregate information well in large societies even when sincere voting is not able to do so. For small (i.e. finite sized) societies, little is known regarding the comparative performance of runoff voting compared to plurality, when both common value and partisan voters co-exist, and the subsequent analysis will start by trying to shed some light on this issue.

Finally, as far as experimental literature is concerned, Bouton et al. (2019) is the unique other paper that studies the comparative performance of the plurality and the runoff rule in a strategic voting context.[1] In contrast to our paper, Bouton et al. (2019) focus on private values settings –which, as we argue next, generate substantially different coordination incentives compared to common value ones– and show that voters exhibit very similar behavior under both systems: Duverger's conjecture does not seem to be confirmed (i.e. the effective number of parties is similar across systems). Our work complements their analysis by demonstrating that the validity of Duverger's prediction can be restored when voters seek to aggregate information rather than conflicting preferences. In fact, under the runoff rule when the majority voters have common values, it might very well be the case that, not only they do not benefit from coordinating blindly behind a majority candidate, but –perhaps surprisingly– they have an interest to further *discoordinate* (i.e. to split more evenly between the two majority candidates than under sincere voting). This never happens with private values. As we find, it is a behavior often adopted by subjects in common value elections, and the one that is responsible for the more fragmented vote (i.e. larger effective number of parties) under the runoff rule. In a way, a combined reading of these papers hints that the stylized fact that runoff systems are associated with a larger number of candidates than plurality systems might be –at least, partially– due to the fact that voters use elections also to aggregate information.

3 Theoretical Analysis

To keep the formal analysis as brief as possible, we consider a model that is compatible with Bouton et al. (2016). This allows us to focus solely on the runoff rule since

[1]Other experimental papers that work on the comparative performance of electoral rules in the laboratory include Bouton et al. (2016), Bol et al. (2016), Kamm (2017), Miller et al. (2018), and Bol et al. (2019).

results regarding the plurality rule are readily available. We have a society composed of a fixed number of voters, given by $N = \{1, \ldots, n\}$. These voters are split into two groups: $m > n/2$ of them are the independent/common-value majority; and $n - m > m/2$ voters compose the partisan minority. The set of alternatives is $\{A, B, C\}$. The partisans enjoy utility equal to one if C wins, and zero otherwise. For simplicity we consider that partisan voters are parametric (they always vote for C whenever C participates in an election, and abstain otherwise). As it is easy to check this is the unique undominated behavior of such voters under both electoral rules that we will consider. Hence, discarding them from the set of instrumental agents comes without any loss of generality and allows us to characterize equilibria without unnecessary notational complexities.

This leaves us with a game among the m independent voters. We assume for simplicity that m is odd and that these players get utility $H > 0$ if X from $\{A, B\}$ wins and X is the correct alternative, utility L s.t. $H > L > 0$ if X from $\{A, B\}$ wins and X is not the correct alternative, and zero otherwise.

The correct alternative is determined as follows: before voters select an alternative, nature moves and a state of the world is drawn from the set $\{a, b\}$. Each state is drawn with the same probability.[2] Alternative A is the correct one if the state is a and alternative B is the correct one if the state is b. Alternatives A and B are the majority alternatives.

After the state is drawn, each voter receives an i.i.d. signal from the set $\{s_a, s_b\}$, according to the following information structure: If the state of the world is a, then

[2] Most of our results qualify to the more general case in which priors are not even. The only difference is that when priors are too asymmetric superiority of the runoff rule compared to the plurality rule can only be established in weak –and no longer in strict– terms as coordination to the ex-ante most likely state of the world becomes, trivially, quite profitable. For more details regarding the effect of having uneven priors one is referred to Kawamura and Vlaseros (2017), and Mengel and Rivas (2017).

s_a is drawn with probability q_a and s_b with probability $1 - q_a$; and if the state of the world is b, then s_a is drawn with probability q_b and s_b with probability $1 - q_b$, with q_a different than q_b.

We consider two voting rules: plurality and runoff voting. Under plurality, voters, after privately observing their signals, simultaneously cast a vote for an alternative; and the most voted alternative is proclaimed winner. Similarly, under runoff voting, voters, after privately observing their signals, simultaneously cast a vote for an alternative. In case there is an alternative that collects an absolute majority of votes it wins the election. In every other case, the two most voted alternatives advance to the second round. In the second round, voters vote between these two alternatives and the alternative that is voted by most of them is proclaimed winner. All ties break with equiprobable draws.

Since partisans will vote for C in the first round and given that they are more than half of the independent ones, C always advances to the second round, if there is one. Therefore, in the second round, all independent voters have a unique undominated action: to vote for the majority alternative that runs against C. We assume that all independent voters employ such undominated actions in the second round. The unambiguous behavior of all voters in the second stage, essentially, leaves us under both rules with a simultaneous game of incomplete information (players know their type –i.e. the signal that they got— but not the type of the rest of the players); and the obvious equilibrium notion is Bayes Nash Equilibrium with a special focus on undominated strategies.

3.1 Equilibria

We first notice that under both rules there are Duvergerian equilibria in undominated strategies. A Duvergerian equilibrium is such that all independent voters coordinate behind the same alternative. Such equilibria guarantee that one of the majority preferred alternatives, A and B, will be elected, but that valuable information is discarded.

Proposition 1 *There exist generic information environments (i.e. open sets of q_a and q_b) such that both the plurality rule and the runoff rule admit Duvergerian equilibria in undominated strategies.*

All proofs can be found in the Appendix.

The fact that both rules support some kind of Duvergerian coordination is known in the literature, especially, in settings with private values (see, e.g., Palfrey, 1988; Myerson and Weber, 1993; Bouton, 2013; Bouton and Gratton, 2015). What is perhaps less obvious is why we may have Duvergerian equilibria in undominated strategies. Consider that we are in a plurality rule election and that an independent voter believes that exactly $n-m$ independent voters will vote for A for both signal realizations and that the remaining $m-(n-m)-1$ will vote for B for both signal realizations. Then, this voter's unique best response is to vote for A independently of which signal she received. Hence, playing A and discarding the signal is an undominated strategy, and therefore, the Duvergerian equilibrium in which all independent voters vote for A is an equilibrium in undominated strategies. Similarly, consider that in the first round of a runoff election an independent voter believes that all other independent voters behave sincerely (they vote for A when they get signal s_a, and for B when they get signal s_b). If $q_a = 1$ (or relatively close to one) and $q_b = 1/2$, then, for any admissible n and m, voting for B is the voter's unique best response after both signals, which establishes that it is an undominated strategy. In fact it is easy to see that as long as the priors and the signal probabilities induce asymmetric posteriors (i.e. post receiving signal s_a the probability that A is the correct alternative is not identical to the probability of B

being the correct alternative upon reception of the signal s_b), then there exists at least one Duvergerian equilibrium in undominated strategies when the society is sufficiently large.

The above result establishes that in most situations Duvergerian outcomes are reasonable predictions of this strategic voting model under both voting procedures. Importantly, if voters coordinate in such equilibria, then information is not aggregated and the welfare of the independent voters is identical –and relatively low— under both rules.

We now turn attention to informative equilibria: equilibria in which independent voters condition their behavior on their private signals.

Proposition 2 *Under both the plurality rule and the runoff rule, there always exist informative equilibria in undominated strategies.*

It is noteworthy that informative equilibria do not always involve sincere voting. In fact, under the same generic condition under which an undominated Duvergerian equilibrium exists, we also have that sincere voting is not an equilibrium under runoff voting if the society is sufficiently large.

The part of this proposition that refers to the plurality rule directly follows from Bouton et al. (2016). The part that concerns the runoff rule, is actually a corollary of the upcoming –and, at first sight, independent— proposition. Overall, it guarantees that under both rules information can be taken into account when deciding, while not always leading to better outcomes compared to Duvergerian –and, hence, uninformative— play. Especially, under plurality rule, informative play presents a very obvious trade–off: when independent voters condition their vote on their information, they have higher chances of giving larger support to the correct alternative rather than to the incorrect one, but they also increase the probability that they divide between the two and the undesired Condorcet loser alternative, C, wins the election. Under runoff voting there are significantly less dangers in this respect: even if independent voters split in the first round, then they can unite in the second round and make sure that the least preferred alternative, C, does not win the election. Therefore, runoff voting should admit better equilibria than the best equilibrium of the plurality rule. The next proposition establishes this intuitive result –to our knowledge, for the first time in the literature— for a society of any fixed size.

Proposition 3 *The runoff rule admits an informative equilibrium that gives to the independent voters a strictly higher expected utility than any equilibrium of the plurality rule.*

This result describes the main advantage of the runoff rule compared to the plurality rule in the present context: independent voters can always do strictly better under the runoff rule than under the plurality rule, and this is true for any admissible parametrization both with respect to the information environment and as far as the size of the electorate is concerned.

Notice though that the advantage of the runoff rule over plurality is not unambiguous. Independent voters need to coordinate to this "good" equilibrium in order to reap the benefits of the two stage rule, and this might not always be easy. Indeed, as we saw in Propositions 1 and 2, there are many cases in which the runoff rule admits undominated Duvergerian equilibria that deliver strictly lower payoffs to the independent voters compared to the best informative equilibrium of the plurality rule. In such situations –which are the canon and not the exception— if independent voters coordinate to an undominated Duvergerian equilibrium under the runoff rule, and to the best informative equilibrium under the plurality rule, then they get a strictly lower utility under runoff voting than under plurality rule!

Consider for instance the example of $q_a = 1$ and $q_b = 1/2$ that we used above. Given this information structure, always voting for B is, under the runoff rule, not only an undominated strategy, but actually *the unique strategy that is a best response to every symmetric pure strategy profile of the other independent voters*. Hence, the Duvergerian equilibrium in which all independent voters vote for B is a theoretically strong prediction under runoff voting, and sincere voting is not even an equilibrium. In terms of payoffs, this Duvergerian equilibrium delivers, in expected terms, (H+L)/2 to each independent voter. Interestingly, with this parametrization, sincere voting is an equilibrium under plurality rule and, for sufficiently low values of L, it delivers a payoff strictly larger than (H+L)/2. Of course, by Proposition 3 the runoff rule admits an equilibrium (in mixed or asymmetric strategies) that is superior in terms of payoffs to the Duvergerian one. But will the voters coordinate to it?

Given that the standard equilibrium refinement used in similar settings (equilibria in undominated strategies) is not sufficient to provide an answer, we turn to the laboratory. We test the comparative performance of these two popular voting procedures in situations in which the theoretical advantages of the runoff voting are less clear, in order to investigate whether the theoretical ambiguity is carried on also in settings of empirical interest or not.

4 The Experiment

4.1 Experimental Design

The experiment took place at the Laboratory for Experimental Economics at the University of Cyprus (UCY LExEcon). A total of 120 subjects were recruited in 8 sessions, with 15 subjects in each session. We conducted 4 different treatments, with 2 sessions per treatment. Average total payment was approximately 13.8 euros and each session lasted about 90 minutes. The experiment lasted for 100 periods, prior to which there were 3 practice periods that aimed at helping the subjects familiarize with the experimental environment. The experiment was designed on z-Tree (Fischbacher, 2007). Final earnings were determined by the sum of the subject's payoffs in 10 randomly selected periods out of the 100. The conversion rate used in the experiment was 1 euro for every 40 points.

At the beginning of each session, the subjects were placed in groups of three. The groups remained unchanged throughout the experiment. All subjects were instrumental voters in all periods and shared a common payoff function. Two artificial subjects were added to them to play the roles of partisan voters, whose behavior was parametric and thus was played directly by the computer. In our model's notation that would be $n = 5$ and $m = 3$. Then, the five members of each group participated in 100 identical voting processes. Note that, the stability of groups is desired in this setup, as we would like to see the extent to which group members can achieve coordination to jointly profitable outcomes.

Prior to the voting stage, each subject had access to a piece of private information regarding an unknown state of nature. Namely, there were two jars, one Blue and one Red, as the ones appearing in Figure 1. Red jar contained two red balls, whereas Blue jar contained one blue and one red ball.

At the beginning of each period, one jar would be selected randomly for each group, with the color of the jar representing the state of nature. Each jar was selected with the same probability and draws were independent across periods. The subjects could not observe the color of the selected jar, but they could observe the color of one ball selected

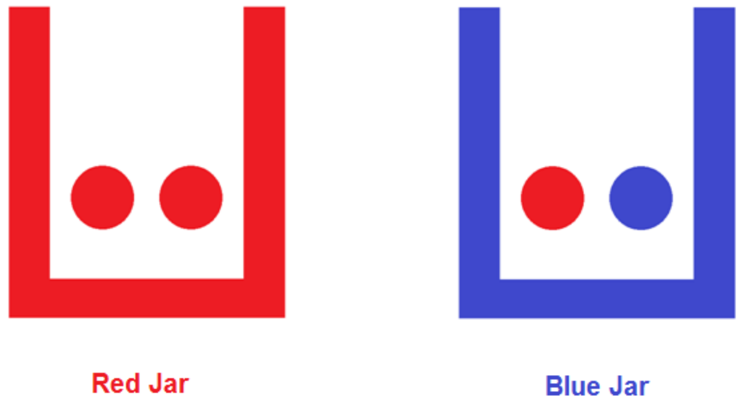


Figure 1: The two jars

uniformly at random from the selected jar. The draws of the balls were independent across subjects and across periods. Therefore, conditional on the selected jar being Red, each subject observed a red ball with probability 1, whereas conditional on the selected jar being Blue, each subject observed a blue or a red ball with probability 1/2 each (i.e. the selected parametrization corresponds to the example used in the previous section).

After the subjects observe the color of their balls, a voting process begins via which the group selects one of three possible colors: Red, Blue or Black. Depending on the treatment, the voting process would either follow plurality rule or runoff rule.

More specifically, under plurality rule, each of the three human subjects had to cast a vote in favor of either Red, Blue or Black color. In addition to these, the computer was adding the two votes of the partisan voters in favor of the Black color. The decision of the group then would be the color that would get the highest number of votes. If two colors were receiving the same number of votes, then one of these two was chosen uniformly at random.

Under runoff rule, voting could potentially consist of two rounds: The first round was the same as in plurality voting, with each human subject selecting one of the three

		Group's Decision		
		Red	Blue	**Black**
Jar's Color	Red	**100**	$\mathbf{L} \in \{\mathbf{10}, \mathbf{70}\}$	**0**
	Blue	$\mathbf{L} \in \{\mathbf{10}, \mathbf{70}\}$	**100**	**0**

Table 1: The payoffs of instrumental voters conditional on the group's decision and the color of the jar. The value of L depended on the treatment.

colors and the computer casting two votes in favor of Black. Now, if one color managed to get absolute majority (at least three votes) in the first round, then this color was selected. If no color could get the majority, then the process was moving to the second round. In the second round, the subjects could vote only in favor of one of the two colors that received the higher number of votes in the first part. By construction, if the second round was reached, then one of the two colors was always Black, hence the two votes of the partisan voters would be cast again in favor of Black. Whichever color would be the one to receive the majority of the votes in the second round would be selected.

For both voting rules, the common payoffs of the three subjects depended on the decision of the group in relation to the state of nature, as presented in Table 1. Namely, if the group's decision matched the color of the jar each subject would receive $H = 100$ points, if the group's decision was the Blue or the Red color, but did not match the color of the jar then each subject would receive $L \in \{10, 70\}$ point (depending on the treatment) and finally, if the group's decision was the Black color then subjects would receive 0 points.

Therefore, overall we conducted four different treatments, by varying the voting rule (runoff versus plurality) and the value of L (10 versus 70). The variation in the latter parameter changes the order of payoffs between the Duvergerian and the sincere equilibrium in elections under plurality rule. The subjects of each session participated in a single treatment.

Treatment	Voting Process	(H, L)	Sessions	Subjects	Groups
P10	Plurality	(100,10)	2	30	10
P70	Plurality	(100,70)	2	30	10
R10	Runoff	(100,10)	2	30	10
R70	Runoff	(100,70)	2	30	10

Table 2: The four treatments, changing the voting rule and the value of L.

4.2 Theoretical arguments based on parameter values

In this part we present some relevant theoretical arguments that hold for the particular parameter values we used in our experiment. Table 3 contains several strategy profiles that are of interest, some of which are equilibria, whereas others are not.

Under plurality rule, both Duvergerian strategy profiles and sincere voting are equilibria in both treatments. In fact, sincere equilibrium for $L = 10$ and Duvergerian equilibria for $L = 70$ can be shown to yield the maximum payoff among all strategy profiles. There is no other equilibrium in pure strategies. We have identified some other equilibria in mixed strategies (not necessarily all), which however yield inferior payoffs to instrumental voters. This suggests that trying to coordinate towards some more sophisticated profile might actually be harmful for the voters.

Under runoff rule, both Duvergerian equilibria are still sustained, but sincere voting is no longer an equilibrium in either of the treatments. This is a targeted feature of our experimental setup, which is due to the asymmetric nature of the signal structure we have used. In fact, voting always in favor of Blue (irrespective of the observed signal) is a best–response towards any symmetric pure strategy profile of the other two voters, which makes the Duvergerian equilibrium in which everyone votes always in favor of Blue a strong candidate to be observed. Interestingly, sincere voting is not an equilibrium, despite yielding always a higher payoff than either Duvergerian equilibrium. There are three more equilibria, two that involve mixed strategies for either two or three voters and one in pure strategies, all of which are more efficient than both sincere voting and the Duvergerian equilibria. The equilibrium in pure strategies is asymmetric and involves one voter voting always in favor of Blue and the other two voters voting sincerely. The advantage of this strategy profile is that it reduces the number of draws of blue balls that leads to the selection of Blue when the chosen jar is Blue to 1/2 instead of 2/3, without affecting the probability of selecting the correct color when the jar is Red (as both sincere voters will vote in favor of Red). Apparently, this advantage no longer works with plurality rule, as it increases the likelihood of Black's election, without compensating enough.

The equilibrium analysis verifies the main theoretical observations of Section 2 and will provide useful intuition for the analysis of the experimental data that follows. More specifically, we have verified the following: Duvergerian equilibria are sustained in all four treatments and are in undominated strategies (Proposition 1), the efficient asymmetric pure strategy equilibrium for the runoff rule and the sincere voting equilibrium for the plurality rule are obviously informative and in undominated strategies (Proposition 2), and the efficient asymmetric pure strategy equilibrium of the runoff rule yields a higher payoff than any equilibrium of plurality rule (Proposition 3).

In addition to these, we have three more observations that make the experimental analysis empirically relevant: (i) For $L = 10$, the most efficient equilibrium of the plurality rule (sincere) yields a higher payoff than the least efficient equilibrium of runoff (Duvergerian). For $L = 70$ the most efficient of plurality coincides with the least efficient of runoff (Duvergerian in both), thus yielding equal payoffs. Therefore, although runoff rule seems in general to outperform plurality rule, it remains ambiguous whether the voters would manage to coordinate in profiles strategy profiles that would yield higher payoffs under runoff rule compared to plurality. (ii) Sincere voting is not

Voting Process	Strategy Profile	Payoff ($L = 10$)	Payoff ($L = 70$)	Equilibrium
Runoff	Duvergerian	55	85	YES
Runoff	Sincere	77.5	92.5	NO
Runoff	Symmetric Mixed	79.7959	93.2653	YES
Runoff	Asymmetric Mixed	80.3125	93.4375	YES
Runoff	1B, 2 sincere	88.75	96.25	YES
Plurality	Duvergerian	55	85	YES
Plurality	Sincere	67.1875	76.5625	YES
Plurality	Fully Mixed	–	50.9252	YES
Plurality	Symmetric Mixed	47.81	61.0629	YES

Table 3: Strategy profiles and payoffs for the parameter values and voting processes used in the experiment. Runoff: (i) Symmetric Mixed: $P(R|r) = 6/7$ and $P(B|b) = 1$ (ii) Asymmetric Mixed: One player votes sincerely, the other two mix $P(R|r) = 3/4$ and $P(B|b) = 1$. No other equilibria except those presented. Plurality: (i) Fully Mixed: $P(R|r) \approx 0.37$ and $P(B|b) \approx 0.11$. Exists only for $(H, L) = (100, 70)$. It is the unique fully mixed equilibrium, even when allowing for asymmetric strategy profiles. (ii) Symmetric Mixed: For $(H, L) = (100, 10)$, $P(R|r) \approx 0.34$ and $P(B|b) = 1$ and for $(H, L) = (100, 70)$, $P(R|r) \approx 0.54$ and $P(B|b) = 1$. Each of these is the unique equilibrium in which all players mix when observing r, even when allowing for asymmetric strategy profiles. Other equilibria may exist, yet it can be shown that sincere in $(H, L) = (100, 10)$ and Duvergerian in $(H, L) = (100, 70)$ are the strategy profiles (not just equilibria) that yield the highest payoffs to the voters.

an equilibrium in either treatment of the runoff rule. In fact, all efficient equilibria involve either/both mixed or/and asymmetric strategy profiles, which are less intuitive and coordination to them seems far from trivial in an anonymous setting. (iii) Finally, the relative efficiency of sincere and Duvergerian equilibria in plurality is reversed from one treatment to the other, i.e. sincere is more efficient for $L = 10$, Duvergerian is more efficient for $L = 70$.

4.3 Testable Hypotheses

We are now ready to state our main empirical hypotheses. The major empirical question is the relative efficiency of the two voting rules.

Hypothesis 1 *For each* $(H, L) \in \{(100, 10), (100, 70)\}$*, individual profits are higher under runoff rule than under plurality rule.*

Recall that, the motivation for the experimental investigation of the runoff rule was that sincere voting might not constitute an equilibrium, despite this being rather efficient. However, we have shown the existence of equilibria that might be less intuitive, yet they are even more efficient than sincere voting. Hence, another plausible question regarding runoff rule is whether voters manage to spot this efficient behavior and outperform both sincere voting and Duvergerian outcomes.

On the contrary, under plurality rule, either Duvergerian or sincere voting equilibria are efficient, depending on the treatment. But, it is unclear whether experimental subjects would actually grasp the difference in incentives among the two treatments that involved plurality rule.

Hypothesis 2

1. *Under runoff rule, for each* $(H, L) \in \{(100, 10), (100, 70)\}$*, individual profits are higher than what sincere voting or Duvergerian equilibria would predict.*

2. *Under plurality rule, for* $(H, L) = (100, 10)$*, individual profits are higher than what Duvergerian equilibria would predict.*

3. *Under plurality rule, for* $(H, L) = (100, 70)$*, individual profits are higher than what sincere voting would predict.*

Another prevalent difference of the two rules is that under runoff rule, voters do not face a significant threat of Black being selected (unless, of cource, some instrumental voter chooses the dominated strategy of voting in favor of Black).

Therefore, the need to coordinate their votes towards a common candidate already from the first round of the election is not really important, which in turn suggests that the votes are distributed over a larger number of parties (colors). We measure the effective number of parties through the Laakso and Taagepera (1979) index: i.e. as $1/\sum_i s_i^2$ where s_i is the share of votes cast in favor of color $i \in \{red, blue, black\}$. The more evenly distributed the votes are across colors, the higher the value of the index and analogously, the more concentrated the votes towards a single color, the lower the value of the index.

Hypothesis 3 *For each* $(H, L) \in \{(100, 10), (100, 70)\}$*, the effective number of parties is higher under runoff rule than under plurality rule.*

Our last main hypothesis deals with the effect of experience on performance. Namely, given the existence of multiple equilibria in all treatments and the fact that groups are fixed throughout the experiment, it is natural to expect coordination to improve over time and hence profits from strategic voting to increase.

Hypothesis 4 *Gains from strategic voting improve over periods for all treatments.*

Notice though that considering simply the evolution of profits over periods might be misleading, because profits in each period are strongly affected by the realizations of different draws. Thus, even if the voters of a group manage to coordinate in the best way possible, their profits might not reflect this, due to unfavorable draws. Hence, we believe it is perhaps more informative to consider actual profits in comparison to the profits that sincere voting would yield –which we think of as a natural focal point– and therefore we will measure gains as the difference between those two quantities.

5 Experimental Results

5.1 Preliminary Results

We start by presenting some descriptive statistics on the behavior of the experimental subjects. Table 4 contains the frequencies of different outcomes for each treatment. Elections of Black in treatments R10 and R70 could occur only in periods in which some subject voted in favor of Black, whereas under plurality rule it could be also the outcome of a draw in case of a tie between two colors.

Decisions were much more accurate when the jar was Red (as it can be seen in Table 5), with the effect being much stronger in P10 and P70. This can be explained by looking at the aggregate voting behavior conditional on the observed signal, where it seems that subjects tended to vote more sincerely under plurality rule compared to runoff (see Table 6), which in turn favored correct decisions when the jar was Red, as all voters received the same signal, which also matched the color of the jar.

Treatment	Correct	Incorrect	Black	Ties
P10	537	148	315	497
P70	568	182	250	447
R10	782	198	20	659
R70	786	208	6	678

Table 4: Distribution of outcomes per treatment, out of 1000 observations (10 groups, 100 periods). Correct: Selected color matches the state. Incorrect: Selected color does not match the state, but is either Blue or Red. Black: Selected color is Black.

Treatment	Red Jar (n=509)				Blue Jar (n=491)			
	Correct	Incorrect	Black	Ties	Correct	Incorrect	Black	Ties
P10	393	18	98	157	144	130	217	340
P70	445	4	60	100	123	178	190	347
R10	461	46	2	292	321	152	18	367
R70	462	43	4	316	324	165	2	362

Table 5: Distribution of outcomes per treatment conditional on the color of the jar, out of 1000 observations in total. Correct: Selected color matches the state. Incorrect: Selected color does not match the state, but is either Blue or Red. Black: Selected color is Black.

Nevertheless, a significant proportion of the votes (10-20% depending on the treatment) were not sincere, with the observation being particularly predominant in elections under runoff rule and conditional on a subject having observed a red ball (see Table 6 for the first round and Table 7 for the second one). In fact, the relative frequency of not sincere behavior in runoff is close to the predictions of the equilibria that are more efficient than sincere voting and in particular the mixed equilibria, both of which consider either two or all three players to vote sincerely when observing a blue ball (as observed in the data) and occasionally vote insincerely when observing a red ball (recall Table 3). This is a first indication that subjects indeed grasped the opportunity of additional gains from coordinating in a more sophisticated way compared to sincere voting.

Under plurality rule, intuition is less clear, as there is a substantial number of observations in which voters voted insincerely despite observing a blue ball, thus despite knowing with certainty that the color of the jar was Blue. This observation is more prevalent in P70, where we also observe a higher frequency of sincere voting from subjects who obseved red balls. This could be an indication of either a higher inclination of voters towards the Duvergerian equilibrium where all vote for Red in P70, or towards the Duvergerian equilibrium where all vote for Blue in P10. Answering this question requires a more careful analysis than just looking on aggregate values.

Regarding P10, it is also worth commenting on the much higher frequency of votes in favor of Black compared to the other treatments. It turns out that the vast majority of these votes came from a single subject who voted in favor of Black 33 times in 100 periods. We have not excluded this group from our analysis, hence all results are presented considering all groups, but we have also repeated all the tests excluding it and our qualitative results remain the same.

		$(L = 10)$			$(L = 70)$		
		red	blue	black	red	blue	black
Plurality	r	1938	300	25	2092	167	4
	b	74	648	15	139	595	3
Runoff	r	1764	498	1	1728	534	1
	b	12	724	1	23	714	0

Table 6: Distribution of voting decisions during the first voting round conditional on observed signal, aggregated over all groups of the same treatment. Each box contains 3000 observations (3 subjects, 10 groups, 100 periods) in total.

Finally, Table 8 contains the average per period profit for each treatment, in comparison to what the voters would obtain in each treatment by voting sincerely, coordinating to one of the Duvergerian equilibria or coordinating to the ex–ante optimal theoretical equilibrium. At a first glance, it is striking the difference in profits across treatments

		red	black	blue	black
($L = 10$)	r	1032	4	437	2
	b	96	8	394	4
($L = 70$)	r	1111	2	431	3
	b	111	0	376	0

Table 7: Distributions of voting decisions during the second voting round conditional on observed signal, aggregated over all groups of each treatment of runoff rule.

Treatment	Observed	Duverger (Red)	Duverger (Blue)	Sincere	Best
P10	55.18	55.81	54.19	67.14	67.14
P70	69.54	85.27	84.73	75.78	85.27
R10	80.18	55.81	54.19	78.94	98.65
R70	93.16	85.27	84.73	92.98	96.55

Table 8: Average per period profits for each treatment. Profits of Duverger (Red/Blue), Sincere and Best are calculated based on the realizations of jars, balls and draws in the experiment, thus are slightly different from the theoretical expected payoffs calculated before. "Best" corresponds to the best theoretical equilibrium in each treatment, which is Sincere for P10, Duverger (Red) for P70 and the asymmetric in which one voter votes always in favor of Blue and the other two vote sincerely in R10 and R70.

with different electoral rules, as well as that in both treatments of plurality rule profits are way lower than those of sincere voting and in treatments of runoff rule are slightly higher than those of sincere voting.

5.2 Main Results

We can now present the main experimental results, which are related to the comparison of the two electoral rules and the observed efficiency of each of them separately. Recall that the groups were fixed throughout the experiment, which means that for each treatment we have ten fully independent data points. It is also apparent that observations within each group will exert some dependency between them. For this reason, unless otherwise stated, all our tests are performed at a group level, by pooling together all observations of the said group (in ways that are specified in each case). Moreover, we used the same draws for one group of each treatment, which allows us to consider our observations as matched and use the respective tests as well.

Result 1 *For each* $(H, L) \in \{(100, 10), (100, 70)\}$ *profits are higher under runoff rule than under plurality rule.*

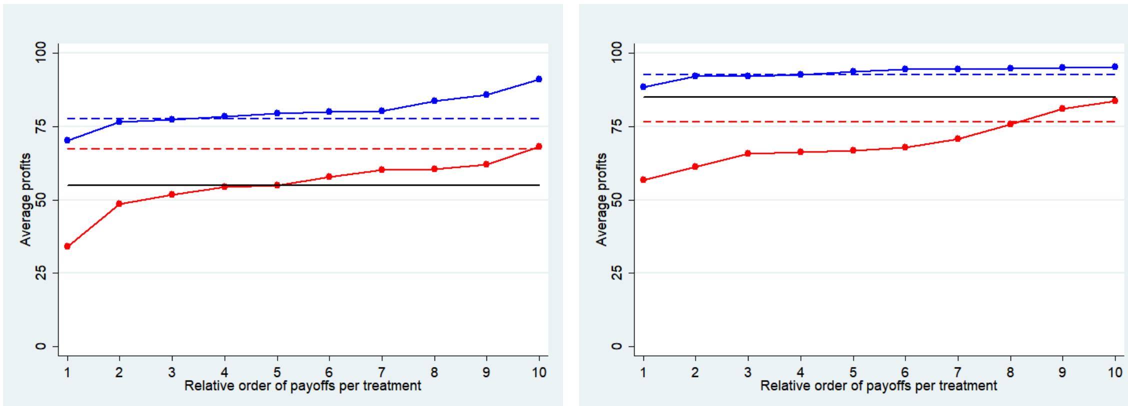
This is the first and most fundamental result of the paper and we find very strong evidence in favor of it. Our main comparison is at a group level, where we consider the average per period profit a voter receives over the course of the 100 periods.[3]

As presented in Figure 2, it is striking that even the best–performing group of P10 (P70 respectively) achieved a lower average profit than the worst–performing group of R10 (R70 respectively). All differences are strongly statistically significant according to several tests.[4]

The result could be predicted by observing the aggregate choice distribution in Table 6. Recall that the ambiguity regarding the relative outcome was due to the possibility of behavior coordinating towards Duvergerian equilibria in runoff rule, which does not seem to occur often. In fact, aggregate behavior, as well as aggregate profits, suggest a behavior closer to the one predicted by the mixed equilibria, which are quite efficient for both treatments of the runoff rule. Managing to reach such efficient levels of coordination under runoff rule is then expected to guarantee higher profits.

[3]Apparently, the results are the same if we consider individual profits. For each treatment, the lowest average profit a subject made under runoff rule was higher than the highest average profit a subject made in the respective treatment under plurality rule. As an additional robustness check, we considered a hypothetical scenario for plurality rule, in which Black lost all draws when there was a tie, keeping everything else unchanged. Even in this extremely favorable for plurality rule scenario, runoff rule yields higher average profits, which are statistically significant for $L = 10$ ($p < 0.01$ in all relevant tests for $L = 10$, $p \approx 0.15$ for $L = 70$).

[4]($L = 10$) Wilcoxon: $z = -2.803$, $p = 0.0051$, Sign Test: $p = 0.001$, T-test (matched): $t = -8.6787$, $p = 0.0000$, Mann-Whitney: $z = -3.780$, $p = 0.0002$, T-test (unmatched): $t = -7.3038$, $p = 0.000$. ($L = 70$) Wilcoxon: $z = -2.805$, $p = 0.005$, Sign Test: $p = 0.001$, T-test (matched): $t = -8.3921$, $p = 0.000$, Mann-Whitney: $z = -3.781$, $p = 0.0002$, T-test (unmatched): $t = -8.6171$, $p = 0.000$.



(a) Treatments R10 and P10 ($L = 10$)

(b) Treatments R70 and P70 ($L = 70$)

Figure 2: Average per period profits of each group in increasing order. Solid blue (resp. red) curve corresponds to profits under runoff (resp. plurality) rule. Dashed blue (resp. red) line corresponds to the expected profits with sincere voting under runoff (resp. plurality) rule. Solid black line corresponds to expected profit of Duvergerian equilibrium.

The next set of results is related to the comparison of observed profits with those predicted by Duvergerian equilibria and sincere voting, as described in Hypothesis 2. We use again average per period profits aggregated over the course of 100 periods for each group. For Duvergerian equilibria and sincere voting we use the actual profits that these strategy profiles would yield given the realizations of our experiment. A quick view of aggregate profits that will be compared to each other has already been presented in Table 8. We find mixed evidence in terms of support to our Hypothesis.

Result 2

1. *Under plurality rule, for* $(H, L) = (100, 70)$ *profits are lower than those of either Duvergerian equilibrium, whereas for* $(H, L) = (100, 10)$ *profits are not significantly different than those of either Duvergerian equilibrium.*

2. *Under plurality rule, for each* $(H, L) \in \{(100, 10), (100, 70)\}$ *profits are lower than those of sincere voting.*

3. *Under runoff rule, for each* $(H, L) \in \{(100, 10), (100, 70)\}$ *profits are higher than those of either Duvergerian equilibrium.*

4. *Under runoff rule, for each* $(H, L) \in \{(100, 10), (100, 70)\}$ *profits are not significantly different than those of sincere voting.*

5. *Under runoff rule, for* $(H, L) = (100, 10)$ *profits are higher than those of sincere voting when considering only the last 50 periods.*

The results of all tests related to Result 2 are presented in Table 9.

Under runoff rule subjects outperform Duvergerian equilibria in both treatments, which is expected given that they successfully resist from coordinating to those sub-optimal equilibria. Moreover, they perform slightly better than sincere voting, yet the difference is not statistically significant. In fact, for $L = 10$ they perform significantly better during the last 50 periods.[5] The result is not really unexpected, as the payoff difference between sincere voting and the more efficient (but less intuitive) equilibria is much smaller in R70 compared to R10, which means that observing significant differences would require players to coordinate very efficiently towards some not really intuitive equilibria. Even for R10 this would be rather challenging, but as it appears after a certain learning period the majority of groups manage to achieve this high level of coordination to efficient outcomes.

Under plurality rule, subjects underperform compared to sincere voting in both treatments, despite this being an equilibrium in these cases. This suggests that subjects erroneously intend to coordinate to some strategy profile more sophisticated and

[5]For the last 50 periods, we have also compared observed profits with those of sincere voting at the period level. All previous results carry through and we get additionally a significant difference at the 10% level for R70.

	Duverger (Red)			Duverger (Blue)		
	T–test	Wilcoxon	Sign–test	T–test	Wilcoxon	Sign–test
P10	-0.2237 (.4140)	0.459 (.6465)	(.3770)	0.2976 (.6136)	0.357 (.7213)	(.6230)
P70	-6.5480 (.0001)	-2.803 (.0051)	(.0010)	-5.1783 (.0003)	-2.701 (.0069)	(.0107)
R10	18.7696 (.0000)	2.803 (.0051)	(.0010)	10.1746 (.0000)	2.803 (.0051)	(.0010)
R70	10.7271 (.0000)	2.805 (.0050)	(.0010)	11.5818 (.0000)	2.812 (.0049)	(.0010)
	Sincere			Sincere (> 50 periods)		
	T–test	Wilcoxon	Sign–test	T–test	Wilcoxon	Sign–test
P10	-4.1037 (.0013)	-2.803 (.0051)	(.0010)	-3.3217 (.0045)	-2.757 (.0058)	(.0020)
P70	-3.3147 (.0045)	-2.497 (.0077)	(.0020)	-1.8542 (.0484)	-1.939 (.0525)	(.0195)
R10	0.7111 (.2475)	0.459 (.6465)	(.6230)	3.0283 (.0071)	2.129 (.0333)	(.0625)
R70	0.2660 (.3981)	0.103 (.9182)	(.6367)	1.4000 (.0975)	1.277 (.2014)	(.2539)

Table 9: Profits refer to the average per period profits aggregated over the course of 100 periods for each group. Observed profits are compared to the profits that either of the Duvergerian equilibria or sincere voting would yield. For sincere voting we repeat the same test considering only the last 50 periods respectively. The number reported under T-test is the value of t and for the Wilcoxon signed-rank test is the value of z, while p-values are reported in parentheses. For the T-test and the Sign-test p-values correspond to the relevant one-sided test.

profitable than sincere voting, and this leads them to inferior outcomes. This pattern is not improved, even if one considers only the last 50 periods of the experiment for each group and yields naturally the question on what makes subjects so prone to look for more sophisticated and risky forms of coordination, when simple and intuitive solutions would perform sufficiently well. In particular, it is quite puzzling why a substantial number of times subjects who observed a blue ball voted in favor of Red in both treatments of plurality rule (see Table 6).

Result 3 *For each* $(H, L) \in \{(100, 10), (100, 70)\}$ *the effective number of parties is higher under runoff rule than under plurality rule.*

Recall that we measure the effective number of parties (ENP) as $1/\sum_i s_i^2$ where s_i is the share of votes cast in favor of color $i \in \{red, blue, black\}$.[6]. We provide

[6]The results are identical if we measure effective number of parties as $1 - \sum_i s_i^2$.

some graphical evidence in Figure 3, which contains the average values of ENP per treatment, with the 95% confidence intervals considering as unit of measure the average value of ENP for one group over the course of 100 periods. Using these averages as our independent observations, we also perform several tests that support the finding that the differences across treatments are statistically significant for both $L = 10$ and $L = 70$, with the effect being slighlty larger for $L = 70$.[7]

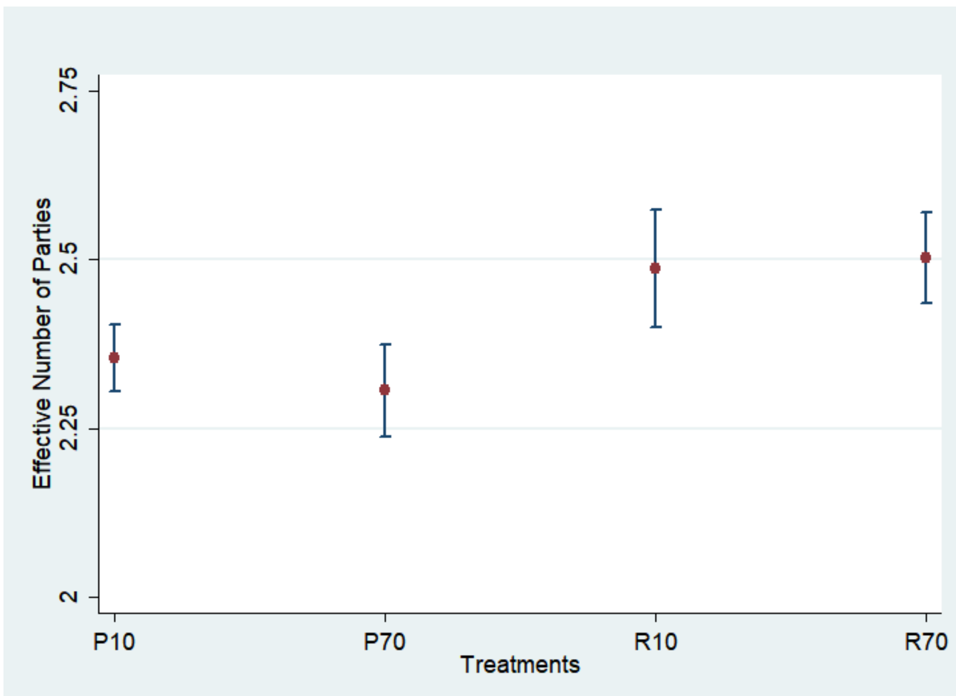


Figure 3: Average effective number of parties per treatment. As unit of measurement is considered the average effective number of parties over the 100 periods for each group.

Result 4 *Observed profits are increasing over periods relative to the profits sincere voting would yield.*

[7]($L = 10$) Wilcoxon: $z = -1.784$, $p = 0.0745$, Sign Test: $p = 0.0547$, T-test (matched): $t = -2.1750$, $p = 0.0288$, Mann-Whitney: $z = -2.496$, $p = 0.0126$, T-test (unmatched): $t = -2.5953$, $p = 0.0091$. ($L = 70$) Wilcoxon: $z = -2.701$, $p = 0.0069$, Sign Test: $p = 0.0107$, T-test (matched): $t = -4.4173$, $p = 0.0008$, Mann-Whitney: $z = -3.028$, $p = 0.0025$, T-test (unmatched): $t = -4.0328$, $p = 0.0004$.

Our last main result is related to the relative performance of observed behavior compared to sincere voting. Under runoff rule, there are equilibria that yield higher profits than sincere voting, but it is not straightforward that the subjects would be able to coordinate towards them. Similarly, under plurality rule, there are opportunities for higher profits in P70, but not in P10. Does coordination improve over time? To check that, we calculate the difference between observed profits and profits that sincere voting would yield and regress this on the experimental period, controlling for dependencies within each group. Then, we repeat the same analysis for each treatment separately. Regression results are presented in Table 10.

Overall, we observe that performance is indeed improving as the experiment progresses and this improvement is present in all treatments except in R70, in which however initial performance is already at the level of sincere voting. In P10 and P70 (where note that coefficients are significantly different than zero only at the 10% level), despite this improvement, profits remain below those sincere voting would yield, whereas in R10 they overpass it substantially. A visualization of the result is presented in Figure 4 in which we plot 10–period moving averages of relative profits, aggregated at treatment level. The trends are apparent in the figure.

6 Conclusion

In this paper we conducted the first experimental comparison of two popular voting rules, when a divided majority seeks to aggregate payoff relevant information. The particular setup was deliberately biased against the favorite (runoff rule) and in favor of the underdog (plurality) and, hence, the overall approach qualifies as an experimental stress test. Our analysis showed that the runoff rule is deservedly considered better than

	P10	P70	R10	R70	All
Period	0.0680*	0.0701*	0.1580***	0.0136	0.0774***
	(0.0346)	(0.0363)	(0.0480)	(0.0204)	(0.0193)
Runoff					6.420***
					(1.923)
Low L					-5.720
					(3.335)
Runoff × Low L					6.780
					(3.789)
Constant	-15.39***	-9.778***	-6.735	-0.505	-10.15***
	(3.485)	(1.786)	(3.052)	(1.565)	(1.834)
N	1000	1000	1000	1000	4000

Standard errors in parentheses, * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 10: Difference between observed profits and profits sincere voting would yield regressed on experimental period, with standard errors clustered at a group level. The first four columns show the results for each treatment separately. The last column includes all treatments and controls for election rule and value of L.

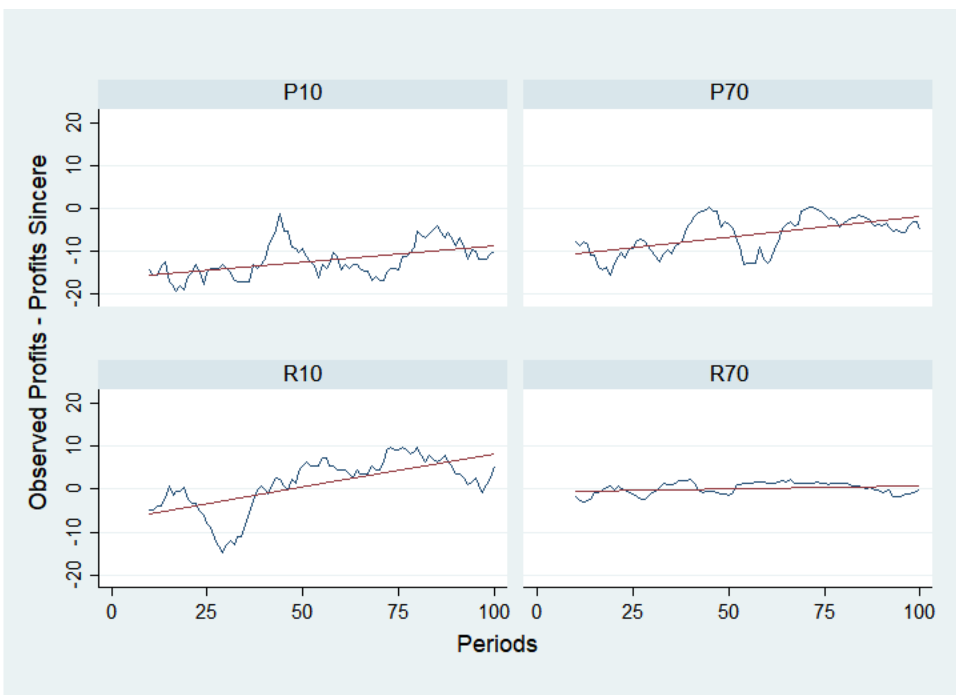


Figure 4: Ten–period moving average of the difference between observed profits and profits sincere voting would yield, aggregated at treatment level.

the plurality rule, since, even in these unfavorable scenarios, allowed majority voters to better coordinate and enjoy higher payoffs compared to plurality.

Also, our study provides the first experimental validation of Duverger's law and hypothesis in a setup with a common value majority. This finding is of arguably independent appeal since alternative studies in environments with private values do not find strong evidence in support of Duverger's predictions (Bouton et al., 2019). Naturally, the issue of whether runoff voting is superior to plurality in alternative setups needs to be further explored. Would the identified advantages carry over to cases in which we have multiple majority alternatives? Are the results robust to considering that majority voters are biased (i.e. have both common and private values)? Is runoff-voting better than plurality also when voters exhibit group-based reasoning (Feddersen and Sandroni, 2006)? Despite the fact that an answer to these questions is beyond the scope of the present analysis, it is evident that they constitute interesting and promising avenues for future research.

Appendix

Proof of Proposition 1: The argument supporting Proposition 1 was provided in text.

Proof of Proposition 2: We know that the plurality rule admits an informative equilibrium in undominated strategies by Bouton et al. (2016). By Proposition 3 (its proof is presented below) we know that the runoff rule admits an equilibrium that is strictly better than the best equilibrium of the plurality rule. Since, uninformative equilibria –equilibria in which independent players do not condition their actions on private information— are bound to deliver an expected utility to the independent voters that is weakly lower compared to the best Duvergerian equilibrium, and given that both voting rules trivially support the existence of both Duvergerian equilibria, it follows that the best equilibrium under the runoff rule cannot be an uninformative one. Hence, an informative equilibrium should always exist. Evidently, this equilibrium (or one of them, if there are many that deliver the same highest expected utility) is in undominated strategies since, if it weren't, the players who use weakly dominated strategies could trivially change them with undominated ones. This would not reduce the payoff of the independent voters and, hence, would still be an equilibrium.

Proof of Proposition 3: Since both these rules admit Duvergerian equilibria, it follows that equilibrium existence is not an issue. Assume that $(s_1, s_2, ..., s_n)$ is the best equilibrium under plurality (it maximizes the ex-ante expected utility of the independent voters). Since Duvergerian equilibria deliver strictly positive expected utility to the independent voters, the best equilibrium must involve the election of some majority alternative with strictly positive probability.

If the best equilibrium is such that that only one majority alternative –say, A— has a positive probability of being elected, then it must be the Duvergerian one that delivers A with certainty. This must be so because A with certainty is better for the independent voters compared to any proper lottery between A and C.

If both A and B win with positive probability, then it must be the case that strictly less than $n - m$ voters vote for A (B) with probability one. If at least $n - m$ voters vote for A with certainty, then B gets at most $n - 2(n - m)$ votes which are always strictly less than $n - m$ (because we have assumed that $n - m > m/2$), contradicting the assumption that B wins with positive probability. Since, neither A nor B are voted with certainty by more than $n - m$ voters each, it follows that C wins with positive probability. In particular, we have that C is the plurality–but not the majority– winner with strictly positive probability.

Now that we have pinned down the two possible kinds of strategy profiles that may constitute the best equilibrium under plurality rule, we are ready to argue why the runoff always admits a strictly better one. Since our runoff–voting game is formally defined as a simultaneous game among m common value players, the results of McLennan (1998) fully apply. Hence, any strategy profile that maximizes the expected utility of the independent voters should also constitute an equilibrium of the game. The compactness of the (mixed extension) of the strategy space guarantees that there is such a strategy profile. Therefore, if we show that there exists a strategy profile that, under the runoff rule, provides to the independent voters an expected utility that is strictly larger compared to the expected utility of the best equilibrium of the plurality rule, then we will know that the runoff rule also admits a "strictly better" equilibrium.

Indeed, if we are in the first case (i.e. if the best equilibrium of the plurality rule is a Duvergerian one), then under the runoff rule, it is possible to construct a strategy that assigns a strictly larger utility to the independent voters compared to Duvergerian coordination. Consider that $\frac{m-1}{2}$ independent voters vote for A for any signal realization, $\frac{m-1}{2}$ independent voters vote for B for any signal realization, and the remaining independent voter employs an informative strategy such that the ex-ante (i.e. before state and signal realization) probability that she votes for the correct alternative is strictly larger than $\frac{1}{2}$. Then, the expected utility of independent voters should be strictly larger compared to the utility that they would expect in any Duvergerian equilibrium. To see why such a strategy trivially exists observe that if $q_a > q_b$, then sincere voting is such a strategy; and if $q_a < q_b$, then voting for A upon observing signal s_b and voting for B upon observing signal s_a leads to the correct alternative more often than half of the times (i.e. the frequency of correct play in a Duvergerian equilibrium). Since, such strategy profiles also never result to C winning, we conclude that there exists an equilibrium that is strictly better for the independent voters compared to the Duvergerian ones.

If we are in the second case (i.e. if the best equilibrium of the plurality rule involves the election of C with positive probability), then independent voters can use the strategy profile of plurality's best equilibrium under the runoff rule, and enjoy a strictly larger expected utility: by using the same strategy profile they elect the same majority alternative when this alternative gets elected under plurality and a majority alternative in all eventualities in which C is the plurality but not the majority winner.

This concludes the argument, and we have that for every admissible parametrization, the runoff rule admits an equilibrium that is strictly better for the independent voters compared to the best equilibrium of the plurality rule. *QED*

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