University of Cyprus
Department of Economics

Working Paper 08-2016

The effect of entry on R&D networks

Emmanuel Petrakis and Nikolas Tsakas

Department of Economics, University of Cyprus, P.O. Box 20537, 1678 Nicosia, Cyprus
Tel.: +357-22893700, Fax: +357-22895028, Web site: http://www.ucy.ac.cy/econ/en

The Effect of Entry on R&D Networks *

Emmanuel Petrakis[1] and Nikolas Tsakas[2]

[1]Department of Economics, University of Crete

[2]Department of Economics, University of Cyprus

September 14, 2016

Abstract

We investigate the effect of potential entry on the formation and stability of R&D networks considering farsighted firms. We show that the presence of a potential entrant often alters the incentives of incumbent firms to establish an R&D link. In particular, incumbent firms may choose to form an otherwise undesirable R&D collaboration in order to deter the entry of a new firm. Moreover, an incumbent firm may refrain from establishing an otherwise desirable R&D collaboration, expecting to form a more profitable R&D link with the entrant. Finally, potential entry may lead an inefficient incumbent to exit the market. We also perform a welfare analysis and show that market and societal incentives are often misaligned.

Keywords: R&D Networks, Entry, Farsighted Stability

JEL Classification: D85, L24, O33

*We are indebted to Sanjeev Goyal, David Minarsch and José Luis Moraga–Gonzalez, as well as seminar participants at ASSET 2015, Oligo Workshop 2016, CRETE 2016 and GAMES 2016 for useful comments and suggestions. All remaining errors are our sole responsibility. Part of this project was carried out while Nikolas Tsakas was at SUTD–MIT International Design Center at Singapore University of Technology and Design supported by grant IDG31300110.

Emails: petrakis@uoc.gr (Emmanuel Petrakis), tsakas.nikolaos@ucy.ac.cy (Nikolas Tsakas)

1. Introduction

1.1. Motivation

R&D partnerships are the different modes of inter–firm collaborations in which several independent economic agents agree to share part of their R&D activities and outcomes (Hagedoorn, 2002). The collection of R&D partnerships between firms that operate in the same or in related industries forms an R&D network. R&D networks have attracted a lot of attention recently due to the substantial increase in the number of inter–firm collaborations. Such collaborations arise in most of the industries, but are more common in those industries where the technological development is relatively quick (König, 2013) and make the importance of R&D vital, as for example the pharmaceutical (Roijakkers and Hagedoorn, 2006), computer and military industries. In addition, there is strong evidence of substantial entry and exit rates in the R&D networks of several manufacturing sectors (Tomasello et.al, 2016).[1]

This paper investigates the effect of potential entry on the formation and stability of R&D networks, as well as on market outcomes and social welfare. A main question addressed is whether and how the presence of potential entrants alters the incentives of incumbent firms to form R&D collaborations. In this way we will get a better understanding of why certain network structures are more likely to be sustainable than others in the long-run. Moreover, we will be able to identify novel strategies used either by incumbent firms that seek to deter entry, or by potential entrants that seek to enforce accommodation. For incumbent firms, these strategies may be particularly crucial in industries with a small number of firms, where entry deterrence can secure for them larger market shares and profits. For entrant firms, these strategies may be particularly beneficial in highly asymmetric industries in which efficient incumbent firms may be willing to collaborate with entrants in order to marginalize or even to induce exit of their inefficient rivals.

It is apparent that the study of the effects of potential entry on the formation of R&D networks is of vast importance. It is a decisive stepping stone in the process of identifying those features which guide the investment and collaboration decisions of the firms. To the best of our knowledge, this is the first paper that considers how potential entry can shape the incentives for R&D collaborations between firms, leading them to establish links they would otherwise find unprofitable, as well as to prevent the establishment of collaborations that would otherwise be profitable.

[1]Using a large database of publicly announced R&D alliances, the paper investigates empirically the evolution of R&D networks and the process of alliance formation in several manufacturing sectors over the period 1986-2009 and identifies a rise-and-fall dynamics property of R&D networks, with a peak in the mid-nineties.

1.2. Setting and Results

We consider the simplest setup with potential entry. There are two incumbent firms with (possibly) different initial marginal production costs and a potential entrant. Each firm produces a brand of an horizontally differentiated good. Under R&D collaborations, each firm shares part of its R&D results with its collaborators, i.e., a link between two firms generates a given level of R&D spillovers. The two incumbent firms first decide whether to establish an R&D link and then the entrant decides whether to enter the industry or not. If the entrant stays out, the incumbent firms choose simultaneously their R&D efforts and outputs. If entry occurs, all the firms in the industry meet sequentially in a random order in pairs and decide whether to form R&D collaborations or not. An R&D link is established only if both firms agree on it. This generates a dynamic phase of discussions that ends either if a complete network has been formed, or if after a complete round of discussions no new R&D collaboration has been agreed upon. After the end of the R&D collaboration discussions phase, all firms decide simultaneously their R&D efforts and outputs.

We postulate that firms are *farsighted* when they decide to establish an R&D link between them. That is, during their discussions about the establishment of an R&D link, the firms are able to anticipate networks that may arise in the future as a consequence of their current decisions. Therefore, a firm will refrain from forming an R&D collaboration yielding a higher payoff under the current network, as long as it realizes that the establishment of such an R&D link will induce a new network in which it will obtain a lower payoff than that it will end up without the R&D link. This notion of farsightedness is an adaptation of the Dutta et.al (2005)'s model, which has been studied extensively in the literature and possesses several interesting properties.[2] The behavior of farsighted firms is in sharp contrast with what is implied by the static notion of pairwise stability. In the latter, firms are assumed to act myopically ignoring future modifications of the network structure due to their current decisions.[3]

Evidently, under potential entry, the static notion of pairwise stability is not adequate. We instead use a dynamic process of network formation in which each pair of firms is allowed to reconsider forming an R&D collaboration as long as a new R&D link has been established between another pair of firms. The only restriction we impose is that any two firms that have established an R&D link cannot break it afterwards. This is reasonable as partnerships between firms are usually official and protected by

[2]See for instance Herings et.al (2009, 2010a,b); Mauleon et.al (2014) and references therein.

[3]This distinction is presented clearly in Example 2.2: In a setup with three firms, two of them establish an R&D link and isolate the third one, despite the fact that each one of them would prefer to become the hub in a star network. This is because both firms anticipate that by opting towards a star network will eventually lead to a complete network in which both will be worse–off. This reasoning is not captured by pairwise stability, and thus the unconnected network cannot be sustained in equilibrium when firms act myopically.

binding contracts, hence breaking an already established R&D link may induce significant costs for them. Assuming farsighted firms, i.e., firms that are highly sophisticated when taking decisions regarding their long-term planning, that engage in such a dynamic network formation process is, to our opinion, the most appropriate way to analyze R&D networks under potential entry. In this way, we can capture the dynamic nature of an entry process.

In the symmetric case in which all firms have initially equal marginal costs, it is evident that their incentives are also identical. Under no potential entry, both incumbent firms would opt for establishing an R&D link, since the benefit from reduced marginal costs due to their R&D activities outweighs the loss due to fiercer competition. Under potential entry though, incumbent firms may establish an R&D link in order to deter the entry of the new firm. This occurs whenever competition is intense, i.e. goods are not too differentiated, and R&D spillovers take intermediate values. In this case, the potential entrant remains isolated in an unconnected network and decides not to enter even if the entry cost is nil. This result is in line with Goyal and Moraga (2001). For lower R&D spillovers (and high intensity of competition), the incumbent firms still establish an R&D link, but entry is now accommodated with the new firm being isolated in the unconnected network. Finally, for the rest of the parameters, entry is again accommodated with the complete being the equilibrium network structure.

We further compare our equilibrium network structures with those arising under pairwise stability and identify important differences. In particular, the unconnected network arises more often in equilibrium under farsighted stability compared to pairwise stability, in which there is higher tendency towards the complete network. This is to be expected as farsighted firms are able to achieve their most preferred outcome more often than myopic firms. In fact, we obtain a more general result that holds under asymmetric costs too: If two out of the three firms obtain their maximum payoffs in the unconnected network in which they are connected, then this is the unique equilibrium network. This result is in line with previous findings in farsighted network formation and reveals that farsightedness can act as a coordination device for some firms, helping them to reach their maximum payoffs.

Under asymmetric marginal costs, there are often different incentives across firms. In particular, under no potential entry, an inefficient incumbent firm always wants to establish an R&D link with the efficient one, but the opposite need not be true anymore. In particular, the efficient incumbent has incentives to form an R&D collaboration only when its partner-rival is sufficiently efficient, otherwise it prefers to stay unconnected. As is shown, the incentives of the efficient incumbent are those that will be particularly affected by the potential entry.

Considering a potential entrant that has initially the same marginal cost as that of the efficient incumbent, we identify conditions under which the latter, anticipating entry, refrains from establishing an otherwise profitable R&D link with the inefficient incumbent. This is because in a triopoly it is optimal for the two efficient firms to establish an R&D link and isolate the inefficient one, thus forming an unconnected network. A farsighted efficient incumbent correctly anticipates entry and also knows that the entrant will not establish an R&D link with the inefficient incumbent. As a consequence, the efficient incumbent and the entrant are able to achieve their highest profits in the unconnected network that arises in the unique equilibrium. In a sense, the efficient incumbent prefers to wait for a more desirable partner instead of committing to an R&D collaboration that, without being harmful, would not be the optimal one. Under these conditions, entry is accommodated and the inefficient incumbent is marginalized in the market.

Yet, when competition is fierce relative to the benefits of R&D spillovers, the efficient incumbent is willing to form an R&D collaboration with a rather inefficient counterpart – that would otherwise be ignored – in order to deter the entry of the new firm. Under these circumstances, an R&D link between any two firms would force the third one to exit the market. Therefore, the efficient incumbent, in order to stay in the market, needs to establish an R&D link with the inefficient one at the outset of the game. On the top of that, it has the chance, via the R&D collaboration, to choose its preferred partner-rival in the ensuing duopoly. As a matter of fact, when competition is fierce, it is better for the efficient incumbent to compete (and cooperate) with a less efficient firm - the inefficient incumbent -, since this would allow it to enjoy a larger market share and profits. Here too, the incentives of the efficient incumbent are altered due to potential entry. Under these conditions, entry of a more efficient new firm is deterred as a result of the R&D collaboration of the two incumbents.

Moreover, exit from the market may become unavoidable for the inefficient incumbent. In fact, when competition is intense and R&D spillovers are low, entry cannot be deterred and the efficient incumbent prefers to establish an R&D link with the entrant rather than with its inefficient counterpart. Then due to fierce competition, the isolated firm is forced to exit the market. Note that in this case the incentives of the efficient incumbent are not altered due to potential entry. In fact, under no threat of entry, the efficient incumbent would have not established an R&D link with the inefficient one, yet the latter would have produced a positive quantity in the market.

Finally, we perform welfare analysis and show that there is often misalignment between market and societal incentives. Interestingly, there are parameter values under which an unconnected network leads to higher welfare than a complete network. This holds when competition is intense and R&D spillovers are rather low. The reasoning is that in a complete network firms' profits erode due to intense competition and, due to low R&D spillovers, consumer surplus does not increase much. As a result, the unconnected network in which competition is softer leads to higher welfare than the complete network. Nevertheless, in the symmetric case, the parameter space in which the unconnected network is welfare superior is substantially smaller than that in which it arises as an equilibrium network structure. Under asymmetric marginal costs, these parameter spaces are, to a major extent, distinct, confirming a strong misalignment between market and societal incentives. This gives room to an interesting discussion about potential policy measures that will give firms incentives to opt for welfare maximizing network formation.

1.3. Related Literature

The formation of R&D collaborations has been studied quite extensively in the recent years both in economics and in other disciplines, such as management and strategy. Extensive work has been done in management focusing on the formation of research and development alliances, especially in industries with lots of entry and exit (see for instance Clood et.al, 2006; Frankort, 2014; Frankort et.al, 2015; Hagedoorn, 2002; Roijakkers and Hagedoorn, 2006). This is mainly because there is widespread empirical evidence showing a significant increase in such collaboration agreements, mostly in industries with rapid technological development. Moreover, empirical results confirm our main motivation, showing that entry and exit is often observed in these industries across time (see Tomasello et.al, 2016 and in a more general setup focusing on growth Acemoglu et.al, 2012).

The formation of R&D collaborations is often observed even between firms that compete in the same markets. The aim of these collaborations can be either the joint reduction of production costs (see Goyal and Moraga, 2001; Goyal and Joshi, 2003; Goyal et.al, 2008; König, 2013; König et.al, 2014), or the growth of knowledge in each firm as a result of the combination of existing knowledge in each collaborator (see König et.al, 2011, 2012), or else as the participation in an innovation contest (see Czarnitzki et.al, 2008; Marinucci and Vergote, 2011). Most of these papers study the stability of R&D networks, as well as their effects on firms' profits and on welfare.

A large part of the literature on R&D networks, including the present paper, has focused on modelling R&D collaborations as tools that deterministically lead to the joint reduction of production costs. In the seminal paper of Goyal and Moraga (2001), the authors study the stability of symmetric R&D networks in which the agreements lead to marginal cost reduction. They find that if the firms compete in the same market, the individual R&D effort declines in the level of collaborative activity, whereas the level of cost reduction and social welfare are maximized for an intermediate level of collaboration. Using pairwise stability,[4] they show that the complete network is always stable, in the sense that no firm wishes to break any link. Furthermore, for three-firm asymmetric networks with

[4] A network is pairwise stable if no pair of nodes wants to create a new link bilaterally, and no node wants to destroy one of its existing links unilaterally.

spillovers between unconnected firms,[5] they show that the complete network is stable for any level of spillovers, whereas the empty and the star networks are never stable. They also identify conditions under which the unconnected network is also stable. Note that the latter result has a flavour of entry deterrence, since it highlights that two firms, via their R&D collaboration, can potentially force the third one not producing at all. Yet, pairwise stability is a static solution concept that cannot capture a by nature dynamic process of network formation in the presence of potential entrants. Our contribution is to propose farsighted stability as the appropriate solution concept that identifies stable networks when incumbent firms face potential entry. In contrast to Goyal and Moraga (2001), we show that when firms are farsighted, the network that arises in equilibrium is unique. Nontheless, analyzing network structures with the use of pairwise stability concept too, we are able to highlight important differences between environments with myopic and farsighted firms.

A large body of the R&D networks literature followed Goyal and Moraga (2001). Goyal and Joshi (2003) consider a quite similar framework, while focusing more on the properties of the marginal cost and extending their analysis to non–regular networks. Goyal et.al (2008) turn their attention to the difference between in-house research activities that each firm performs individually and joint projects with other collaborating firms. They obtain necessary and sufficient conditions on the profit function that guarantee that investment on in–house and joint projects are complementary. More recently, König (2013) extends Goyal and Moraga (2001) to general network structures with no ex ante restrictions. The author studies the dynamic endogenous formation of the network and finds that the probability of observing a certain network structure is the same as that of a random graph with a certain probability, which depends on the parameters of the model. König et.al (2011) and König et.al (2012) follow a slightly different approach, by assuming that the R&D networks lead to knowledge sharing between connected firms. They show that, depending on the marginal cost of collaboration, stable networks can be the empty network, the complete network, or else a combination of star subnetworks and disconnected cliques. Finally, König et.al (2014), using a similar model to Goyal and Moraga (2001), provide equilibrium quantities and payoffs for any number of firms and various exogenously given network structures. The authors though do not deal with stability issues. Yet, as their model is very rich, allowing for different levels of competition and R&D spillovers, we follow it closely in our analysis. We differ from the above literature as our focus is on the effects of potential entry on network formation.

There is also an extensive literature on R&D networks in patent race contests (see for instance Goyal and Joshi, 2006; Joshi, 2008; Martin, 1995, 2002; Stein, 2008; Marinucci and Vergote, 2011) which, to some extent, is related to our paper. The main theme in this literature is to identify

[5] For a similar analysis regarding upstream R&D networks see Kesavayuth et.al (2015).

conditions under which firms form collaborations in order to guarantee higher probability of success in patent race contests. Within this literature, Marinucci and Vergote (2011) is the only paper that considers R&D collaborations as a potential barrier to entry. Yet, entry in this setup refers to the participation in a contest, with the environment described by an all–pay auction. Contrary to our case, the authors consider pairwise stability as their equilibrium concept and find that the complete network is not always pairwise stable, with the stable networks leading some firms to drop out of the race. This last feature has also been supported empirically in a quite different environment by Hochberg et.al (2010).

Following the seminal work of Bloch (1995), there is a stream of literature focusing on the formation of clusters of R&D cooperation (see also Dawid and Hellmann, 2016), inside which firms are assumed to exchange information with all other members, while intending to maximize individual profits. A particular feature of these analyses is that they restrict the types of potential network structures that may arise in equilibrium.

In a rather different setup, collaborations may constitute bilateral market-sharing agreements, usually under the form of research joint ventures, leading to cartel formations (see Belleflamme and Peitz, 2010, and references therein). The role of research joint ventures as a barrier to entry has been analyzed in O'Sullivan (2013). There is though a crucial difference between the analysis of R&D networks and research joint ventures (RJVs). Under RJVs firms form collaborations and choose strategies to maximize total profits, while in R&D networks each firm maximizes its own profits.

In general, the relation between entry and R&D investment has been subject to a long debate, starting with the seminal paper of Arrow (1962). The author has shown that entry leads to lower incentives for the incumbent firms to invest as compared to those of the outsiders. Recently, this has been questioned by Czarnitzki et.al (2008), where it is shown that under entry pressure all firms tend to invest less, but leading firms tend to invest more than the average firm. Nevertheless, these papers focus on the effect of entry on individual R&D investment, without considering the formation of R&D networks as in our case.

The remainder of the paper is organized as follows. Section 2 presents the model and discusses the R&D collaboration formation mechanism when firms are farsighted. Section 3 identifies equilibrium networks in the symmetric case, i.e., when incumbent firms are equally efficient with the potential entrant. Section 4 analyzes the asymmetric case in which one of the incumbent firms is less efficient than the other incumbent and the potential entrant. Section 4 performs a welfare analysis and discusses discrepancies between networks arising in equilibrium and those that maximize total welfare. Finally, Section 5 provides concluding remarks.

2. The Model

We consider an industry with two incumbent firms, *firm 1* and *firm 2*, and one potential entrant, *firm 3*. Each firm produces a brand of an horizontally differentiated good. In line with König et.al (2014), the inverse demand function of firm i is,

$$p_i = \bar{A}_i - q_i - \rho \sum_{j \neq i} q_j, \tag{1}$$

where $\bar{A}_i$ denotes the size of firm i's market and $\rho \in (0,1)$ is the degree of substitutability between the products of any two firms i and j, $i, j \in \{1,2,3\}$, $i \neq j$. The higher the value of ρ, the more substitutable the products i and j are.

Firms are endowed with constant returns to scale technologies and compete in quantities in the market. Firm i's initial marginal (and unit) cost is $\overline{c}_i$. Firm i, by exerting an R&D effort level e_i, can decrease its marginal cost to $\overline{c}_i - e_i$.[6] R&D costs are $\frac{1}{2}e_i^2$, i.e., they are increasing, at an increasing rate, with the effort level (D'Aspremont and Jacquemin (1988)). Moreover, firm i can decrease its marginal cost by forming R&D collaborations with rival firms. Under a collaborative agreement, i.e., *a network link*, the involved firms commit to share part of their R&D results. In particular, let $\phi \in (0,1)$ be the share of R&D effort that a firm exchanges with each of its collaborators. Letting $a_{ij} \in \{0,1\}$ be the index that indicates whether firms i and j have a link ($a_{ij} = 1$) or not ($a_{ij} = 0$), the marginal cost of firm i is given by,

$$c_i = \bar{c}_i - e_i - \phi \sum_{j \neq i} a_{ij} e_j \tag{2}$$

This formulation of cost reduction has been used extensively in the literature of R&D networks (see e.g. Goyal and Moraga, 2001; Goyal et.al, 2008; König, 2013). It can be perceived as a process of improving already existing cost-reducing technologies. Notice that collaborations are considered to be mutual ($a_{ij} = a_{ji}$), in the sense that one firm cannot "steal" information from its rival. Moreover, our formulation disregards any potential knowledge spillovers between firms that are not directly linked. Therefore, given its network of collaborations, firm i's profits are given by,

$$\begin{aligned} \pi_i &= (p_i - c_i)q_i - \frac{1}{2}e_i^2 & (3) \\ &= \left(\mu_i - q_i - \rho \sum_{j \neq i} q_j + e_i + \phi \sum_{j \neq i} a_{ij} e_j \right) q_i - \frac{1}{2}e_i^2, & (4) \end{aligned}$$

[6] In line with the bulk of the related literature, we assume that the R&D process is deterministic. Yet, we are aware that in many real word situations the outcome of the R&D effort is uncertain.

where $\mu_i \equiv \bar{A}_i - \bar{c}_i$ is a measure of the relative "efficiency" of firm i. A firm with a higher μ_i either has initially a lower marginal cost or it faces a larger market.

In the sequel, we assume that $\bar{A}_i$ and $\bar{c}_i$ are sufficiently large to ensure that both equilibrium prices and marginal costs (after firms' R&D efforts) remain strictly positive. Existence of such lower bounds is guaranteed by Assumption 2 that follows. We also make two additional simplifying assumptions:

Assumption 1: $\mu_1 = \mu_3 = 1$ and $\mu_2 = \alpha \leq 1$.

Assumption 2: $\phi < \frac{1}{2} + \rho$

Assumption 1 tells us that incumbent firm 1 is equally efficient with the potential entrant firm 3, while incumbent firm 2 may be less efficient than the other two firms. In this way, we can focus on incumbents R&D collaborations as a mechanism that may deter entry of a more efficient firm.[7] We consider both the symmetric case ($\alpha = 1$), in which all firms are equally efficient and the asymmetric case ($\alpha < 1$), in which incumbent firm 2 is less efficient than the other two firms.

Assumption 2 guarantees that in equilibrium prices and marginal costs are positive under all circumstances. It requires that spillovers are not too high when the degree of substitutability among firms' products is low $\left(\rho \leq \frac{1}{2}\right)$. If this does not hold, then there exist cases in which firms end up producing at zero marginal cost, irrespectively of the initial value of $\bar{c}_i$. Note that when $\rho > \frac{1}{2}$, there is no restriction on the level of spillovers.

Finally, we assume that entry and exit costs are *nil*. This is done deliberately, because our focus is not on the role of entry costs as a potential barrier to entry, but instead on the importance of R&D collaborations between incumbents on this. In a similar vein, we want to abstract from exit costs preventing the exit of an inefficient incumbent firm and instead concentrate on the role of R&D collaboration between an efficient incumbent firm and a potential entrant on inducing exit.

2.1. Timing

The timing of the game is as follows. In the first stage, the two incumbent firms decide whether or not to form an R&D collaboration (using the mechanism described below). Upon observing their action, the potential entrant decides whether to enter or not in the market. If the entrant decides not to enter, then the two incumbent firms choose simultaneously their quantities and R&D effort levels, profits are then realized and the game ends. If the entrant decides to enter, then the three firms form their network of R&D collaborations (again using the mechanism described below). After the network is formed, all three firms choose simultaneously their quantities and R&D effort levels, profits are then realized and the game ends.

[7] Considering $\mu_3 = \beta$, with $\beta \gtreqless 1$, complicates a lot the analysis, without adding much to our insights. Nevertheless, we provide a few simple intuitive arguments on how the results are expected to change if we relax Assumption 1.

Note that, the only advantage that incumbent firms have in this scenario is the opportunity to form an R&D collaboration before the entry of the new firm. As we will see, this advantage can be used either to deter entry of the new firm or to protect each of the incumbent firms from ending up in a worse equilibrium network after entry occurs.[8]

2.2. R&D Collaboration Formation

The mechanism of formation of R&D collaborations is the most crucial element of our model. In the bulk of the existing literature, the number of firms is considered to be fixed (see among others Goyal and Moraga, 2001; Goyal and Joshi, 2003). Under fixed number of firms, it is reasonable to focus on equilibrium networks that satisfy some notion of *stability*. A few alternative notions of stability have been used in the literature, such as pairwise stability,[9] Nash stability (Dutta and Mutuswami (1997); Dutta et.al (1998)) and strong stability (Jackson and van den Nouweland (2005)), all of which refer to static conditions and ignore the dynamics that lead to a particular network structure.[10] These stability notions are, however, inadequate to capture the dynamics of an entry model, as firms in the latter are sophisticated enough to predict the future consequences of their present decisions for the network structure.

For this reason, we consider a mechanism of network formation that has the form of a dynamic game with farsighted players. Networks with farsighted players have been studied theoretically in several instances, see for instance Dutta et.al (2005); Herings et.al (2009, 2010a,b), showing a number of interesting features. However, in the context of R&D networks, they have been considered only by Mauleon et.al (2014) in a simplified setting with exogenously fixed outcome of R&D and with a fixed number of firms.

The mechanism proceeds as follows: Given an initial network g, among all the links that are still not established one is chosen at random to be discussed. If it is mutually beneficial for both involved firms to create the link, then the link is established. A crucial assumption here is that if two firms agree to create an R&D collaboration link, then this link cannot be removed later on.[11]

[8] The game has been intentionally constructed in this way, since we want to abstract from advantages of incumbent firms stemming from their involvement in R&D for a longer time. In fact, many of these intuitions are explained in the part of our analysis where efficiencies are considered to be asymmetric among firms.

[9] The notion of pairwise stability, introduced by Jackson and Wolinsky (1996) and first adapted to R&D networks by Goyal and Moraga (2001), is the most commonly used in this literature. For this reason we provide some comparisons with our results.

[10] For a dynamic model of network formation with myopic agents see Jackson and Watts (2002), which has been applied to R&D networks by Dawid and Hellmann (2014). Both papers focus on the notion of stochastic stability.

[11] In fact, this is not restrictive at all. Network collaborations often require a certain level of commitment between the two interacting parties. If two firms enter into a discussion and agree on signing an agreement to exchange information,

If those firms do not reach an agreement, then another link is chosen at random to be discussed. Without loss of generality, we assume that the new link is different from those that have already been discussed. This is because rational farsighted firms would never choose differently in two identical situations. The process is repeated until either two firms i and j agree on creating a link ij or all potential links have been discussed unsuccessfully. In the former case, the process is repeated considering as initial the network $g \cup \{ij\}$. In the latter case, the process is terminated and the equilibrium network is g.

When a link is chosen to be discussed, each of the involved firms agrees to its establishment if its expected payoff is strictly higher with the link than without it. In case of indifference, we assume that the firm agrees to establish the link unless its potential collaborator is not going to produce in the market.[12] This assumption seems quite reasonable, as such an indifference may arise only in the following three cases. First, if a firm knows that there will be a unique equilibrium network to be reached irrespectively of its current decision. As will be shown later on, this can only be the complete network. This implies that the link under discussion will eventually be formed and thus, there is no reason why firms should meet twice to discuss a link they will form anyway. Second, if a firm knows that it can reach the same network structure by forming a link with another firm that does not participate in the current discussion. In this case, as firms do not care about the identity of otherwise identical R&D collaborators, there is no particular reason for going to a future meeting in order to achieve something that it can be achieved today. Third, in some cases, an inefficient firm will not produce neither when it is isolated in an unconnected network, nor when it is peripheral in a star network. This makes an efficient firm indifferent between the two network structures. A link with a firm not operating in the market is then meaningless and any arbitrarily small link cost would be sufficient to break this indifference; hence, a firm would opt not to create a link with a non–operating firm.

The following remark establishes that our dynamic network formation game ends after a finite number of rounds, therefore a farsighted (forward looking) firm is able to solve the game by backward induction. Implicitly, the equilibrium concept we use is subgame perfect equilibrium.

Remark 1. For a population of n firms, starting from the empty network, the described mechanism of network formation reaches an equilibrium after at most $\binom{n}{2}!$ rounds of discussion. $\triangleleft$

It is worth noticing that the equilibria of our dynamic mechanism may not coincide with the set

it is reasonable to consider that breaking this agreement incurs a cost. Therefore, if firms are uncertain whether an R&D collaboration will be beneficial for them, they may wait and establish a link in a later round of communication. Note though that there is no uncertainty in our setup.

[12]Clearly, a firm not producing in the market will not spend on R&D effort too.

of pairwise stable networks. This happens because in pairwise stability, firms are considered to be myopic and short-sighted. They do not take into account that creating a link, although it seems beneficial given the current network structure, may lead to a series of link creations that will end up in a future network structure in which they will find themselves worse-off.

Our mechanism can also be visualized as a coordination device for firms that would like to commit to a commonly beneficial strategy, but could not be protected from unilateral deviations if either the decisions were made simultaneously or there was a limited number of communication rounds. This idea is captured more clearly in the following remark.

Remark 2. Given a population of three farsighted firms i, j, k, if the unconnected network with link ij yields the unique highest possible payoffs to both i and j, then this is the unique equilibrium network. $\triangleleft$

This result is a direct consequence of the farsightedness of the firms. The firms i and j that can reach their maximum payoff are able to coordinate and use a strategy that leads them to their preferred network structure equilibrium. In fact, they will reject any collaboration offer from firm k and create a link between them the first time their link is chosen to be discussed. Moreover, as this link leads to their maximum payoffs, they have no incentive to deviate. Note that, this is not the only case where the unconnected network arises in equilibrium. Nevertheless, this observation allows us to identify the equilibrium network structures in some cases without having to compare all the possible firms' payoffs.

The following example describes the dynamics of network formation for a given artificial payoff structure, as an attempt to clarify the mechanism described above. The reasoning exposed below will be very similar for our analysis and main results.

Example: Consider three identical firms 1, 2 and 3 playing a network formation game in which the payoffs are determined exclusively by the network structure and are given in the following Table,

UO	E	SP	C	UI	SC
0	1	2	3	4	5

Table 1: Payoffs (ad–hoc) for different network structures

where UO and UI denote respectively the isolated firm and the connected firms in the unconnected network, E and C denote respectively the firms in an empty and a complete network, and SP and SC denote respectively a peripheral and a central firm in a star network.

Let 12 be the first link to be discussed – analogous to 1 and 2 being the two incumbent firms and 3 the potential entrant. If the two firms agree to form a collaboration then link 13 is chosen to be discussed. Now, firm 1 seems to have an incentive to form a collaboration with firm 3 and become the center of the star. Clearly, firm 3 also has an incentive to collaborate with firm 1 since being the isolated firm in an unconnected network is its worst case scenario. However, if firm 1 agrees on the collaboration, then when link 23 comes to be discussed, both firms 2 and 3 have incentives to form a link and pass from being peripheral in a star network to be parts of a complete network. Therefore, a farsighted firm 1 anticipates this continuation of the game and rejects the link 13. The same reasoning applies to the discussion of the link 23, in which case firm 2 rejects it. Therefore, in equilibrium only firms 1 and 2 are connected, while firm 3 is the isolated firm of the unconnected network.

Note that the equilibrium network is not pairwise stable, since a myopic firm 1 would have accepted to create the link 13, although in the long–run this would have lead to the formation of the complete network, making thus firm 1 worse–off. In fact, given the payoff structure in Table 1, the complete is the only pairwise stable network.[13]

Finally, it becomes apparent why our network formation mechanism can be considered as a coordination device for two firms that wish to form a "coalition" against the third one. Namely, if after the creation of link 12, firm 3 was able to offer simultaneous take–it–or–leave it offers to firms 1 and 2, then this would have made them face a "prisoner's dilemma" situation that would eventually have led to the formation of the complete network. The same would have happened if the offers were sequential, but there was only one round of discussion. In that case the first firm to receive an offer would have been obliged to accept it, since otherwise it would have ended up being a peripheral firm in a star network.

The previous analysis is quite similar to that in our model setup. The fact that payoffs are deterministic allows us to rank networks and compute equilibrium outcomes. Moreover, under efficiency asymmetries, we are able to eliminate the impact of randomness on the order of link discussion. This happens because under efficiency asymmetries, firms have preferences not only over network structures but also over which firm to be linked with.

2.3. Preliminaries

We will use some preliminary analysis directly from König et.al (2014). In our model, as we assume away entry and exit costs, zero production of the potential entrant is interpreted as entry deterrence, whereas zero production of an incumbent is interpreted as exit from the market.

In particular, consider all n firms that produce (strictly) positive quantities and define the following matrices: $\mathbf{A}$ is the $n \times n$ matrix, with $n \in \{1, 2, 3\}$, that represents the network of established

[13] Note that this is only one of the potential equilibria that satisfy Nash stability.

links, with elements $a_{ij} = 1$ if there exists an R&D collaboration between i and j and $a_{ij} = 0$ if there does not exist one; $\mathbf{I}_n$ is an $n \times n$ identity matrix; and $\mathbf{B}$ is an $n \times n$ matrix with zero diagonal elements and ones everywhere else, that describes the competition between firms.[14] Moreover, $\boldsymbol{\mu}$ is the $n \times 1$ vector of firms' efficiencies. Then the equilibrium $n \times 1$ vector of quantities $\mathbf{q}$, and the respective R&D efforts and profits of the firms that produce positive quantities are given by

$$\begin{aligned} \mathbf{q} &= (\mathbf{I} + \rho\mathbf{B} - \phi\mathbf{A})^{-1}\boldsymbol{\mu} && (5) \\ e_i &= q_i && (6) \\ \pi_i &= \frac{1}{2}q_i^2 && (7) \end{aligned}$$

Equation (7) turns out to be very helpful, since it allows us to compare equilibrium quantities instead of equilibrium profits. Nevertheless, it still remains to identify which firms produce strictly positive quantities in each of the possible network structures.

3. The symmetric case ($\alpha = 1$)

In this section we consider that both the incumbent firms and the potential entrant are equally efficient. Without loss of generality, we assume that $\mu_i = A_i - c_i = 1$ for $i = 1, 2, 3$. We will proceed to determine the equilibrium network structures for different values of the parameters ρ and ϕ. For the sequel, it is useful to define $k = \rho - \phi$, with $k \in (-1, 1)$, where k is a measure of the intensity of competition relative to the benefits from R&D links. The higher the value of k, the fiercer is the competition compared to the benefits from R&D collaboration. By Assumption 2, $k > -1/2$.[15] All algebraic manipulations and proofs are relegated to the Appendix.

3.1. No entry

We start by determining the equilibrium quantities of the incumbent firms in a duopoly, i.e., in case that no entry occurs. There are two possible scenarios: with and without an R&D collaboration link between incumbent firms. The equilibrium quantities are

$$q^{D,C} = \frac{1}{1+k} \quad \text{and} \quad q^{D,E} = \frac{1}{1+\rho} \tag{8}$$

[14] König et.al (2014) allow firms to operate in different markets, leading to richer forms of matrix $\mathbf{B}$.

[15] If $k \leq -1/2$, the R&D collaborations provide a relatively larger benefit than the loss due to the additional competition they induce, which leads firms to opt for higher production. In that case, absent any upper bounds induced by $\overline{A_i}$ and $\overline{c_i}$, the firms would produce infinitely much. This means that no matter how high these upper bounds are, some of them will eventually bind.

where D refers to duopoly, and C and E refer to complete and empty networks, respectively.

Clearly, it is beneficial for identical incumbent firms to form an R&D link (since $\rho > k$), i.e., the complete network is preferred to the empty one. As we will see below, this is not anymore the case under efficiency asymmetries.

3.2. Potential Entry

Under potential entry, the market structure is often a triopoly. With all firms equally efficient, each produces a positive quantity as long as the network structure is either the complete, or the empty or the star network. In a complete and an empty network, the equilibrium quantities are, respectively

$$q^C = \frac{1}{1+2k} \quad \text{and} \quad q^E = \frac{1}{1+2\rho} \tag{9}$$

Note that $k > -1/2$ guarantees that $q^C > 0$ (see also Footnote above). Again, the complete network is preferred to the empty one. Moreover, a duopolistic market structure (no entry) is preferred by incumbent firms as long as the network is empty, $q^{D,E} > q^E$. This is also true under a complete network, but only if $k > 0$, i.e., when the benefits from R&D collaborations is weak relative to the intensity of competition.

In a star network S – where SC and SP stand for center and periphery respectively –, the equilibrium quantities are

$$q^{SC} = \frac{1+\rho-2k}{1+\rho-2k^2} \quad \text{and} \quad q^{SP} = \frac{1-k}{1+\rho-2k^2}. \tag{10}$$

Assumption 2 implies that $1+\rho-2k^2 > 0$; thus, equilibrium quantities are always positive.

In an unconnected network U – where UI and UO stand for insider and outsider respectively – the equilibrium quantities are

$$q^{UI} = \frac{1-\rho}{1+k-2\rho^2} \quad \text{and} \quad q^{UO} = \frac{1+k-2\rho}{1+k-2\rho^2}, \tag{11}$$

as long as $\phi < 1-\rho$.[16] Under these circumstances, all firms produce positive quantities in the unconnected network.

By contrast, if $\phi \geq 1-\rho$, the isolated firm does not produce at all ($q^{UO} = 0$) and the other firms behave as duopolists, i.e., $q^{UI} = q^{D,C}$. The isolated firm is then excluded from the market.[17] Interestingly, this is the only case in which, absent of any additional costs, entry can be deterred. Essentially, the two incumbent firms agree ex–ante to form an R&D collaboration link that does not

[16] In fact, $1+k-2\rho^2 > 1+k-2\rho$ for $\rho \in (0,1)$, and $1+k-2\rho > 0$ if and only if $\phi < 1-\rho$.

[17] This result can be seen as complementary to Goyal and Moraga (2001), in which the authors assume that $\phi = \rho = 1$, but allow for spillovers among unconnected firms.

allow the entry of the new firm and ensures them duopoly profits. By forming the link, the two connected incumbents reduce substantially their marginal costs and in turn, produce so much to make it unprofitable for the isolated firm to start producing.

Nevertheless, if $\phi \geq 1-\rho$ and spillovers are sufficiently large, the incumbent firms may prefer to accommodate entry and move to a complete network, instead of deterring the entry of the new firm. It should be noted that if firms were not farsighted, they may have ended up accommodating entry for low values of spillovers too, by forming a complete network, despite the fact that they would prefer to have deterred entry.[18]

To determine the equilibrium outcome of the game, we need to compare firms' equilibrium quantities (and thus profits) under alternative scenarios. Table 2 contains the ranking of equilibrium quantities produced by each firm under various network structures for different parameters values (The exact expressions can be found in the Appendix). With a slight abuse of notation, we substitute equilibrium quantities for the position in the network they refer to, e.g. q^{SC} with SC. Figure 1 illustrates the various parameter regions. Remember that by assumption 2, $k > -1/2$. Hence, the red area in Figure 1 includes parameter values that are non-permissible.

A	$\phi \geq 1-\rho$		$\phi > \rho$		$C > SC > UI > SP > E > UO$
B			$\rho - 1/2 < \phi < \rho$		$UI > SC > C > SP > E > UO$
C			$\phi < \rho - 1/2$		$UI > SC > C > E > SP > UO$
D	$\phi < 1-\rho$	$\rho < 1/2$	$\phi > \frac{2\rho(1-\rho)}{1-2\rho}$		$C > SC > SP > UI > E > UO$
E			$\rho < \phi < \frac{2\rho(1-\rho)}{1-2\rho}$		$C > SC > UI > SP > E > UO$
F			$\phi < \rho$		$SC > C > UI > SP > E > UO$
G		$\rho > 1/2$	$\phi > \rho - 1/2$	$\phi < \frac{1-\rho}{2\rho}$	$SC > UI > C > SP > E > UO$
H				$\phi > \frac{1-\rho}{2\rho}$	$UI > SC > C > SP > E > UO$
I			$\phi < \rho - 1/2$	$\phi < \frac{1-\rho}{2\rho}$	$SC > UI > C > E > SP > UO$
J				$\phi > \frac{1-\rho}{2\rho}$	$UI > SC > C > E > SP > UO$

Table 2: Comparison of profits under alternative network structures: the symmetric case.

A few important remarks can be made by inspecting Table 2. First, C and SC always yield higher profits than SP, E and UO; moreover, apart from parameter region D, UI yields higher profits than SP, E and UO too. Second, UO is the worst possible scenario for a firm, even if it remains active in the market. Third, the fact that C is always preferred to SP implies that the star network can never arise in equilibrium. In addition, the latter observation together with the fact that both UI

[18]This might also be the case if there was only a fixed number of discussion rounds, which would lead to coordination problems.

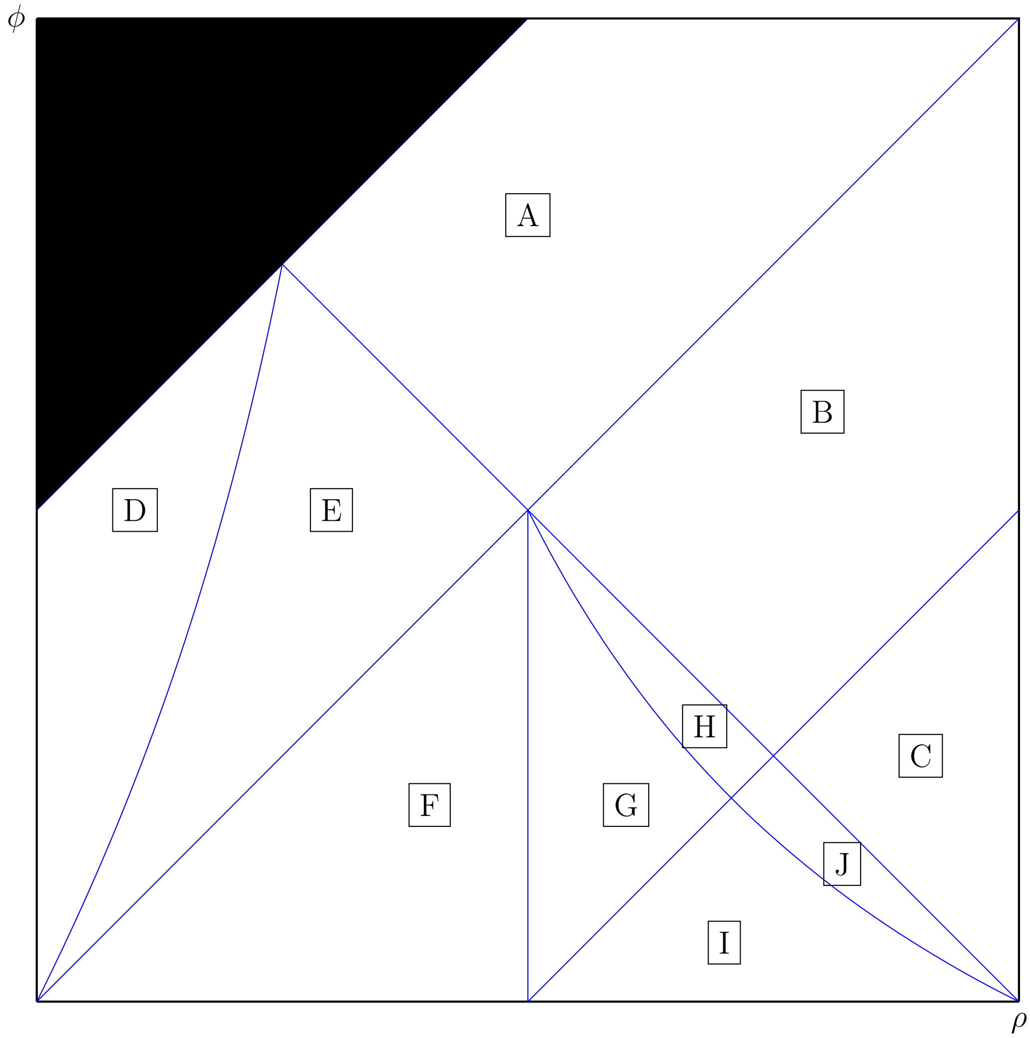


Figure 1: The various regions with different profit rankings.

and C are always preferred to E imply that the empty network cannot arise in equilibrium either. Therefore, it is enough to restrict attention to the conditions under which the unconnected and the complete are the equilibrium networks. In fact, as it can be seen in Figure 2, which summarizes the equilibrium outcomes, different profit rankings lead to the same equilibrium network structure when the firms are farsighted. Our results are qualitatively similar to those under pairwise stability. In this case too, only the complete and under certain conditions, the unconnected network are stable. Yet, as we will see below, the particular equilibrium outcomes vary depending on the equilibrium concept employed. The following Proposition establishes the uniqueness of the equilibrium outcome in the symmetric case for each parameter constellation and also characterizes the equilibrium outcomes for different parameter values (see Figure 2 for an illustration).

Proposition 3.1. *For each $(\phi, \rho) \in (0,1)^2$ that satisfy Assumption 2 there is a unique equilibrium outcome that leads to the following entry decision and network structures:*

- *The two incumbent firms always form a link before the (potential) entry of the new firm.*
- *If $\rho \leq 1/2$, or if $\rho > 1/2$ and $\phi \geq \rho$, then the new firm enters and a complete network is formed.*
- *If $\rho > 1/2$ and $1 - \rho \leq \phi < \rho$, then the new firm does not enter.*
- *If $\rho > 1/2$ and $\phi < 1 - \rho$, then the new firm enters, but the network remains unconnected.*

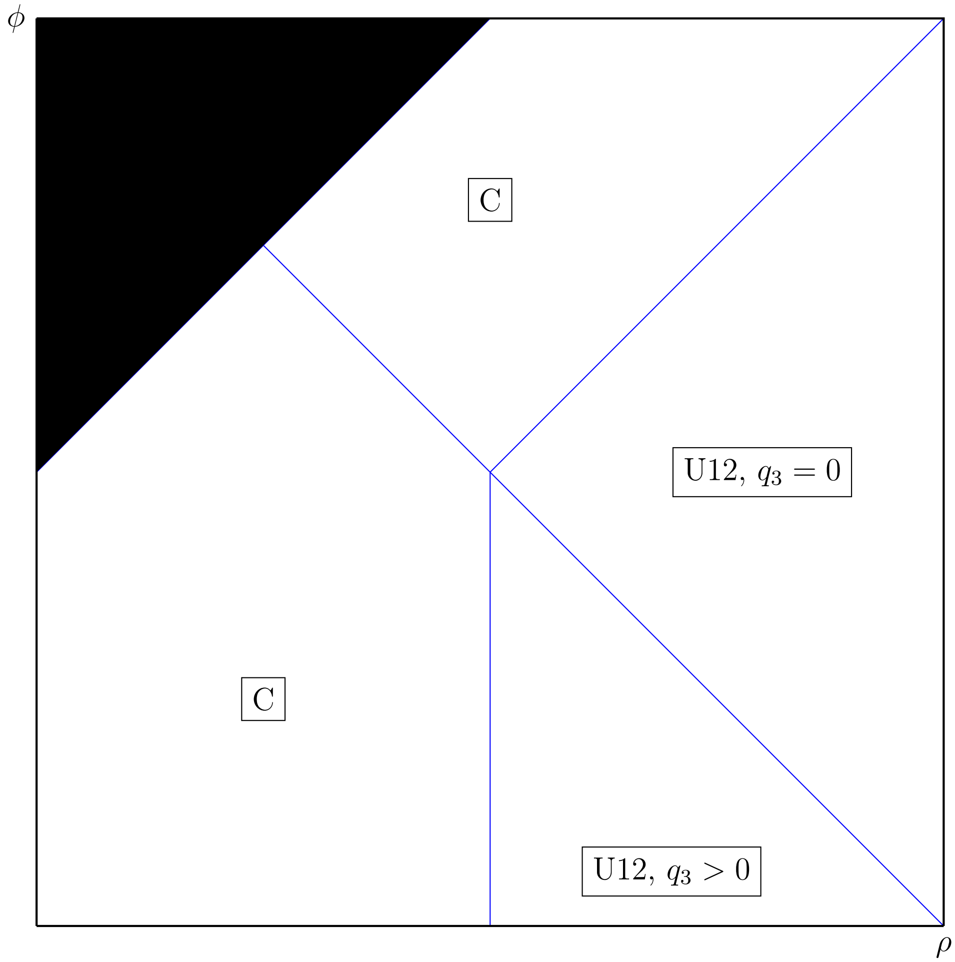


Figure 2: Equilibrium networks and quantities for each pair of parameters (ρ, ϕ).

The intuition behind this result is as follows. An R&D collaboration between the incumbent firms is beneficial for each of them both under the presence and the absence of a potential entrant. In the case with no entry and with equally efficient firms, the increase in profits due to the cost reduction generated by the collaborator's R&D effort more than compensates for its decrease due to the stronger competition induced by the reduction of the rival's cost as a result of this R&D collaboration. A similar reasoning applies under potential entry, and the empty network is thus dominated by the complete network. In particular, when competition is relatively weak (low ρ), the complete is the equilibrium network independently of the strength of spillovers. Moreover, the same holds even if competition is strong (high ρ), provided that spillovers are sufficiently high compared to competition's intensity (high ϕ). In these cases, the incumbent firms form an R&D collaboration link initially and subsequently form links with the potential entrant, accommodating thus the entry of the new firm.

Yet, the unconnected network in which the incumbent firms form initially an R&D link, with the new firm remaining isolated, is sometimes preferable by the incumbent firms compared to a complete network. In particular, if competition is strong (high ρ) and spillovers are low compared to the intensity of competition (low ϕ), then the unconnected is the equilibrium network. In the latter, the isolated firm either produces zero and stays out of the market (for very low values of ϕ), or it produces a positive, albeit lower than that of the connected firms, quantity (for intermediate values of ϕ). In these cases, the incumbent firms form an R&D link before the (potential) entry of the new firm in order each to avoid ending up being an isolated firm in an unconnected network. Further, the fact that firms are farsighted allows the unconnected network to remain stable even in cases in which being located at the center of a star network is preferable by an incumbent firm. This is because the incumbent firm at the center anticipates that the peripheral incumbent will form an R&D link with the entrant, which would eventually lead to the formation of a complete network that is less preferable than the unconnected one.

As mentioned above, for certain parameter values, R&D collaboration between incumbent firms deters the entry of the new firm. Yet, the possibility of entry does not alter the incentives of incumbent firms to form a link, since the latter would be beneficial even without the threat of entry. As will become apparent later on, this is no more true under the presence of efficiency asymmetries.

It should be stressed that the equilibrium networks differ from those under pairwise stability. In particular, the complete network is *always* pairwise stable, yet it is not always an equilibrium network when firms are farsighted. In fact, when $\rho > 1/2$ and $\phi < \rho$, the complete network does not arise in equilibrium. This is not too surprising because farsighted firms are able to predict the final network structure that their current decision will induce.

Our findings are in line with Goyal and Moraga (2001), in which it is shown how an unconnected network can lead an isolated firm not producing in the market. Nevertheless, in the current setting, apart from focusing on farsighted firms, we show that entry deterrence depends crucially on the relation between R&D spillovers and the intensity of competition. In fact, the incumbent firms often prefer to accommodate entry, by forming or not R&D collaborations with the new firm, although they would be able to deter its entry to the market.[19]

A final observation – following from Remark 2 – is that whenever the unconnected network gives the highest possible profits to the collaborating firms, then this is the unique equilibrium network irrespectively of the order in which firms discuss potential links after entry. More specifically, the optimal strategy for either of these two firms is to establish a link with the other the first time they meet to discuss. Without efficiency asymmetries, a firm is indifferent regarding who its collaborator will be, therefore the unique R&D link that is established is the one between the two firms that meet first, which in our case are the two incumbent firms. Introducing efficiency asymmetries may break this indifference and alter the network arising in equilibrium. Nevertheless, this observation turns out to be very useful since it will allow us to find firms' equilibrium strategies, without having to compare their profits in all possible scenarios.

4. Asymmetric Firms ($\alpha < 1$)

In the previous section, we showed that an R&D collaboration can be a barrier to entry for a new firm. However, the possibility of entry did not alter the incentives of the incumbent firms to form an R&D collaboration, as the latter would be preferable even without potential entry.

In this section, we consider two incumbent firms differing in their efficiencies and highlight a number of cases in which the presence of a potential entrant may indeed change their incentives to form, or not, an R&D collaboration. In particular, we present a case in which in the absence of potential entry, the more efficient incumbent is willing to establish an R&D link with the less efficient one; yet, the presence of an efficient entrant makes the efficient incumbent to reject this R&D link and wait to form an R&D collaboration with the entrant. We also present a case in which in the absence of potential entry, the more efficient incumbent prefers not to establish an R&D link with the less efficient one; yet, the presence of a potential entrant alters its incentives, since by establishing this R&D link can deter the entry of a more efficient rival. Finally, we present a case in which entry triggers the exit of the less efficient incumbent from the market.[20]

[19] This result seems to be driven by the fact that incumbent firms are equally efficient. However, this is not necessary, as it would be sufficient for the incumbent firms to be sufficiently close in terms of efficiency. In the opposite case (see Proposition 4.3 below), the two efficient firms form an R&D collaboration, thus isolating the inefficient firm and forcing it out of the market. If the two efficient firms are the two incumbents, this strategy can deter the entry of an inefficient new firm.

[20] Our analysis can be extended to provide a full characterization of equilibrium network structures over the three–dimensional parameter space (α, ρ, ϕ), but only at a considerable computational cost.

Formally, consider two incumbent firms 1 and 2 with $\mu_1 = 1$ and $\mu_2 = \alpha < 1$, respectively, and a potential entrant with $\mu_3 = \beta \lesseqgtr 1$. We refer to firms 1 and 2 as the efficient and the inefficient incumbent, respectively. For keeping the analysis tractable and the exposition simple, we will focus on $\beta = 1$ and then briefly discuss how our results may change for other values of β.

4.1. No entry

We start by determining the equilibrium quantities of the incumbent firms in a duopoly, i.e., in case that no entry occurs. There are two possible scenarios: with and without an R&D collaboration link between incumbent firms.

Without R&D collaboration between incumbent firms, the equilibrium quantities are

$$q_1^{D,E} = \frac{1-\alpha\rho}{1-\rho^2} \quad \text{and} \quad q_2^{D,E} = \frac{\alpha-\rho}{1-\rho^2} \quad \text{if } \alpha > \rho \tag{12}$$

and $q_1^{D,E} = 1$ and $q_2^{D,E} = 0$ otherwise. Clearly, if the efficiency difference is large (small α), only the efficient incumbent produces the monopoly quantity in the market.

Under R&D collaboration, the equilibrium quantities are

$$q_1^{D,C} = \frac{1-\alpha k}{1-k^2} \quad \text{and} \quad q_2^{D,C} = \frac{\alpha-k}{1-k^2} \quad \text{if } \alpha > k, \tag{13}$$

and $q_1^{D,C} = 1$ and $q_2^{D,C} = 0$ otherwise. Again, for large differences in efficiency the efficient incumbent is a monopolist producer in the market.

It is easy to see that for all $k < \alpha < 1$, $q_2^{D,C} > q_2^{D,E}$. Hence, the inefficient incumbent always wishes to establish an R&D link. However, this is not always the case for the efficient one, which prefers to form an R&D collaboration only if $q_1^{D,C} > q_1^{D,E}$, or else, only if $\max\left\{\rho, \frac{\rho+k}{1+k\rho}\right\} < \alpha < 1$. If the latter condition holds and there is no potential entry, an R&D collaboration is established between the incumbent firms. As we will see below, the efficient firm's incentives to establish an R&D link may change under the threat of entry.

Finally, notice that for $k > 0$, i.e., for high intensity of competition relative to R&D spillovers, $q_1^{D,C}$ is decreasing in α. That is, an efficient incumbent prefers to compete against a less rather than a more efficient rival, provided that there is no threat of entry. As we will see below, this turns out to be important for the R&D collaboration decision of the efficient incumbent under potential entry.

4.2. Potential entry

Under potential entry, efficiency asymmetries lead to further complications. First, the equilibrium quantities in a given network structure often differ across firms. Second, star networks may have different centers: $S1$ with center the efficient incumbent firm and $S2$ with center the inefficient one. And third, unconnected networks may have different isolated firms: $U12$ in which isolated firm is the entrant and $U13$ in which isolated firm is the inefficient incumbent. To deal with it, some additional notation is needed. Let q_f^{Si} be the equilibrium quantity of firm f in a star network with firm i as a center, and q_f^{Uij} be the equilibrium quantity of firm f in the unconnected network with unique link ij. Using Equation (5), the equilibrium quantities under various networks and parameter values are obtained and reported in Table 3 in Appendix A.

As already mentioned, the full characterization of equilibrium networks for all parameter values, albeit possible, comes at a considerable computational cost without adding much to our insights. For this reason, we will focus on three interesting cases which satisfy the conditions of Remark 2. In the following three Propositions, we establish conditions ensuring that the unconnected networks $U13$ and $U12$ are the unique optimal network structures for firms 1 and 3 and for firms 1 and 2, respectively; as a consequence, these are also the unique equilibrium network structures. The dynamics that lead to these equilibrium networks are those described below Remark 2. In fact, the two firms that want to create the unconnected network can do so by rejecting any proposal from the third firm and establishing an R&D link between them the first time they meet to discuss.

Proposition 4.1. *Let $(\rho, \phi) \in (0,1)^2$ be such that $\rho(2-\rho) - \sqrt{\rho^2(1-\rho)^2 + \rho} < \phi < \min\{\rho, 1-\rho\}$. Then if $\max\left\{\frac{2\rho}{1+k}, \frac{k+\rho}{1+k\rho}\right\} < \alpha < \min\left\{\frac{k+2\rho+2\rho^2}{1+\rho+k+2k\rho}, 1\right\}$ and $\beta = 1$, the unique equilibrium network is unconnected, with the efficient incumbent firm and the entrant forming an R&D collaboration and the isolated inefficient incumbent firm being active in the market.*

Proposition 4.1 describes a scenario in which all three firms produce in an unconnected network and in which the efficient incumbent firm prefers to establish an R&D collaboration with the efficient entrant rather than with the inefficient incumbent. Notice that under no threat of entry, the efficient incumbent would have established an R&D link with the inefficient incumbent in this parameter region (since $\alpha > \frac{k+\rho}{1+k\rho}$). Nevertheless, as the potential entrant is a better partner (since $q_1^{U13} > q_1^{U12}$), the efficient incumbent prefers to allow its entry and subsequently form an R&D collaboration with him. It should be stressed that the incentives of the efficient incumbent and the equally efficient entrant are the same. Therefore, the former does not fear that upon its entry the new firm will establish an R&D collaboration with the inefficient incumbent. Figure 3 illustrates in the (ρ, ϕ) space the different regions, with their respective values of α, in which entry occurs and the efficient incumbent forms an R&D collaboration with the entrant. Notice that for those parameter values under which all firms produce in all networks, the unconnected network is sustained in equilibrium more often under the presence of efficiency asymmetries, and in particular when competition is rather fierce (high ρ). This is not surprising since lower (average) efficiency is expected to be associated

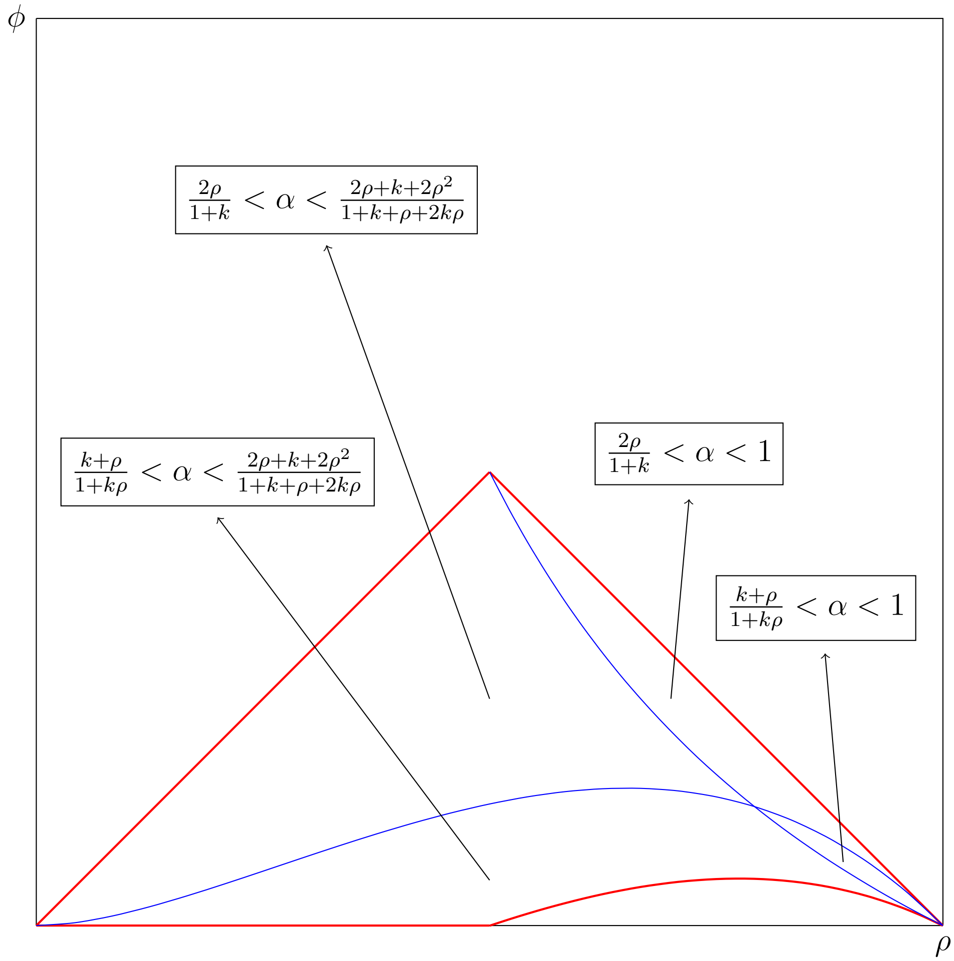


Figure 3: Two–dimensional projection of the parameter values under which Proposition 4.1 holds. The area surrounded by the red curve illustrates the parameter values (ρ, ϕ) that satisfy the first condition. This area is divided by the blue curves in four regions, each of which corresponds to one of the inequalities regarding α of the second condition.

with lower overall connectivity in equilibrium.

Proposition 4.2. *Let $(\rho, \phi) \in (0,1)^2$ be such that $1-\rho^2 < \phi < \rho$ and $\rho > \frac{\sqrt{5}-1}{2}$. Then if $\rho < \alpha < \frac{k+\rho}{1+k}$ and $\beta = 1$, the unique equilibrium network is unconnected, with the two incumbent firms forming an R&D collaboration and the potential entrant staying out of the market.*

Proposition 4.2 describes another interesting scenario in which the incentives of the incumbent firms are altered due to the possibility of entry. In particular, the inefficient incumbent firm is now too inefficient (as $k = \rho - \phi > 0$, we have $a < \frac{\rho+k}{1+k} < \frac{\rho+k}{1+k\rho}$) to make the efficient incumbent unwilling to form an R&D link under no threat of entry. On the other hand, the inefficient incumbent is sufficiently efficient (as $1 - \rho^2 < \phi$, we have $a > \rho > \frac{1+k-\rho}{\rho}$) to ensure that upon entry of an equally efficient firm, the equilibrium network would be unconnected with one of the three firms been kicked

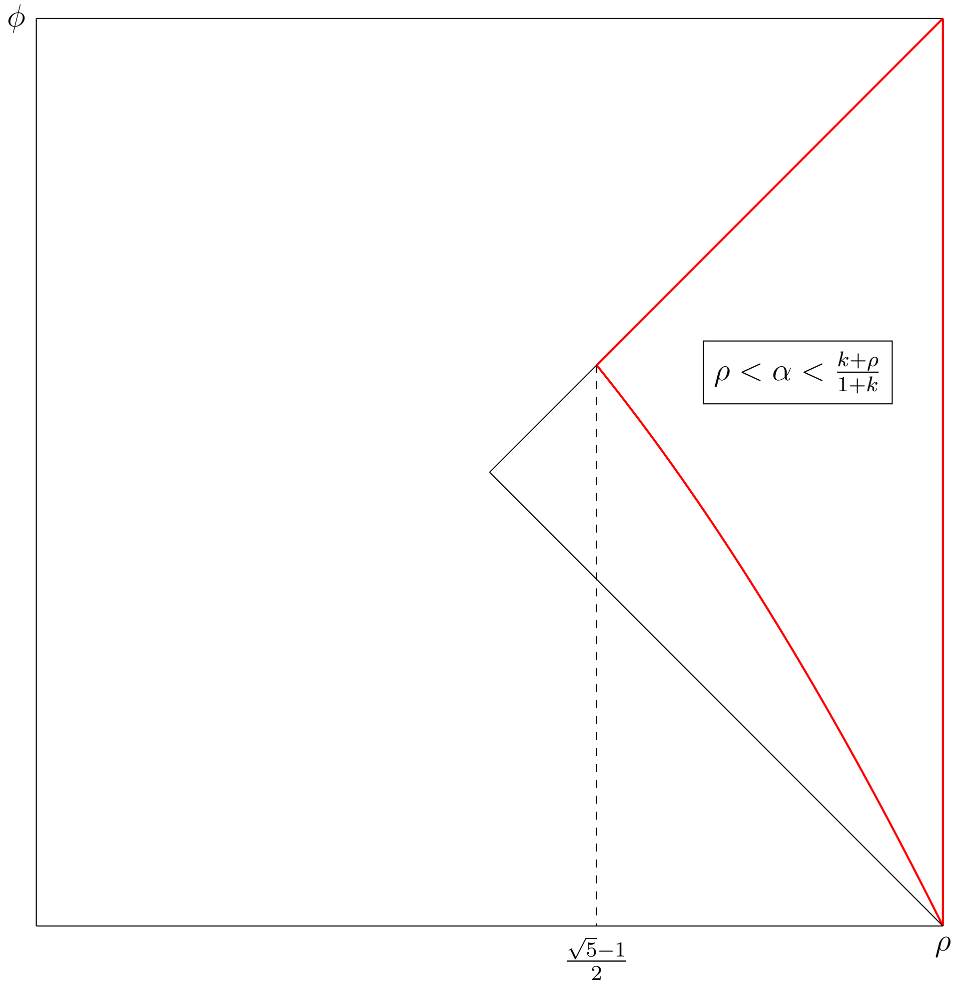


Figure 4: Two–dimensional projection of the parameter values under which Proposition 4.2 holds. The area surrounded by the red curve illustrates the parameter values (ρ, ϕ) that satisfy the first two conditions. The black lines correspond to the two diagonals and illustrate the relationship with the respective area in the symmetric case for which entry is deterred.

out of the market. At first sight, this implies that the efficient incumbent would face the risk of exclusion from the market in case the other two firms were forming an R&D collaboration. But in that case, why are the two efficient firms not excluding the inefficient incumbent from the market? Equation (13) tells us that if the efficient incumbent could choose with whom to compete against in a duopoly market, it would choose the less efficient firm as long as $k > 0$, which is true in our case. Therefore, the efficient incumbent prefers to establish an otherwise unwanted R&D collaboration with the inefficient one to prevent the entry of a tougher competitor (the potential entrant). In addition, the condition on ρ (i.e., $\rho > \frac{\sqrt{5}-1}{2}$) ensures that there is sufficiently high competition to prevent the firms from having incentives to form a complete network, whereas the condition on ϕ ensures that R&D collaboration between the two incumbent firms can prevent the entry of the new firm.

Figure 4 illustrates the (ρ, ϕ) region, with its respective values of α, in which the incumbent firms establish an R&D link and entry is deterred. Note that entry deterrence occurs for higher values of α too, but in that case the incentives of the efficient incumbent are not altered by potential entry. Moreover, entry deterrence is the driving force behind the incumbents' strategy in this case, because when entry occurs anyway ($\alpha < \frac{1+k-\rho}{\rho}$), the efficient incumbent would be unwilling to stick to an R&D collaboration with a less efficient partner, at least to a non-exclusive one.

Proposition 4.3. *Let $(\rho, \phi) \in (0,1)^2$ be such that $\phi < \rho(1-\rho)$. Then, if $\frac{k+\rho}{1+\rho} < \alpha < \frac{k+\rho}{1+k}$ and $\beta = 1$ the unique equilibrium network is unconnected, with the efficient incumbent firm and the entrant forming an R&D collaboration and the inefficient incumbent firm exiting the market.*

In this scenario, in contrast to the previous ones, the incentives of the efficient incumbent are not altered by potential entry. The efficient incumbent has no incentives to form an R&D link with the inefficient incumbent either with or without potential entry. Under no threat of entry, the inefficient incumbent would be able to produce a positive quantity. Yet, the presence of an efficient entrant induces an R&D collaboration with the efficient incumbent, making the environment too competitive for the inefficient incumbent and thus forcing him out of the market. Figure 5 illustrates the (ρ, ϕ) region, with its respective values of α, in which exit of the inefficient incumbent occurs and the efficient incumbent forms an R&D collaboration with the entrant.[21]

Discussion: How our main results are expected to change with the relative efficiency of the potential entrant, i.e., with the parameter β, is worth discussing here. In the main analysis we have assumed that $\beta = 1$. What would happen in case that $\beta > 1$? Of course, this crucially depends on how much more efficient the entrant is. An interesting question here is whether two inefficient incumbent firms can deter the entry of a more efficient new firm. It can be shown that this occurs as long as the entrant is not too efficient.[22]

Next, considering that $\beta < 1$ does not seem to qualitatively affect our results, as long as $\beta > \alpha$. This is because the incentives of each firm are similar to those already discussed. Namely, the efficient incumbent still prefers to stay in a duopoly (no entry) with the least efficient of the other two firms, but if entry is unavoidable then it prefers to form an R&D link with the more efficient rival or even

[21] Note that Figures 3 and 5 might give the impression that the areas corresponding to Propositions 4.1 and 4.3 may coincide, which would be in contrast with the claim that the equilibrium is unique. This is not however the case because these areas, although they may coincide on their (ρ, ϕ) projection, they do not coincide on the third dimension that corresponds to α.

[22] The exact condition for entry deterrence is $1 < \beta \leq \frac{(1+\alpha)\rho}{1-k^2}$, which can also be sustained as an equilibrium outcome for $k > 0$. Detailed analysis of this case is available upon request.

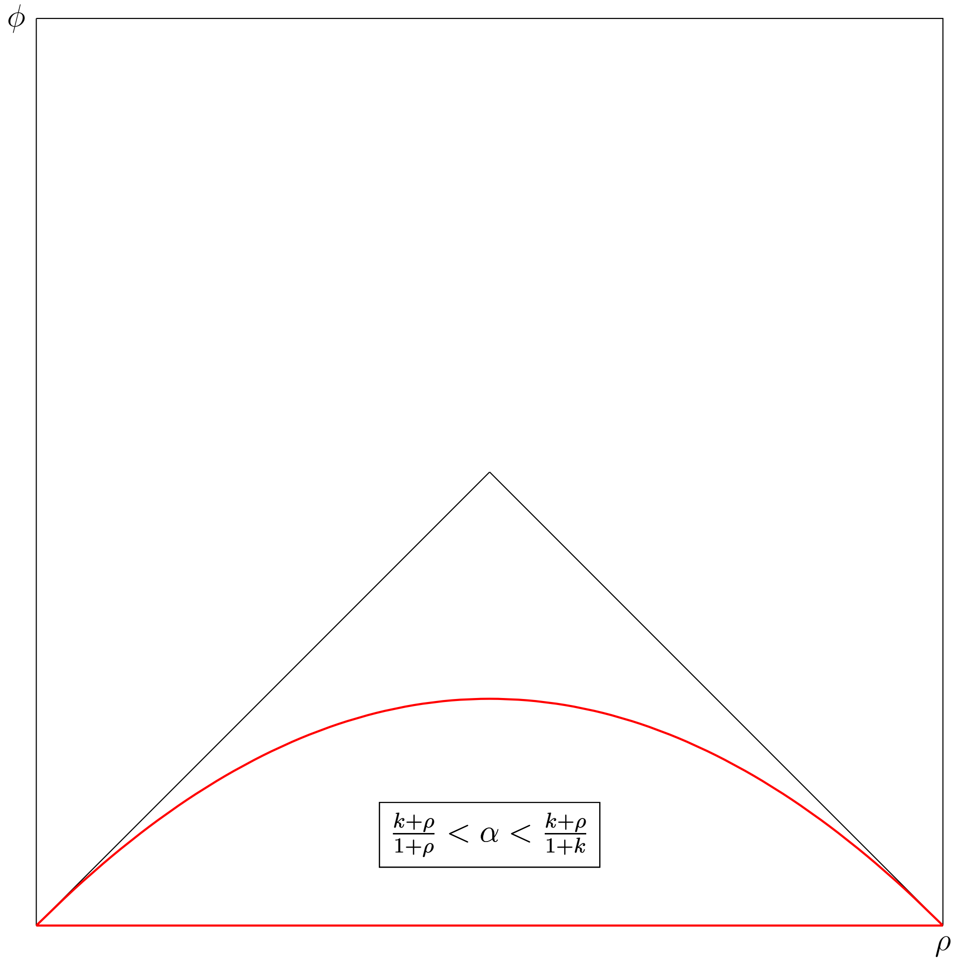


Figure 5: Two–dimensional projection of the parameter values under which Proposition 4.3 holds. The area surrounded by the red curve illustrates the parameter values (ρ, ϕ) that satisfy the first condition. The black lines correspond to the two diagonals and show the relationship with the respective area of Proposition 4.1.

with both rivals. This argument is the main driving force behind the scenarios discussed in the previous three Propositions and is expected to hold also as long as $\alpha < \beta < 1$.

Finally, if $\alpha = 1$ and $\beta < 1$, i.e., two efficient incumbents face a less efficient potential entrant, the equilibrium outcome may differ (although the calculations are identical to those already made). Of particular interest is the case that is analogous to the one described in Proposition 4.2, in which an efficient incumbent prefers to form a link with the inefficient one, in order to leave an efficient entrant out of the market. When both incumbent firms are equally efficient, they face a risk when they decide to follow such a strategy. This is because the incumbent that will exit the market will be determined randomly depending on which one of them will be the first to discuss with the entrant. Depending on its risk aversion (not modelled here though), an incumbent may risk or not its exit from the market in its effort to seek higher profits. A high risk aversion will make the incumbent firms to stick to an R&D collaboration, a strategy though that may be suboptimal for both of them.

5. Welfare Analysis

In this section we examine the effect of potential entry on social welfare. Contrary to conventional wisdom that points out that the presence of more firms in the market is expected to be welfare enhancing, we find circumstances under which entry deterrence leads to higher social welfare. This holds mainly in cases in which the intensity of competition is high relative to the level of spillovers (high k). Intuitively, the larger number of firms under entry leads to fiercer competition whose positive effect on consumer surplus is outweighed by its negative effect on firms' profits.

Social welfare is the sum of consumer surplus and firms' profits, which after some manipulations is given by

$$W = CS + \Pi = \left(\frac{1}{2} \sum_{i=1}^{n} q_i^2 + \frac{\rho}{2} \sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} q_i q_j \right) + \frac{1}{2} \sum_{i=1}^{n} q_i^2 = \sum_{i=1}^{n} q_i^2 + \frac{\rho}{2} \sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} q_i q_j,$$

with $n = 3$ under entry and $n = 2$ under no entry. Note that social welfare is increasing in the quantity produced by each individual firm. Yet, these quantities crucially depend on the R&D links formed among firms, i.e., on the network structure. In what follows, we graphically present the networks that maximize social welfare in the (ρ, ϕ) space for different values of α.[23]

Consider first the symmetric case ($\alpha = 1$). Figure 6 depicts the welfare maximizing network structures for each (ρ, ϕ) pair. Note first that neither the star nor the empty network maximize welfare for any value of the parameters. Moreover, the complete network does not always lead to maximum welfare. In fact, if spillovers are low and competition is intense (dark grey area in Figure 6), the unconnected network is welfare maximizing. Interestingly, in the bulk of this area, the unconnected network in which the entry of the new firm is deterred is the equilibrium network too. In the rest of the area, the new firm enters, yet the network remains unconnected in equilibrium (compare Figures 2 and 6). It should be noted that in the dark grey area the market and societal incentives are aligned. Nevertheless, there is a large parameter region in which the complete network is welfare maximizing, but it does not arise in equilibrium.

In the asymmetric case ($\alpha < 1$), the welfare analysis turns out to be much more complicated. In what follows we will briefly discuss a few scenarios that contain as special cases the parameter values considered in the three Propositions of the previous section.

[23] All files that are necessary for the reproduction of the graphs are available upon request.

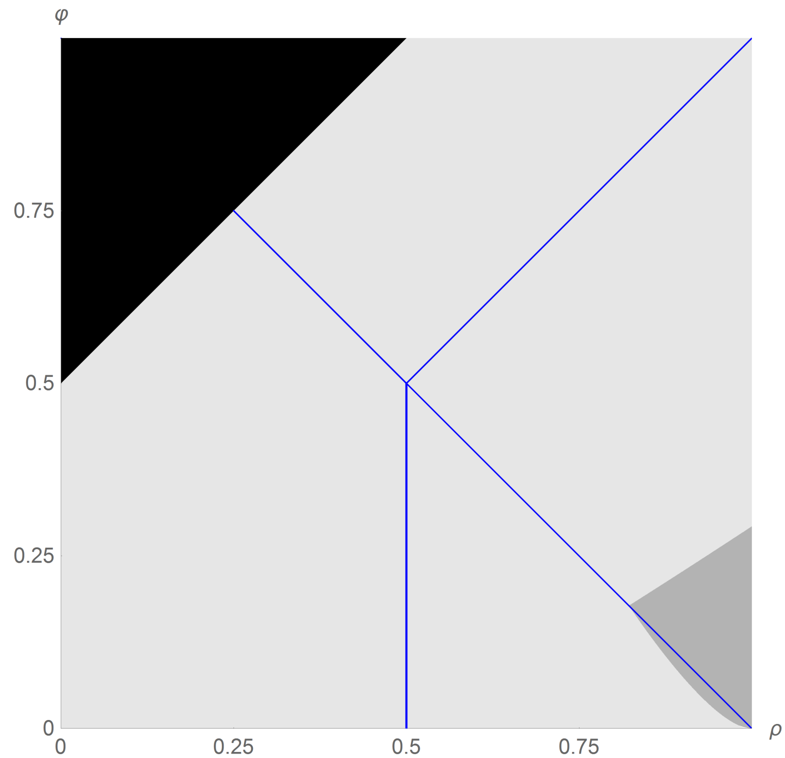
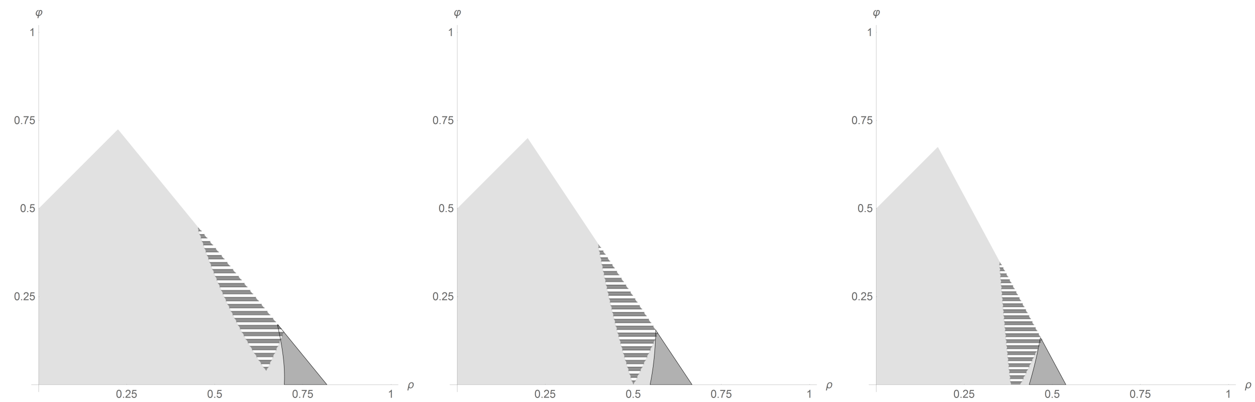


Figure 6: Welfare maximizing network structures. In light (dark) grey area the complete (unconnected) network maximize welfare. The blue lines are the same as in Figure 2.

Figure 7 illustrates for three different values of α, the (ρ, ϕ) region in which all three firms produce independently of the network structure. These include the parameter values that give rise to Proposition 4.1. As expected, this area shrinks as the inefficient incumbent becomes less efficient, i.e., as the value of α decreases. Again, in the light and dark grey areas, the complete and the unconnected $U13$ networks maximize welfare, respectively. Intuitively, unless R&D collaborations are very ineffective (low spillovers) and competition is fierce, a larger number of R&D collaborations is welfare enhancing, which is captured by the light grey areas in the three subfigures. It is only in the extreme cases in the lower right corner (low ϕ, high ρ) that an unconnected network leads to higher welfare. Yet, the unconnected is the equilibrium network in the dashed area (which corresponds to the parameter values under which Proposition 4.1 holds) that hardly intersects with the dark grey area. Therefore, the unconnected equilibrium network is typically suboptimal in terms of social welfare due to its lower connectivity compared to the complete network. This is mainly due to the farsightedness of the firms. Essentially, the efficient incumbent and the entrant are able to ensure higher profits by rejecting additional R&D collaborations, which comes though at the expense of both the inefficient incumbent's profits and the consumer surplus. Clearly, there is misalignment between market and societal incentives which becomes more serious as α decreases.

Figure 8 illustrates for three different values of α, the (ρ, ϕ) region in which the isolated firm



(a) $a = 0.9$ (b) $a = 0.8$ (c) $a = 0.7$

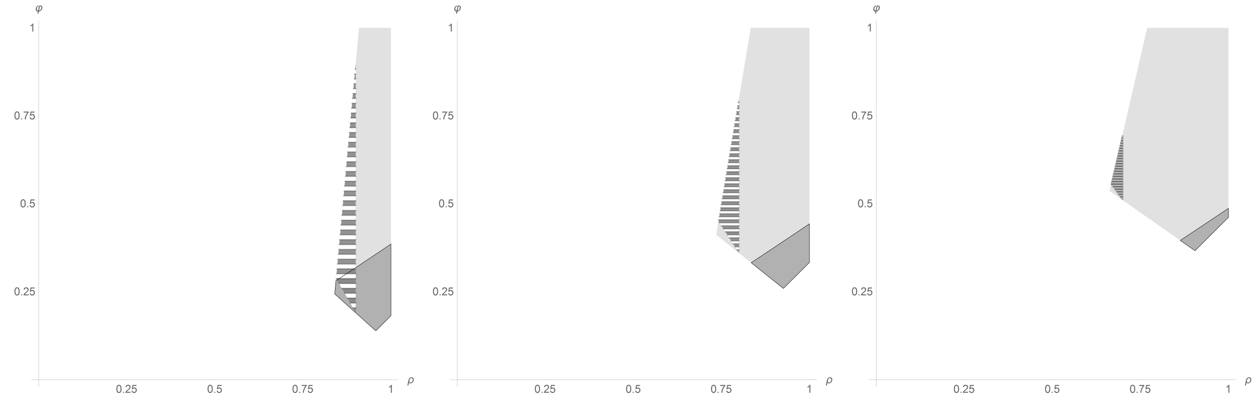
Figure 7: Parameter values for which all firms are active for different values of α. In the light (dark) grey area, the C ($U13$) network maximizes welfare. The dashed area illustrates the region described in Proposition 4.1 in which $U13$ is the equilibrium network with entry accommodation.

does not produce in an unconnected network.[24] These include the parameter values that give rise to Proposition 4.2, i.e., the (ρ, ϕ) region in which the equilibrium network is $U12$ and the entry of the new firm is deterred. Again, light and dark grey areas correspond to parameter values under which C and $U13$ network maximizes welfare, respectively, and the dashed areas corresponds to the parameter values under which Proposition 4.2 holds. It is apparent that the equilibrium network $U12$ *never* maximizes welfare, that is, market and societal incentives are never aligned here. Our finding suggests that in cases in which competition is intense and spillovers are large, entry deterrence is expected to harm welfare. Intuitively, when competition is intense, firms have incentives to form an exclusive R&D collaboration, leaving the unconnected firm out of the market, in order to soften competition. In addition, due to high spillovers, the connected firms do not need to put a lot of R&D effort. The latter makes easier their R&D collaboration, but reduces total quantity produced in the market. Overall, the increase in profits of the incumbent firms by softer competition in a $U12$ equilibrium network does not outweigh the reduction in consumer surplus due to lower total production. As a consequence, welfare is not maximized.

Finally, Figure 9 illustrates for three different values of α, the (ρ, ϕ) region in which entry cannot be deterred, that is, the entrant is active in a $U12$ network, and in which an isolated inefficient incumbent in a $U13$ network is forced to exit the market.[25] These include the parameter values that give rise to Proposition 4.3. Recall that in this case the efficient incumbent, knowing that entry cannot be deterred, prefers to establish an R&D collaboration with the entrant and force the inefficient

[24] In fact, we consider the parameter values under which the inefficient firm is also inactive in E and $S1$ networks.

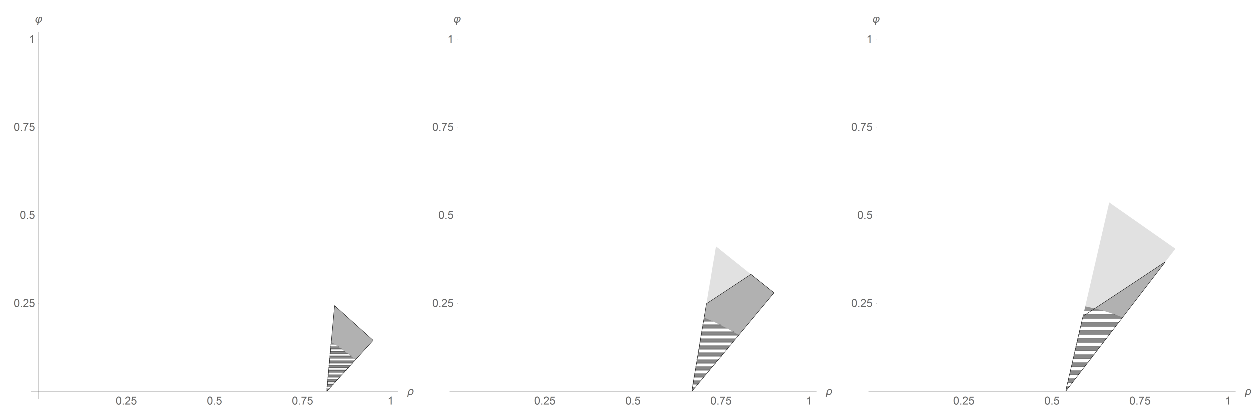
[25] Again, we consider the parameter values under which the inefficient firm is also inactive in E and $S1$ networks.



(a) $a = 0.9$ (b) $a = 0.8$ (c) $a = 0.7$

Figure 8: Parameter values for which the isolated firm is inactive on an unconnected network. In the light (dark) grey area, the C ($U13$) network maximizes welfare. The dashed area illustrates the region described in Proposition 4.2 in which $U12$ is the equilibrium network and entry is deterred.

incumbent out of the market. As above, light and dark grey areas correspond to parameter values under which C and $U13$ network maximizes welfare, respectively, and the dashed area corresponds to the parameter values under which Proposition 4.3 holds. We observe from Figure 9 that market and societal incentives are often aligned. In particular, the equilibrium network $U13$ is the one that also maximizes welfare in most cases. This implies that the presence of the inefficient incumbent in the market could not lead to higher welfare, as the two efficient firms would invest less in R&D in this case.



(a) $a = 0.9$ (b) $a = 0.8$ (c) $a = 0.7$

Figure 9: Parameter space in which the inefficient incumbent is inactive on E, $S1$ and $U13$ networks. In the light (dark) grey area, the C ($U13$) network maximizes welfare. The dashed area illustrates the region described in Proposition 4.3 in which $U13$ is the equilibrium network and exit is induced.

6. Concluding Remarks

In this paper we have investigated the formation and stability of R&D networks under potential entry. To this end, we have introduced two novel elements in the R&D network literature. First, we have postulated that firms are farsighted when they decide to form an R&D collaboration and second, we have adopted a dynamic network formation process. We believe that both those elements are particularly adequate for the analysis of R&D networks under potential entry which is a dynamic phenomenon by its nature.

Our analysis has shed light on the effects of entry on equilibrium R&D networks as well as on market outcomes and welfare. We have found that potential entry may alter the incentives of incumbent firms to form an R&D collaboration. In particular, if the potential entrant is equally efficient with the two incumbent firms, the latter establish a link and manage to deter entry as long as competition is sufficiently intense compared to the R&D spillovers. Moreover, two incumbent firms differing in their marginal costs, in order to deter entry of an efficient new firm, agree on an otherwise unprofitable R&D collaboration as long as competition is intense relative to R&D spillovers and the inefficient incumbent is quite inefficient. In addition, we have found that an efficient incumbent may refrain from establishing an otherwise profitable R&D link with its inefficient counterpart, as it anticipates the entry of an efficient new firm which is a better R&D partner. Finally, the presence of an efficient potential entrant may trigger the exit of an inefficient incumbent from the market.

We have fully characterized the network arising in equilibrium and its welfare properties for the case in which all firms have initially equal marginal costs. We have found that the unconnected network arises often in equilibrium, yet in most of these cases it is the complete network that maximizes welfare. We have also compared our equilibrium networks with those arising with the use of the static notion of pairwise stability (a concept that has been extensively used in the relevant literature) and have identified important differences. In addition, under asymmetric marginal costs, we have provided a partial characterization of equilibrium networks and their welfare implications in order to pinpoint a few interesting situations that arise due to presence of a potential entrant. In this case too, we have found that market and societal incentives for network formation are far from being aligned. This opens up an interesting discussion about potential policy measures to induce the formation of welfare superior network structures.

Our paper is the first to study network formation under potential entry and provides a useful benchmark for understanding the complex effects of entry in more general settings. In fact, a natural extension of our model would be to investigate the effect of entry on arbitrary network structures and identify the properties of networks that guarantee stability and efficiency. The presence of more than one potential entrant may also play a crucial role and it is worth analyzing too.

References

Acemoglu, D., Akcigit, U., Bloom, N. & Kerr, W.R. (2012). Innovation, reallocation and Growth. *NBER Working Paper Series*, WR no. 18993.

Arrow, K. (1962). Economic Welfare and the Allocation of Resources for Invention, in *The Rate and Direction of Innovatice Activity*, Edited by R.Nelson, Princeton University Press, Princeton.

Belleflamme, P. and Peitz, M. (2010). Industrial Organization: Markets and Strategies, Cambridge University Press, Cambridge.

Bloch, F. (1995). Endogenous Structures of Association in Oligopolies. *The RAND Journal of Economics*, 26(3): 537–556.

Czarnitzki, D., Etro, F. and Kraft, K. (2008). The effect of entry on R&D investment of leaders: Theory and empirical evidence. *Discussion Paper No.08-078*, ZEW, Switzerland.

Clood, M., Hagedoorn, J. and Roijakkers, N. (2006). Trends and Patterns in Interfirm R&D Networks in the Global Computer Industry: An Analysis of Major Developments, 1970-1999. *The Business History Review*, 80(4): 725–746.

d'Aspremont, C. and Jacquemin, A. (1988). Cooperative and noncooperative R&D in duopoly with spillovers. *The American Economic Review*, 78(5): 1133–1137.

Dawid, H. and Hellmann, T. (2014). The evolution of R&D networks. *Journal of Economic Behavior and Organization*, 105: 158–172.

Dawid, H. and Hellmann, T. (2016). R&D Investments under Endogenous Cluster Formation. *Center for Mathematical Economics University of Bielefeld Working Paper Series*, WR no. 555.

Dutta, B., Ghosal, S. and Ray, D. (2005). Farsighted network formation. *Journal of Economic Theory*, 122(2): 143–164.

Dutta, B. and Mutuswami, S. (1997). Stable Networks. *Journal of Economic Theory*, 76(2): 322–344.

Dutta, B., van den Nouweland, A. and Tijs, S. (1998). Link formation in cooperative situations. *International Journal of Game Theory*, 27(2): 245–256

Frankort, H. (2014). Open Innovation Norms and Knowledge Transfer in Interfirm Technology Alliances: Evidence from Information Technology, 1980–1999, in Ron Adner, Joanne E. Oxley, Brian S. Silverman (eds.), Collaboration and Competition in Business Ecosystems (Advances in Strategic Management, Volume 30), Emerald Group Publishing Limited, 239–282.

Frankort, H., Hagedoorn, J. and Letterie, W. (2015). Learning horizon and optimal alliance formation. *Computational and Mathematical Organization Theory*, 1–25.

Goyal, S. and Joshi, S. (2003). Networks of collaboration in oligopoly. *Games and Economic Behavior*, 43(1): 57–85.

Goyal, S. and Joshi, S. (2006). Unequal Connections. *International Journal of Industrial Organization*, 34: 319–349.

Goyal, S. and Moraga-Gonzalez, J.L. (2001). R&D networks. *RAND Journal of Economics*, 32(4): 686–707.

Goyal, S., Konovalov, A. and Moraga-Gonzalez, J.L. (2001). Hybrid networks. *Journal of the European Economic Association*, 6(6): 1309–1338.

Hagedoorn, J. (2002). Inter–firm R&D partnerships: an overview of major trends and patterns since 1960. *Research Policy*, 31(4): 477–492.

Herings P.J.J., Mauleon, A. and Vannetelbosch, V. (2009). Farsightedly Stable Networks *Games and Economic Behavior*, 67(2): 526–541.

Herings P.J.J., Mauleon, A. and Vannetelbosch, V. (2010a). A Characterization of Farsightedly Stable Networks. *Games*, 1: 226–241.

Herings P.J.J., Mauleon, A. and Vannetelbosch, V. (2010b). Coalition Formation Among Farsighted Agents. *Games*, 1: 286–298.

Hochberg, Y.V., Ljungqvist, A. and Lu, Y. (2010). Networking as a Barrier to Entry and the Competitive Supply of Venture Capital. *Journal of Finance*, 65(3): 829–859.

Jackson, M. and van den Nouweland, A. (2005). Strongly stable networks. *Games and Economic Behavior*, 51(2): 420–444.

Jackson, M. and Watts, A. (2002). The Evolution of Social and Economic Networks. *Journal of Economic Theory*, 106(2): 265–295.

Jackson, M. and Wolinsky, A. (1996). A Strategic Model of Social and Economic Networks. *Journal of Economic Theory*, 71: 44–74.

Joshi, S. (2008). Endogenous Formation of Coalitions in a Model of a Race. *Journal of Economic Behavior and Organization*, 65: 62–85.

Kesavayuth, D., Manasakis, C. and Zikos, V. (2015). Endogenous R&D Networks among Upstream Firms. *Managerial and Decision Economics*, 37(2): 79–94.

König, M., Battiston, S., Napoletano, M. and Schweitzer, F. (2011). Recombinance knowledge and the evolution of innovation networks. *Journal of Economic Behavior and Organization*, 79(3): 201–219.

König, M., Battiston, S., Napoletano, M. and Schweitzer, F. (2012). The efficiency and stability of R&D networks. *Games and Economic Behavior*, 75: 694–713.

König, M. (2013). Dynamic R&D networks. *Working Paper*, Department of Economics, University of Zurich.

König, M., Liu, X. and Zenou, Y. (2014). R&D Networks: Theory, Empirics and Policy Implications. *University of Zurich Working Paper Series*, WR no. 142.

Marinucci, M. and Vergote, W. (2011). Endogenous Network Formation in Patent Contests and its Role as a Barrier to Entry. *Journal of Industrial Economics*, 59(4): 529–551

Martin, S. (1995). R&D Joint Ventures and Tacit Product Market Collusion. *European Journal of Political Economy*, 11: 733–741.

Martin, S. (2002). Spillovers, Appropriability and R&D. *Journal of Economics*, 75: 1–32.

Mauleon, A., Sempere–Monerris, J.J. and Vannetelbosch, V. (2014). Farsighted R&D Networks. *Economics Letters*, 125: 340–342

O'Sullivan, P. (2013). Research Joint Ventures: A Barrier to Entry?. *Working Paper*, Department of Economics, Finance and Accounting, National University of Ireland, Maynooth.

Roijakkers, N. and Hagedoorn, J. (2006). Inter–firm R&D partnering in pharmaceutical biotechnology since 1975: Trends, patterns and networks. *Research Policy*, 35(3): 431–446.

Stein, J.C. (2008). Conversations among Competitors. *American Economic Review*, 98: 2150–2162.

Tomasello, M.V., Napoletano, M., Garas, A. and Schweitzer, F. (2016). The Rise and Fall of R&D Networks. *SSRN Working Paper*, id. 2749230.

Appendix

A. Equilibrium Quantities in Different Networks

Throughout the calculations we assume that A_i, c_i are all sufficiently large, so that neither the market is exhausted, nor marginal zero is eliminated. This leads to some cases where no equilibrium can be achieved because the benefits from collaborations overcome the negative effect of competition, which means that absent of other constraints firms would like to increase their quantities ever more.

The calculations are made initially for general values of α and β and are subsequently imposed the restrictions $\alpha \leq 1$ and $\beta = 1$. This is done in order to facilitate future work on more general values of the parameters. Moreover, note that allowing the possibility for corner solutions for the produced quantities does not affect the fact that profit is strictly increasing in production, i.e. $\pi_i = \frac{1}{2}q_i^2$, therefore in the calculation of equilibria we can keep focusing on quantities instead of profits.

Throughout our calculations we observe the following issue which simplifies our work. The FOC for e_i are $\frac{\partial \pi_i}{\partial e_i} = q_i - e_i \leq 0$ and $e_i \frac{\partial \pi_i}{\partial q_i} = e_i(q_i - e_i) = 0$. Notice that they do not depend on the decision of the competitor and impose that $q_i = e_i \geq 0$. If $q_i > 0$ and $e_i = 0$ then the inequality is violated, whereas if $q_i = 0$ and $e_i > 0$ the equality constrained is violated. Therefore, they are either both equal to zero, in which case both constraints are trivially satisfied, or they are both strictly positive, in which case the equality requires $q_i = e_i$ in order to be satisfied. The intuition behind this result is that the marginal cost of R&D, namely e_i, can never exceed the marginal benefit from R&D, namely q_i, therefore they are either both positive and equal, or $e_i = 0$, in which case q_i cannot be positive, because if this was the case then positive R&D effort would also be preferable. Given this result, we can substitute the equilibrium R&D effort in the FOC of q_i's and focus only on equilibrium quantities produced. To keep calculations as short as possible we present directly the FOC after having substituted $q_i = e_i$. Finally, recall that $k = \rho - \phi \in (-1, 1)$.

Table 3 summarizes the results regarding the produced quantities for different networks and parameter values.

Network	α	(ρ,ϕ)	q_1	q_2	q_3
Duop.					
E	$a \leq \rho$		1	0	$-$
	$a > \rho$		$\frac{1-\alpha\rho}{1-\rho^2}$	$\frac{\alpha-\rho}{1-\rho^2}$	$-$
C	$a \leq k$		1	0	$-$
	$a > k$		$\frac{1-\alpha k}{1-k^2}$	$\frac{\alpha-k}{1-k^2}$	$-$
Triop.					
E	$a \leq \frac{2\rho}{1+\rho}$		$\frac{1}{1+\rho}$	0	$\frac{1}{1+\rho}$
	$a > \frac{2\rho}{1+\rho}$		$\frac{1-\alpha\rho}{1+\rho-2\rho^2}$	$\frac{(1+\rho)\alpha-2\rho}{1+\rho-2\rho^2}$	$\frac{1-\alpha\rho}{1+\rho-2\rho^2}$
C	$a \leq \frac{2k}{1+k}$		$\frac{1}{1+k}$	0	$\frac{1}{1+k}$
	$a > \frac{2k}{1+k}$	$k > -1/2$	$\frac{1-\alpha k}{1+k-2k^2}$	$\frac{(1+k)\alpha-2k}{1+k-2k^2}$	$\frac{1-\alpha k}{1+k-2k^2}$
S1	$a \leq \frac{k+\rho}{1+k}$		$\frac{1}{1+k}$	0	$\frac{1}{1+k}$
	$a > \frac{k+\rho}{1+k}$	$1+\rho > 2k^2$	$\frac{1+\rho-k-k\alpha}{1+\rho-2k^2}$	$\frac{k\rho-k+k^2-\rho+(1-k^2)\alpha}{(1-\rho)(1+\rho-2k^2)}$	$\frac{1+k\rho-k-k^2-(\rho-k^2)\alpha}{(1-\rho)(1+\rho-2k^2)}$
S2	$a \leq \frac{2k}{1+\rho}$		$\frac{1}{1+\rho}$	0	$\frac{1}{1+\rho}$
	$a > \frac{2k}{1+\rho}$	$1+\rho > 2k^2$	$\frac{1-\alpha k}{1+\rho-2k^2}$	$\frac{(1+\rho)\alpha-2k}{1+\rho-2k^2}$	$\frac{1-\alpha k}{1+\rho-2k^2}$
U12	$\alpha \leq \frac{k+\rho}{1+\rho}$		$\frac{1}{1+\rho}$	0	$\frac{1}{1+\rho}$
	$\frac{k+\rho}{1+\rho} < \alpha < \frac{1+k-\rho}{\rho}$	$1+k > 2\rho^2$	$\frac{1+k\rho-\rho-\rho^2-(k-\rho^2)\alpha}{(1-k)(1+k-2\rho^2)}$	$\frac{(1-\rho^2)\alpha+k\rho+\rho^2-k-\rho}{(1-k)(1+k-2\rho^2)}$	$\frac{1+k\rho-\rho-k^2-(\rho-k\rho)\alpha}{(1-k)(1+k-2\rho^2)}$
	$\alpha \geq \frac{1+k-\rho}{\rho}$		$\frac{1-\alpha k}{1-k^2}$	$\frac{\alpha-k}{1-k^2}$	0
U13	$\alpha \leq \frac{2\rho}{1+k}$		$\frac{1}{1+k}$	0	$\frac{1}{1+k}$
	$\alpha > \frac{2\rho}{1+k}$	$1+k > 2\rho^2$	$\frac{1-\alpha\rho}{1+k-2\rho^2}$	$\frac{(1+k)\alpha-2\rho}{1+k-2\rho^2}$	$\frac{1-\alpha\rho}{1+k-2\rho^2}$

Table 3: Produced quantities in different networks given parameter values α, ρ, ϕ.

A.1. No Entry

A.1.1. Complete Network

$$\pi_1 = (1 - q_1 - \rho q_2 + e_1 + \phi e_2)q_1 - \frac{1}{2}e_1^2$$
$$\pi_2 = (\alpha - q_2 - \rho q_1 + e_2 + \phi e_1)q_2 - \frac{1}{2}e_2^2$$

such that $q_1, e_1, q_2, e_2 \geq 0$. The first order conditions are the following:

$$1 - q_1 - kq_2 \leq 0 \tag{1}$$
$$q_1(1 - q_1 - kq_2) = 0 \tag{2}$$
$$\alpha - q_2 - kq_1 \leq 0 \tag{3}$$
$$q_2(\alpha - q_2 - kq_1) = 0 \tag{4}$$

To solve the system we consider two cases for each variable $q_i = 0$ and $q_i > 0$ respectively.

1. $q_1 = 0$ and $q_2 = 0$: In this case directly fails Inequality 1.
2. $q_1 = 0$ and $q_2 > 0 \Rightarrow kq_2 \geq 1$ and $q_2 = \alpha$ by Inequality 1 and Equality 4 respectively. But $\alpha \in (0,1)$ and $k \in (-1,1)$, therefore $kq_2 < 1$ the argument fails.
3. $q_1 > 0$ and $q_2 = 0 \Rightarrow q_1 = 1$ and $kq_1 \geq \alpha$ by Equality 2 and Inequality 3 respectively. Hence, this holds for $\alpha \leq k$, which note that can only be true whenever $k > 0$.
4. $q_1 > 0$ and $q_2 > 0 \Rightarrow q_1 + kq_2 = 1$ and $kq_1 + q_2 = \alpha$ by Equality 2 and Equality 4 respectively. This is a linear system that has a unique solution for all $k \in (-1,1)$, which is: $q_1 = \frac{1-\alpha k}{1-k^2}$ and $q_2 = \frac{\alpha-k}{1-k^2}$. For this solution to be valid, it needs to be the case that $q_1, q_2 > 0$. Note that the denominator is always positive, as well as the numerator of q_1 (since $\alpha \in (0,1)$ and $k \in (-1,1)$). Hence, the only necessary condition is that $\alpha > k$.

Overall the solution to the problem is the following:

- For $\alpha \leq k \Rightarrow q_1 = e_1 = 1$ and $q_2 = e_2 = 0$.
- For $\alpha > k \Rightarrow q_1 = e_1 = \frac{1-\alpha k}{1-k^2}$ and $q_2 = e_2 = \frac{\alpha-k}{1-k^2}$.

And for the special case of $\alpha = 1$:

- $q_1 = e_1 = q_2 = e_2 = \frac{1}{1+k}$

A.1.2. Empty Network

$$\pi_1 = (1 - q_1 - \rho q_2 + e_1)q_1 - \frac{1}{2}e_1^2$$
$$\pi_2 = (\alpha - q_2 - \rho q_1 + e_2)q_2 - \frac{1}{2}e_2^2$$

such that $q_1, e_1, q_2, e_2 \geq 0$. The first order conditions are the following:

$$1 - q_1 - \rho q_2 \leq 0 \tag{1}$$
$$q_1(1 - q_1 - \rho q_2) = 0 \tag{2}$$
$$\alpha - q_2 - \rho q_1 \leq 0 \tag{3}$$
$$q_2(\alpha - q_2 - \rho q_1) = 0 \tag{4}$$

To solve the system we consider two cases for each variable $q_i = 0$ and $q_i > 0$ respectively.

1. $q_1 = 0$ and $q_2 = 0$: In this case directly fails Inequality 1.
2. $q_1 = 0$ and $q_2 > 0 \Rightarrow \rho q_2 \geq 1$ and $q_2 = \alpha$ by Inequality 1 and Equality 4 respectively. But $\alpha \in (0,1)$ and $\rho \in (0,1)$, therefore $\rho q_2 < 1$ the argument fails.
3. $q_1 > 0$ and $q_2 = 0 \Rightarrow q_1 = 1$ and $\rho q_1 \geq \alpha$ by Equality 2 and Inequality 3 respectively. Hence, this holds for $\alpha \leq \rho$.
4. $q_1 > 0$ and $q_2 > 0 \Rightarrow q_1 + \rho q_2 = 1$ and $\rho q_1 + q_2 = \alpha$ by Equality **??** and Equality 4 respectively.

This is a linear system that has a unique solution for all $\rho \in (0,1)$, which is: $q_1 = \frac{1-\alpha\rho}{1-\rho^2}$ and $q_2 = \frac{\alpha-\rho}{1-\rho^2}$. For this solution to be valid, it needs to be the case that $q_1, q_2 > 0$. Note that the denominator is always positive, as well as the numerator of q_1 (since $\alpha \in (0,1)$ and $\rho \in (0,1)$). Hence, the only necessary condition is that $\alpha > \rho$.

Overall the solution to the problem is the following:

- For $\alpha \leq \rho \Rightarrow q_1 = e_1 = 1$ and $q_2 = e_2 = 0$.
- For $\alpha > \rho \Rightarrow q_1 = e_1 = \frac{1-\alpha\rho}{1-\rho^2}$ and $q_2 = e_2 = \frac{\alpha-\rho}{1-\rho^2}$.

And for the special case of $\alpha = 1$:

- $q_1 = e_1 = q_2 = e_2 = \frac{1}{1+\rho}$

A.2. Potential Entry

A.2.1. Empty Network

$$\pi_1 = (1 - q_1 - \rho q_2 - \rho q_3 + e_1)q_1 - \frac{1}{2}e_1^2$$
$$\pi_2 = (\alpha - q_2 - \rho q_1 - \rho q_3 + e_2)q_2 - \frac{1}{2}e_2^2$$
$$\pi_3 = (\beta - q_3 - \rho q_1 - \rho q_2 + e_3)q_3 - \frac{1}{2}e_3^2$$

such that $q_1, e_1, q_2, e_2, q_3, e_3 \geq 0$. The first order conditions are the following:

$$1 - q_1 - \rho q_2 - \rho q_3 \leq 0 \tag{1}$$
$$q_1(1 - q_1 - \rho q_2 - \rho q_3) = 0 \tag{2}$$
$$\alpha - q_2 - \rho q_1 - \rho q_3 \leq 0 \tag{3}$$
$$q_2(\alpha - q_2 - \rho q_1 - \rho q_3) = 0 \tag{4}$$
$$\beta - q_3 - \rho q_1 - \rho q_2 \leq 0 \tag{5}$$
$$q_3(\beta - q_3 - \rho q_1 - \rho q_2) = 0 \tag{6}$$

To solve the system we consider two cases for each variable $q_i = 0$ and $q_i > 0$ respectively.

1. $q_1 = 0$, $q_2 = 0$ and $q_3 = 0$: In this case directly fails Inequality 1.
2. $q_1 = 0$, $q_2 = 0$ and $q_3 > 0 \Rightarrow \rho q_3 \geq 1$, $\rho q_3 \geq \alpha$ and $q_3 = \beta$ by Inequalities 1 and 3 and Equality 6 respectively. But $\beta = 1$ and $\rho \in (0,1)$ imply $\rho q_3 < 1$ the argument fails.
3. $q_1 = 0$, $q_2 > 0$ and $q_3 = 0 \Rightarrow \rho q_2 \geq 1$, $q_2 = \alpha$ and $\rho q_2 \geq \beta$ by Inequality 1, Equality 4 and Inequality 5 respectively. But $\alpha \in (0,1)$ and $\rho \in (0,1)$ imply $\rho q_2 < 1$ the argument fails.
4. $q_1 = 0$, $q_2 > 0$ and $q_3 > 0 \Rightarrow \rho q_2 + \rho q_3 \geq 1$, $q_2 + \rho q_3 = \alpha$ and $q_3 + \rho q_2 = \beta$ by Inequality 1 and Equalities 4 and 6 respectively. The two equalities induce a linear system with unique solution for all $\rho \in (0,1)$, which is: $q_2 = \frac{\alpha-\beta\rho}{1-\rho^2}$ and $q_3 = \frac{\beta-\alpha\rho}{1-\rho^2}$, where both q_1, q_2 need to be

strictly positive, i.e. $\alpha > \beta\rho$ and $\beta > \alpha\rho$ and satisfy the inequality $\rho q_2 + \rho q_3 \geq 1$. But, $\rho q_2 + \rho q_3 = \rho\left(\frac{\alpha-\beta\rho}{1-\rho^2} + \frac{\beta-\alpha\rho}{1-\rho^2}\right) = \frac{(\alpha+\beta)\rho(1-\rho)}{1-\rho^2} = \frac{(\alpha+\beta)\rho}{1+\rho} \geq 1 \Leftrightarrow \alpha + \beta \geq \frac{1+\rho}{\rho} = 1 + \frac{1}{\rho}$ which cannot hold for $\beta = 1$ and $\alpha < 1$.

5. $q_1 > 0$, $q_2 = 0$ and $q_3 = 0 \Rightarrow q_1 = 1, \rho q_1 \geq \alpha$ and $\rho q_1 \geq \beta$ by Equality 2 and Inequalities 3 and 5 respectively. But the last inequality cannot hold for $\beta = 1$ and $\rho \in (0, 1)$.
6. $q_1 > 0$, $q_2 = 0$ and $q_3 > 0 \Rightarrow q_1 + \rho q_3 = 1, \rho q_1 + q_3 = \beta$ and $\rho q_1 + \rho q_3 \geq \alpha$ by Equalities 2, 6 and Inequality 3. The two equalitiies lead to a linear system with unique solution for all $k \in (-1, 1)$, which is $q_1 = \frac{1-\beta\rho}{1-\rho^2}$ and $q_3 = \frac{\beta-\rho}{1-\rho^2}$ that for $\beta = 1$ leads to $q_1 = q_3 = \frac{1}{1+\rho}$. For this to be a solution it has to hold that $\rho q_1 + \rho q_3 \geq \alpha \Rightarrow a \leq \frac{2\rho}{1+\rho}$.
7. $q_1 > 0$, $q_2 > 0$ and $q_3 = 0 \Rightarrow q_1 + \rho q_2 = 1, \rho q_1 + q_2 = \alpha$ and $\rho q_1 + \rho q_2 \geq \beta$ by Equalities 2, 4 and Inequality 5. The two equalitiies lead to a linear system with unique solution for all $\rho \in (0, 1)$, which is $q_1 = \frac{1-\alpha\rho}{1-\rho^2}$ and $q_2 = \frac{\alpha-\rho}{1-\rho^2}$, which requires $\alpha > \rho$. The solution needs to satisfy $\rho q_1 + \rho q_2 \geq \beta \Rightarrow \rho\frac{1-\alpha\rho+\alpha-\rho}{1-\rho^2} \geq \beta \Rightarrow \frac{(1+\alpha)\rho}{1+\rho} \geq \beta$ which cannot hold for $\alpha < 1$ and $\beta = 1$.
8. $q_1 > 0$, $q_2 > 0$ and $q_3 > 0 \Rightarrow q_1 + \rho q_2 + \rho q_3 = 1, \rho q_1 + q_2 + \rho q_3 = \alpha$ and $\rho q_1 + \rho q_2 + q_3 = \beta$ by Equalities 2, 4 and 6 respectively. The three equalities form a linear system with unique and fully positive solution when $a > \frac{2\rho}{1+\rho}$. The solution is $q_1 = q_3 = \frac{1-\alpha\rho}{1+\rho-2\rho^2}$ and $q_2 = \frac{\alpha(1+\rho)-2\rho}{1+\rho-2\rho^2}$.

Overall the solution to the problem is the following:

- For $\alpha \leq \frac{2\rho}{1+\rho} \Rightarrow q_1 = e_1 = q_3 = e_3 = \frac{1}{1+\rho}$ and $q_2 = e_2 = 0$.
- For $\alpha > \frac{2\rho}{1+\rho} \Rightarrow q_1 = e_1 = q_3 = e_3 = \frac{1-\alpha\rho}{1+\rho-2\rho^2}$ and $q_2 = e_2 = \frac{\alpha(1+\rho)-2\rho}{1+\rho-2\rho^2}$.

And for the special case of $\alpha = 1$:

- $q_1 = e_1 = q_2 = e_2 = q_3 = e_3 = \frac{1}{1+2\rho}$

A.2.2. Complete Network

$$\pi_1 = (1 - q_1 - \rho q_2 - \rho q_3 + e_1 + \phi e_2 + \phi e_3)q_1 - \frac{1}{2}e_1^2$$
$$\pi_2 = (\alpha - q_2 - \rho q_1 - \rho q_3 + e_2 + \phi e_1 + \phi e_3)q_2 - \frac{1}{2}e_2^2$$
$$\pi_3 = (\beta - q_3 - \rho q_1 - \rho q_2 + e_3 + \phi e_1 + \phi e_2)q_3 - \frac{1}{2}e_3^2$$

such that $q_1, e_1, q_2, e_2, q_3, e_3 \geq 0$. The first order conditions are the following:

$$1 - q_1 - kq_2 - kq_3 \leq 0 \tag{1}$$

$$q_1(1 - q_1 - kq_2 - kq_3) = 0 \tag{2}$$

$$\alpha - q_2 - kq_1 - kq_3 \leq 0 \tag{3}$$

$$q_2(\alpha - q_2 - kq_1 - kq_3) = 0 \tag{4}$$

$$\beta - q_3 - kq_1 - kq_2 \leq 0 \tag{5}$$

$$q_3(\beta - q_3 - kq_1 - kq_2) = 0 \tag{6}$$

To solve the system we consider two cases for each variable $q_i = 0$ and $q_i > 0$ respectively.

1. $q_1 = 0$, $q_2 = 0$ and $q_3 = 0$: In this case directly fails Inequality 1.
2. $q_1 = 0$, $q_2 = 0$ and $q_3 > 0 \Rightarrow kq_3 \geq 1$, $kq_3 \geq \alpha$ and $q_3 = \beta$ by Inequalities 1 and 3 and Equality 6 respectively. But $\beta = 1$ and $\rho \in (0,1)$ imply $kq_3 = k\beta < 1$ and the argument fails.
3. $q_1 = 0$, $q_2 > 0$ and $q_3 = 0 \Rightarrow kq_2 \geq 1$, $q_2 = \alpha$ and $kq_2 \geq \beta$ by Inequality 1, Equality 4 and Inequality 5 respectively. But $\alpha \in (0,1)$ and $k \in (-1,1)$ imply $kq_2 < 1$ so the argument fails.
4. $q_1 = 0$, $q_2 > 0$ and $q_3 > 0 \Rightarrow kq_2 + kq_3 \geq 1$, $q_2 + kq_3 = \alpha$ and $q_3 + kq_2 = \beta$ by Inequality 1 and Equalities 4 and 6 respectively. The two equalities induce a linear system with unique solution for all $k \in (-1,1)$, which is: $q_2 = \frac{\alpha - \beta k}{1-k^2}$ and $q_3 = \frac{\beta - \alpha k}{1-k^2}$, where both q_1, q_2 need to be strictly positive, i.e. $\alpha > \beta k$ and $\beta > \alpha k$ and satisfy the inequality $kq_2 + kq_3 \geq 1$. But, $kq_2 + kq_3 = k\left(\frac{\alpha-\beta k}{1-k^2} + \frac{\beta-\alpha k}{1-k^2}\right) = \frac{(\alpha+\beta)k(1-k)}{1-k^2} = \frac{(\alpha+\beta)k}{1+k} \geq 1$ which fails immediately for $k \leq 0$, but even if $k > 0$ it needs $\alpha + \beta \geq \frac{1+k}{k} = 1 + \frac{1}{k}$ which cannot hold for $\beta = 1$ and $\alpha < 1$.
5. $q_1 > 0$, $q_2 = 0$ and $q_3 = 0 \Rightarrow q_1 = 1, kq_1 \geq \alpha$ and $kq_1 \geq \beta$ by Equality 2 and Inequalities 3 and 5 respectively. But the last inequality cannot hold for $\beta = 1$ and $k \in (-1,1)$.
6. $q_1 > 0$, $q_2 = 0$ and $q_3 > 0 \Rightarrow q_1 + kq_3 = 1, kq_1 + q_3 = \beta$ and $kq_1 + kq_3 \geq \alpha$ by Equalities 2, 6 and Inequality 3. The two equalitiies lead to a linear system with unique solution for all $k \in (-1,1)$, which is $q_1 = \frac{1-\beta k}{1-k^2}$ and $q_{=}\frac{\beta-k}{1-k^2}$ that for $\beta = 1$ leads to $q_1 = q_3 = \frac{1}{1+k}$. For this to be a solution it has to hold that $kq_1 + kq_3 \geq \alpha \Rightarrow a \leq \frac{2k}{1+k}$, for which to hold a necessary condition is $k > 0$.
7. $q_1 > 0$, $q_2 > 0$ and $q_3 = 0 \Rightarrow q_1 + kq_2 = 1, kq_1 + q_2 = \alpha$ and $kq_1 + kq_2 \geq \beta$ by Equalities 2, 4 and Inequality 5. The two equalitiies lead to a linear system with unique solution for all $k \in (-1,1)$, which is $q_1 = \frac{1-\alpha k}{1-k^2}$ and $q_2 = \frac{\alpha-k}{1-k^2}$, which requires $\alpha > k$. The solution needs to satisfy $kq_1 + kq_2 \geq \beta \Rightarrow k\frac{1-\alpha k+\alpha-k}{1-k^2} \geq \beta \Rightarrow \frac{(1+\alpha)k}{1+k} \geq \beta$ which cannot hold for $\alpha < 1$ and $\beta = 1$.
8. $q_1 > 0$, $q_2 > 0$ and $q_3 > 0 \Rightarrow q_1 + kq_2 + kq_3 = 1, kq_1 + q_2 + kq_3 = \alpha$ and $kq_1 + kq_2 + q_3 = \beta$ by Equalities 2, 4 and 6 respectively. The three equalities form a linear system with unique and fully positive solution when $k > -1/2$ and $a > \frac{2k}{1+k}$. The solution is $q_1 = q_3 = \frac{1-\alpha k}{1+k-2k^2}$ and $q_2 = \frac{\alpha(1+k)-2k}{1+k-2k^2}$.

In case $k \leq -1/2$ the collaborations provide much larger benefit than the additional competition they induce, which leads firms to want to produce ever more. In that case, absent of upper bounds induced by $\bar{A}_i$ and $\bar{c}_i$ the firms would produce infinitely much. This means that no matter how high these bounds are, some of them will have to bind. Formally, we show that without considering upper bounds for any feasible q_1, q_2, q_3 one can find q_1', q_2', q_3' that lead all firms to higher profits and are still feasible. Let q_1, q_2, q_3 such that $q_1 + kq_2 + kq_3 \geq 1$, $kq_1 + q_2 + kq_3 \geq \alpha$ and $kq_1 + kq_2 + q_3 \geq 1$ and let $q_1 = q_3$. We show that we can always increase both q_1, q_2. Namely, from the first two inequalities we get $q_1 \leq \frac{1}{1+k} - \frac{k}{1+k}q_2$ and $q_2 \leq \alpha - 2kq_1$ and increase q_1 until the first one binds, hence $q_1' = \frac{1}{1+k} - \frac{k}{1+k}q_2 \Rightarrow q_2\frac{1+k-2k^2}{1+k} \leq \alpha - \frac{2}{1+k}$ which is always true with inequality for $k < -1/2$, therefore we can increase q_2 to q_2' until the constraint binds. But an increase in q_2 relaxes the first constraint, which means that one can again increase q_1, which leads to an everlasting cycle of quantity increases and no equilibrium unless either some marginal cost vanishes.

Overall the solution to the problem is the following:

- For $\alpha \leq \frac{2k}{1+k} \Rightarrow q_1 = e_1 = q_3 = e_3 = \frac{1}{1+k}$ and $q_2 = e_2 = 0$.
- For $\alpha > \frac{2k}{1+k}$ and $k > -1/2 \Rightarrow q_1 = e_1 = q_3 = e_3 = \frac{1-\alpha k}{1+k-2k^2}$ and $q_2 = e_2 = \frac{\alpha(1+k)-2k}{1+k-2k^2}$.
- For $\alpha > \frac{2k}{1+k}$ and $k \leq -1/2$ there is no equilibrium unless some marginal cost becomes equal to zero.

And for the special case of $\alpha = 1$:

- For $k > -1/2 \Rightarrow q_1 = e_1 = q_2 = e_2 = q_3 = e_3 = \frac{1}{1+2k}$
- For $k \leq -1/2$ there is no equilibrium unless some marginal cost becomes equal to zero.

A.2.3. Star Network with 1 as a center - S1

$$\pi_1 = (1 - q_1 - \rho q_2 - \rho q_3 + e_1 + \phi e_2 + \phi e_3)q_1 - \frac{1}{2}e_1^2$$
$$\pi_2 = (\alpha - q_2 - \rho q_1 - \rho q_3 + e_2 + \phi e_1)q_2 - \frac{1}{2}e_2^2$$
$$\pi_3 = (\beta - q_3 - \rho q_1 - \rho q_2 + e_3 + \phi e_1)q_3 - \frac{1}{2}e_3^2$$

such that $q_1, e_1, q_2, e_2, q_3, e_3 \geq 0$. The first order conditions are the following:

$$1 - q_1 - kq_2 - kq_3 \leq 0 \tag{1}$$

$$q_1(1 - q_1 - kq_2 - kq_3) = 0 \tag{2}$$

$$\alpha - q_2 - kq_1 - \rho q_3 \leq 0 \tag{3}$$

$$q_2(\alpha - q_2 - kq_1 - \rho q_3) = 0 \tag{4}$$

$$\beta - q_3 - kq_1 - \rho q_2 \leq 0 \tag{5}$$

$$q_3(\beta - q_3 - kq_1 - \rho q_2) = 0 \tag{6}$$

To solve the system we consider two cases for each variable $q_i = 0$ and $q_i > 0$ respectively.

1. $q_1 = 0$, $q_2 = 0$ and $q_3 = 0$: In this case directly fails Inequality 1.
2. $q_1 = 0$, $q_2 = 0$ and $q_3 > 0 \Rightarrow kq_3 \geq 1$, $\rho q_3 \geq \alpha$ and $q_3 = \beta$ by Inequalities 1 and 3 and Equality 6 respectively. But $\beta = 1$ and $k \in (-1, 1)$ imply $kq_3 = k\beta < 1$ that fails.
3. $q_1 = 0$, $q_2 > 0$ and $q_3 = 0 \Rightarrow kq_2 \geq 1$, $q_2 = \alpha$ and $\rho q_2 \geq \beta$ by Inequality 1, Equality 4 and Inequality 5 respectively. But $\alpha \in (0, 1)$ and $k \in (-1, 1)$ imply $kq_2 < 1$ so the argument fails.
4. $q_1 = 0$, $q_2 > 0$ and $q_3 > 0 \Rightarrow kq_2 + kq_3 \geq 1$, $q_2 + \rho q_3 = \alpha$ and $q_3 + \rho q_2 = \beta$ by Inequality 1 and Equalities 4 and 6 respectively. The two equalities form a linear system with unique solution for all $\rho \in (0, 1)$, which is: $q_2 = \frac{\alpha-\beta\rho}{1-\rho^2}$ and $q_3 = \frac{\beta-\alpha\rho}{1-\rho^2}$, where both q_1, q_2 need to be strictly positive, i.e. $\alpha > \beta\rho$ and $\beta > \alpha\rho$ and satisfy $kq_2 + kq_3 \geq 1$. But, $kq_2 + kq_3 = k\left(\frac{\alpha-\beta\rho}{1-\rho^2} + \frac{\beta-\alpha\rho}{1-\rho^2}\right) = \frac{(\alpha+\beta)k(1-\rho)}{1-\rho^2} = \frac{(\alpha+\beta)k}{1+\rho} \geq 1$ which fails immediately for $k \leq 0$, but even if $k > 0$ it needs $\alpha + \beta \geq \frac{1+\rho}{k}$ which cannot hold for $\beta = 1$ and $\alpha < 1$, because it requires $\alpha \geq \frac{1+\rho-k}{k} > 1$.
5. $q_1 > 0$, $q_2 = 0$ and $q_3 = 0 \Rightarrow q_1 = 1, kq_1 \geq \alpha$ and $kq_1 \geq \beta$ by Equality 2 and Inequalities 3 and 5 respectively. But the last inequality cannot hold for $\beta = 1$ and $k \in (-1, 1)$.
6. $q_1 > 0$, $q_2 = 0$ and $q_3 > 0 \Rightarrow q_1 + kq_3 = 1, kq_1 + q_3 = \beta$ and $kq_1 + \rho q_3 \geq \alpha$ by Equalities 2, 6 and Inequality 3. The two equalitiies lead to a linear system with unique solution for all $k \in (-1, 1)$, which is $q_1 = \frac{1-\beta k}{1-k^2}$ and $q = \frac{\beta-k}{1-k^2}$ that for $\beta = 1$ leads to $q_1 = q_3 = \frac{1}{1+k}$. For this to be a solution it has to hold that $kq_1 + \rho q_3 \geq \alpha \Rightarrow \alpha \leq \frac{k+\rho}{1+k}$.
7. $q_1 > 0$, $q_2 > 0$ and $q_3 = 0 \Rightarrow q_1 + kq_2 = 1, kq_1 + q_2 = \alpha$ and $kq_1 + \rho q_2 \geq \beta$ by Equalities 2, 4 and Inequality 5. The two equalitiies lead to a linear system with unique solution for all $k \in (-1, 1)$, which is $q_1 = \frac{1-\alpha k}{1-k^2}$ and $q_2 = \frac{\alpha-k}{1-k^2}$, which requires $\alpha > k$. The solution needs to satisfy $kq_1 + \rho q_2 \geq \beta \Rightarrow \frac{k(1-\alpha k)+\rho(\alpha-k)}{1-k^2} \geq \beta \Rightarrow \alpha(\rho - k^2) \geq 1 + k\rho - k - k^2$, for $\beta = 1$, which fails immediately if $k^2 \geq \rho$ and otherwise requires $\alpha \geq \frac{1+k\rho-k-k^2}{\rho-k^2} > 1$, therefore the argument fails. The last inequality holds because $\frac{1+k\rho-k-k^2}{\rho-k^2} > 1 \Leftrightarrow 1 + k\rho - k - k^2 > \rho - k^2 \Leftrightarrow (1-\rho)(1-k) > 0$ which is always true.

8. $q_1 > 0$, $q_2 > 0$ and $q_3 > 0 \Rightarrow q_1 + kq_2 + kq_3 = 1, kq_1 + q_2 + \rho q_3 = \alpha$ and $kq_1 + \rho q_2 + q_3 = \beta$ by Equalities 2, 4 and 6 respectively. The three equalities form a linear system with unique and fully positive solution when $1 + \rho - 2k^2 > 0$ and $a > \frac{k+\rho}{1+k}$. The solution for $\beta = 1$ is $q_1 = \frac{1+\rho-k-k\alpha}{1+\rho-2k^2}$, $q_2 = \frac{-(1-k)k-(1-k)\rho+(1-k^2)\alpha}{(1-\rho)(1+\rho-2k^2)}$ and $q_3 = \frac{-k(1-\rho)+(1-k^2)-(\rho-k^2)\alpha}{(1-\rho)(1+\rho-2k^2)}$. The two conditions ensure that all quantities are strictly positive. In fact, it is easy to see that the numerator of q_1 is always positive, for the numerator of q_2 the condition is $a > \frac{k+\rho}{1+k}$, whereas for q_3 is $\alpha > \frac{1+k\rho-k-k^2}{\rho-k^2}$ which is weaker than the one for q_2.
   In case $a > \frac{k+\rho}{1+k}$ but $1 + \rho - 2k^2 \leq 0$ the collaborations provide much larger benefit than the additional competition they induce, which leads firms to want to produce ever more. In that case, absent of upper bounds induced by $\bar{A}_i$ and $\bar{c}_i$ the firms would produce infinitely much. This means that no matter how high these bounds are, some of the will have to bind. Formally, we show that without considering upper bounds for any feasible q_1, q_2, q_3 one can find q'_1, q'_2, q'_3 that lead all firms to higher profits and are still feasible. Let q_1, q_2, q_3 such that $q_1 + kq_2 + kq_3 \geq 1$, $kq_1 + q_2 + \rho q_3 \geq \alpha$ and $kq_1 + \rho q_2 + q_3 \geq 1$ and consider the case where the first and the third condition bind. In this case, solving the linear system with respect to q_1, q_3 we get $q_3 = \frac{(1-k)-(\rho-k^2)q_2}{1-k^2}$ and $q_1 = 1 - kq_2 - \frac{k}{1+k} + \frac{k(\rho-k^2)}{1-k^2}q_2$, which after some calculation leads the second condition to the form $\frac{\rho+k}{1+k} + \frac{(1-\rho)(1+\rho-2k^2)}{1-k^2} \leq \alpha$ which is always true with strict inequality because the first term of left hand-side is smaller than the right hand side and the second term is negative. Hence, we could increase q_2 until the condition binds. This will relax the condition for q_1, but it would violate the condition for q_3. Nevertheless, the fact that q_1's condition is relaxed means that we could further increase q_1. If this increase is sufficient to overcome the violation coming from the increase of q_2, then the argument is completed. In particular, for an increase of q_2 equal to δ, the respective increase in q_1 is equal to $-k\delta$ (k is necessary negative if $1 + \rho - 2k^2 \leq 0$.). The effect of these changes in the condition of q_3 should satisfy $k(-k\delta) + \rho\delta \leq 0 \Leftrightarrow k^2 \geq \rho$ which is always true for $1 + \rho - 2k^2 \leq 0$. Therefore, all three quantities can be increased without violating any of the constraints which leads to an everlasting cycle of quantity increases and no equilibrium unless either some marginal cost vanishes.

Overall the solution to the problem is the following:

- For $\alpha \leq \frac{k+\rho}{1+k} \Rightarrow q_1 = e_1 = q_3 = e_3 = \frac{1}{1+k}$ and $q_2 = e_2 = 0$.
- For $\alpha > \frac{k+\rho}{1+k}$ and $1 + \rho - 2k^2 > 0 \Rightarrow q_1 = e_1 = \frac{1+\rho-k-k\alpha}{1+\rho-2k^2}, q_2 = e_2 = \frac{-(1-k)k-(1-k)\rho+(1-k^2)\alpha}{(1-\rho)(1+\rho-2k^2)}$ and $q_3 = e_3 = \frac{-k(1-\rho)+(1-k^2)-(\rho-k^2)\alpha}{(1-\rho)(1+\rho-2k^2)}$
- For $\alpha > \frac{k+\rho}{1+k}$ and $1 + \rho - 2k^2 \leq 0$ there is no equilibrium unless some marginal cost becomes equal to zero.

And for the special case of $\alpha = 1$:

- For $1 + \rho - 2k^2 > 0 \Rightarrow q_1 = e_1 = \frac{1+\rho-2k}{1+\rho-2k^2}, q_2 = e_2 = q_3 = e_3 = \frac{1-k}{1+\rho-2k^2}$.
- For $1 + \rho - 2k^2 \leq 0$ there is no equilibrium unless some marginal cost vanishes.

A.2.4. Star Network with 2 as a center - S2

$$\pi_1 = (1 - q_1 - \rho q_2 - \rho q_3 + e_1 + \phi e_2)q_1 - \frac{1}{2}e_1^2$$
$$\pi_2 = (\alpha - q_2 - \rho q_1 - \rho q_3 + e_2 + \phi e_1 + \phi e_3)q_2 - \frac{1}{2}e_2^2$$
$$\pi_3 = (\beta - q_3 - \rho q_1 - \rho q_2 + e_3 + \phi e_2)q_3 - \frac{1}{2}e_3^2$$

such that $q_1, e_1, q_2, e_2, q_3, e_3 \geq 0$. The first order conditions are the following:

$$1 - q_1 - kq_2 - \rho q_3 \leq 0 \tag{1}$$
$$q_1(1 - q_1 - kq_2 - \rho q_3) = 0 \tag{2}$$
$$\alpha - q_2 - kq_1 - kq_3 \leq 0 \tag{3}$$
$$q_2(\alpha - q_2 - kq_1 - kq_3) = 0 \tag{4}$$
$$\beta - q_3 - \rho q_1 - kq_2 \leq 0 \tag{5}$$
$$q_3(\beta - q_3 - \rho q_1 - kq_2) = 0 \tag{6}$$

To solve the system we consider two cases for each variable $q_i = 0$ and $q_i > 0$ respectively.

1. $q_1 = 0$, $q_2 = 0$ and $q_3 = 0$: In this case directly fails Inequality 1.
2. $q_1 = 0$, $q_2 = 0$ and $q_3 > 0 \Rightarrow \rho q_3 \geq 1$, $kq_3 \geq \alpha$ and $q_3 = \beta$ by Inequalities 1 and 3 and Equality 6 respectively. But $\beta = 1$ and $\rho \in (0, 1)$ imply $\rho q_3 = \rho\beta < 1$ that fails.
3. $q_1 = 0$, $q_2 > 0$ and $q_3 = 0 \Rightarrow kq_2 \geq 1$, $q_2 = \alpha$ and $kq_2 \geq \beta$ by Inequality 1, Equality 4 and Inequality 5 respectively. But $\alpha \in (0, 1)$ and $k \in (-1, 1)$ imply $kq_2 < 1$ so the argument fails.
4. $q_1 = 0$, $q_2 > 0$ and $q_3 > 0 \Rightarrow kq_2 + \rho q_3 \geq 1$, $q_2 + kq_3 = \alpha$ and $q_3 + kq_2 = \beta$ by Inequality 1 and Equalities 4 and 6 respectively. The two equalities form a linear system with unique solution for all $k \in (-1, 1)$, which is: $q_2 = \frac{\alpha-\beta k}{1-k^2}$ and $q_3 = \frac{\beta-\alpha k}{1-k^2}$, where both q_1, q_2 need to be strictly positive, i.e. $\alpha > \beta k$ and $\beta > \alpha k$ and satisfy $kq_2 + \rho q_3 \geq 1$. But, $kq_2 + \rho q_3 = \frac{(\alpha-\beta k)k+(\beta-\alpha k)\rho}{1-k^2} \geq 1 \Leftrightarrow \alpha k(1 - \rho) + \rho - k^2 \geq 1 - k^2 \Leftrightarrow \alpha k \geq 1$ for $\beta = 1$ which fails for $\alpha < 1$ and $k \in (-1, 1)$.
5. $q_1 > 0$, $q_2 = 0$ and $q_3 = 0 \Rightarrow q_1 = 1, kq_1 \geq \alpha$ and $\rho q_1 \geq \beta$ by Equality 2 and Inequalities 3 and 5 respectively. But the last inequality cannot hold for $\beta = 1$ and $\rho \in (0, 1)$.
6. $q_1 > 0$, $q_2 = 0$ and $q_3 > 0 \Rightarrow q_1 + \rho q_3 = 1, \rho q_1 + q_3 = \beta$ and $kq_1 + kq_3 \geq \alpha$ by Equalities 2, 6 and Inequality 3. The two equalitiies lead to a linear system with unique solution for all $\rho \in (0, 1)$, which is $q_1 = \frac{1-\beta\rho}{1-\rho^2}$ and $q{=}\frac{\beta-\rho}{1-\rho^2}$ that for $\beta = 1$ leads to $q_1 = q_3 = \frac{1}{1+\rho}$. For this to be a solution it has to hold that $kq_1 + kq_3 \geq \alpha \Rightarrow \alpha \leq \frac{2k}{1+\rho}$, for which is necessary that $k > 0$.

7. $q_1 > 0$, $q_2 > 0$ and $q_3 = 0 \Rightarrow q_1 + kq_2 = 1, kq_1 + q_2 = \alpha$ and $\rho q_1 + kq_2 \geq \beta$ by Equalities 2, 4 and Inequality 5. The two equalitiies lead to a linear system with unique solution for all $k \in (-1,1)$, which is $q_1 = \frac{1-\alpha k}{1-k^2}$ and $q_2 = \frac{\alpha - k}{1-k^2}$, which requires $\alpha > k$. The solution needs to satisfy $\rho q_1 + kq_2 \geq \beta \Leftrightarrow \rho(1-\alpha k) + k(\alpha - k) \geq \beta(1-k^2)$, hence for $\beta = 1$ we need $\rho - \alpha k \rho + \alpha k - k^2 \geq 1 - k^2 \Leftrightarrow (1-\alpha k)(1-\rho) \leq 0$ which cannot hold for $\alpha < 1$ and $k \in (-1,1)$.
8. $q_1 > 0$, $q_2 > 0$ and $q_3 > 0 \Rightarrow q_1 + kq_2 + \rho q_3 = 1, kq_1 + q_2 + kq_3 = \alpha$ and $\rho q_1 + kq_2 + q_3 = \beta$ by Equalities 2, 4 and 6 respectively. The three equalities form a linear system with unique and fully positive solution when $1 + \rho - 2k^2 > 0$ and $a > \frac{2k}{1+\rho}$. The solution for $\beta = 1$ is $q_1 = q_3 = \frac{1-\alpha k}{1+\rho-2k^2}$ and $q_2 = \frac{(1+\rho)\alpha - 2k}{1+\rho-2k^2}$, which makes apparent the need of the two conditions. In case $a > \frac{2k}{1+\rho}$ but $1 + \rho - 2k^2 \leq 0$ the collaborations provide much larger benefit than the additional competition they induce, which leads firms to want to produce ever more. In that case, absent of upper bounds induced by $\bar{A}_i$ and $\bar{c}_i$ the firms would produce infinitely much. This means that no matter how high these bounds are, some of the will have to bind. Formally, we show that without considering upper bounds for any feasible q_1, q_2, q_3 one can find q_1', q_2', q_3' that lead all firms to higher profits and are still feasible. Let q_1, q_2, q_3 such that $q_1 + kq_2 + \rho q_3 \geq 1$, $kq_1 + q_2 + kq_3 \geq \alpha$ and $\rho q_1 + kq_2 + q_3 \geq 1$ and consider the case where the first and the third condition bind, as well as $q_1 = q_3$. In this case, solving with respect to q_1, q_3 we get $q_1 = q_3 = \frac{1-kq_2}{1+\rho}$, hence substituting in the second inequality this gives, $2k\left(\frac{1-kq_2}{1+\rho}\right) + q_2 \leq \alpha \Leftrightarrow 2k + (1+\rho-2k^2)q_2 \leq \alpha(1+\rho)$ which holds always with strict inequality for $a > \frac{2k}{1+\rho}$ and $1+\rho-2k^2 \leq 0$. Therefore, one can increase q_2, which will lead to relaxation of the binding conditions for q_1, q_3 (notice that $1+\rho-2k^2 \leq 0 \Rightarrow k < 0$), therefore q_1, q_3 could be increased to make the conditions bind again, which will lead again to a non–binding condition for q_2. Hence, absent of other constraints one can increase produced quantities indefinitely.

Overall the solution to the problem is the following:

- For $\alpha \leq \frac{2k}{1+\rho} \Rightarrow q_1 = e_1 = q_3 = e_3 = \frac{1}{1+\rho}$ and $q_2 = e_2 = 0$.
- For $\alpha > \frac{2k}{1+\rho}$ and $1 + \rho - 2k^2 > 0 \Rightarrow q_1 = e_1 = q_3 = e_3 = \frac{1-\alpha k}{1+\rho-2k^2}$ and $q_2 = e_2 = \frac{(1+\rho)\alpha - 2k}{1+\rho-2k^2}$
- For $\alpha > \frac{2k}{1+\rho}$ and $1 + \rho - 2k^2 \leq 0$ there is no equilibrium unless some marginal cost becomes equal to zero.

And for the special case of $\alpha = 1$:

- For $1 + \rho - 2k^2 > 0 \Rightarrow q_1 = e_1 = q_3 = e_3 = \frac{1-k}{1+\rho-2k^2}, q_2 = e_2 = \frac{1+\rho-2k}{1+\rho-2k^2}$.
- For $1 + \rho - 2k^2 \leq 0$ there is no equilibrium unless some marginal cost vanishes.

A.2.5. Unconnected Network with link between 1 and 2 - U12

$$\pi_1 = (1 - q_1 - \rho q_2 - \rho q_3 + e_1 + \phi e_2)q_1 - \frac{1}{2}e_1^2$$
$$\pi_2 = (\alpha - \rho q_1 - q_2 - \rho q_3 + e_2 + \phi e_1)q_2 - \frac{1}{2}e_2^2$$
$$\pi_3 = (1 - \rho q_1 - \rho q_2 - q_3 + e_3)q_1 - \frac{1}{2}e_3^2$$

such that $q_1, q_2, q_3, e_1, e_2, e_3 \geq 0$

The first order conditions are the following:

$$1 - q_1 - kq_2 - \rho q_3 \leq 0 \tag{1}$$
$$q_1(1 - q_1 - kq_2 - \rho q_3) = 0 \tag{2}$$
$$\alpha - q_2 - kq_1 - \rho q_3 \leq 0 \tag{3}$$
$$q_2(\alpha - q_2 - kq_1 - \rho q_3) = 0 \tag{4}$$
$$\beta - q_3 - \rho q_1 - \rho q_2 \leq 0 \tag{5}$$
$$q_3(\beta - q_3 - \rho q_1 - \rho q_2) = 0 \tag{6}$$

To solve the system we consider two cases for each variable $q_i = 0$ and $q_i > 0$ respectively.

1. $q_1 = 0$, $q_2 = 0$ and $q_3 = 0$: In this case directly fails Inequality 1.
2. $q_1 = 0$, $q_2 = 0$ and $q_3 > 0 \Rightarrow \rho q_3 \geq 1$, $\rho q_3 \geq \alpha$ and $q_3 = \beta$ by Inequalities 1 and 3 and Equality 6 respectively. But $\beta = 1$ and $\rho \in (0, 1)$ imply $\rho q_3 = \rho\beta < 1$ and the argument fails.
3. $q_1 = 0$, $q_2 > 0$ and $q_3 = 0 \Rightarrow kq_2 \geq 1$, $q_2 = \alpha$ and $\rho q_2 \geq \beta$ by Inequality 1, Equality 4 and Inequality 5 respectively. But $\alpha \in (0, 1)$ and $k \in (-1, 1)$ imply $kq_2 < 1$ so the argument fails.
4. $q_1 = 0$, $q_2 > 0$ and $q_3 > 0 \Rightarrow kq_2 + \rho q_3 \geq 1$, $q_2 + \rho q_3 = \alpha$ and $q_3 + \rho q_2 = \beta$ by Inequality 1 and Equalities 4 and 6 respectively. The two equalities form a linear system with unique solution for all $\rho \in (0, 1)$, which is: $q_2 = \frac{\alpha - \beta\rho}{1-\rho^2}$ and $q_3 = \frac{\beta - \alpha\rho}{1-\rho^2}$, where both q_1, q_2 need to be strictly positive, i.e. $\alpha > \beta\rho$ and $\beta > \alpha\rho$ and satisfy $kq_2 + \rho q_3 = \frac{(\alpha-\beta\rho)k + (\beta-\alpha\rho)\rho}{1-\rho^2} \geq 1$. For $\beta = 1$ this is the same as $\alpha(k - \rho^2) \geq 1 - \rho^2 - \rho + k\rho$ which we will show that can never be satisfied together with $\rho < \alpha < 1$. This left part of this inequality is required for positivity of q_2. Notice that $1 - \rho^2 - \rho + k\rho > k - \rho^2$, therefore, whenever $1 - \rho^2 - \rho + k\rho > k - \rho^2 > 0$ the inequality requires $\alpha > 1$, whereas when $1 - \rho^2 - \rho + k\rho > 0 > k - \rho^2$ it requires $\alpha < 0$, both of which fail. On the other hand, when $0 > 1 - \rho^2 - \rho + k\rho > k - \rho^2$ it holds that $\frac{1-\rho^2-\rho+k\rho}{k-\rho^2} < \rho$, but we need $\alpha > \rho$, hence the argument fails again. Therefore, this is not a possible solution.
5. $q_1 > 0$, $q_2 = 0$ and $q_3 = 0 \Rightarrow q_1 = 1, kq_1 \geq \alpha$ and $\rho q_1 \geq \beta$ by Equality 2 and Inequalities 3 and 5 respectively. But the last inequality cannot hold for $\beta = 1$ and $\rho \in (0, 1)$.
6. $q_1 > 0$, $q_2 = 0$ and $q_3 > 0 \Rightarrow q_1 + \rho q_3 = 1, \rho q_1 + q_3 = \beta$ and $kq_1 + \rho q_3 \geq \alpha$ by Equalities 2, 6 and Inequality 3. The two equalitiies lead to a linear system with unique solution for all $\rho \in (0, 1)$,

which is $q_1 = \frac{1-\beta\rho}{1-\rho^2}$ and $q_= \frac{\beta-\rho}{1-\rho^2}$ that for $\beta = 1$ leads to $q_1 = q_3 = \frac{1}{1+\rho}$. For this to be a solution it has to hold that $kq_1 + \rho q_3 \geq \alpha \Rightarrow \alpha \leq \frac{k+\rho}{1+\rho}$.

7. $q_1 > 0$, $q_2 > 0$ and $q_3 = 0 \Rightarrow q_1 + kq_2 = 1, kq_1 + q_2 = \alpha$ and $\rho q_1 + \rho q_2 \geq \beta$ by Equalities 2, 4 and Inequality 5. The two equalitiies lead to a linear system with unique solution for all $k \in (-1, 1)$, which is $q_1 = \frac{1-\alpha k}{1-k^2}$ and $q_2 = \frac{\alpha-k}{1-k^2}$, which requires $\alpha > k$. The solution needs to satisfy $\rho q_1 + \rho q_2 \geq \beta \Leftrightarrow \rho(1 - \alpha k + \alpha - k) \geq \beta(1 - k^2)$, hence for $\beta = 1$ we need $\rho(1 + \alpha) \geq 1 + k \Leftrightarrow \alpha \geq \frac{1+k-\rho}{\rho}$, which is always larger than k. Note that this value may be unattainable, since for $\phi < 1 - \rho \Rightarrow \frac{1+k-\rho}{\rho} > 1$.
8. $q_1 > 0$, $q_2 > 0$ and $q_3 > 0 \Rightarrow q_1 + kq_2 + \rho q_3 = 1, kq_1 + q_2 + \rho q_3 = \alpha$ and $\rho q_1 + \rho q_2 + q_3 = \beta$ by Equalities 2, 4 and 6 respectively. The three equalities form a linear system with unique and fully positive solution when $\frac{k+\rho}{1+\rho} < \alpha < \frac{1+k-\rho}{\rho}$, for $\beta = 1$. Notice, that these inequalities may hold at the same time only if $1 + k - 2\rho^2 > 0$. The solution is $q_1 = e_1 = \frac{(1-\rho^2)-(k-\rho^2)\alpha-\rho(1-k)}{(1-k)(1+k-2\rho^2)}$, $q_2 = e_2 = \frac{-(k-\rho^2)+(1-\rho^2)\alpha-\rho(1-k)}{(1-k)(1+k-2\rho^2)}$ and $q_3 = e_3 = \frac{-\rho(1-k)-\rho(1-k)\alpha+(1-k^2)}{(1-k)(1+k-2\rho^2)}$.

Overall the solution to the problem is the following:

- For $\alpha \leq \frac{k+\rho}{1+\rho} \Rightarrow q_1 = e_1 = q_3 = e_3 = \frac{1}{1+\rho}$ and $q_2 = e_2 = 0$.
- For $\alpha \geq \frac{1+k-\rho}{\rho} \Rightarrow q_1 = e_1 = \frac{1-\alpha k}{1-k^2}$, $q_2 = e_2 = \frac{\alpha-k}{1-k^2}$ and $q_3 = e_3 = 0$.
- For $\frac{k+\rho}{1+\rho} < \alpha < \frac{1+k-\rho}{\rho}$ (which needs $\phi < 1 + \rho - 2\rho^2$) $\Rightarrow q_1 = e_1 = \frac{(1-\rho^2)-(k-\rho^2)\alpha-\rho(1-k)}{(1-k)(1+k-2\rho^2)}$, $q_2 = e_2 = \frac{-(k-\rho^2)+(1-\rho^2)\alpha-\rho(1-k)}{(1-k)(1+k-2\rho^2)}$ and $q_3 = e_3 = \frac{-\rho(1-k)-\rho(1-k)\alpha+(1-k^2)}{(1-k)(1+k-2\rho^2)}$.

And for the special case of $\alpha = 1$:

- For $\phi \geq 1 - \rho \Rightarrow q_1 = e_1 = q_2 = e_2 = \frac{1}{1+k}$ and $q_3 = e_3 = 0$
- For $\phi < 1 - \rho \Rightarrow q_1 = e_1 = q_2 = e_2 = \frac{1-\rho}{1+k-2\rho^2}$ and $q_3 = e_3 = \frac{1+k-2\rho}{1+k-2\rho^2}$.

A.2.6. Unconnected Network with link between 1 and 3 - U13

$$\pi_1 = (1 - q_1 - \rho q_2 - \rho q_3 + e_1 + \phi e_3)q_1 - \frac{1}{2}e_1^2$$
$$\pi_2 = (\alpha - \rho q_1 - q_2 - \rho q_3 + e_2)q_2 - \frac{1}{2}e_2^2$$
$$\pi_3 = (1 - \rho q_1 - \rho q_2 - q_3 + e_3 + \phi e_1)q_1 - \frac{1}{2}e_3^2$$

such that $q_1, q_2, q_3, e_1, e_2, e_3 \geq 0$. The first order conditions are the following:

$$1 - q_1 - \rho q_2 - kq_3 \leq 0 \tag{1}$$

$$q_1(1 - q_1 - \rho q_2 - kq_3) = 0 \tag{2}$$

$$\alpha - q_2 - \rho q_1 - \rho q_3 \leq 0 \tag{3}$$

$$q_2(\alpha - q_2 - \rho q_1 - \rho q_3) = 0 \tag{4}$$

$$\beta - q_3 - kq_1 - \rho q_2 \leq 0 \tag{5}$$

$$q_3(\beta - q_3 - kq_1 - \rho q_2) = 0 \tag{6}$$

To solve the system we consider two cases for each variable $q_i = 0$ and $q_i > 0$ respectively.

1. $q_1 = 0$, $q_2 = 0$ and $q_3 = 0$: In this case directly fails Inequality 1.
2. $q_1 = 0$, $q_2 = 0$ and $q_3 > 0 \Rightarrow kq_3 \geq 1$, $\rho q_3 \geq \alpha$ and $q_3 = \beta$ by Inequalities 1 and 3 and Equality 6 respectively. But $\beta = 1$ and $k \in (-1, 1)$ imply $kq_3 < 1$ and the argument fails.
3. $q_1 = 0$, $q_2 > 0$ and $q_3 = 0 \Rightarrow \rho q_2 \geq 1$, $q_2 = \alpha$ and $\rho q_2 \geq \beta$ by Inequality 1, Equality 4 and Inequality 5 respectively. But $\alpha \in (0, 1)$ and $\rho \in (0, 1)$ imply $kq_2 < 1$ so the argument fails.
4. $q_1 = 0$, $q_2 > 0$ and $q_3 > 0 \Rightarrow \rho q_2 + kq_3 \geq 1$, $q_2 + \rho q_3 = \alpha$ and $q_3 + \rho q_2 = \beta$ by Inequality 1 and Equalities 4 and 6 respectively. The two equalities form a linear system with unique solution for all $\rho \in (0, 1)$, which is: $q_2 = \frac{\alpha - \beta\rho}{1-\rho^2}$ and $q_3 = \frac{\beta - \alpha\rho}{1-\rho^2}$, where both q_1, q_2 need to be strictly positive, i.e. $\alpha > \beta\rho$ and $\beta > \alpha\rho$ and satisfy $\rho q_2 + kq_3 = \frac{(\alpha-\beta\rho)\rho + (\beta-\alpha\rho)k}{1-\rho^2} \geq 1 \Leftrightarrow \alpha\rho(1-k) - \beta(\rho^2 - k) \geq 1 - \rho^2$ which leads for $\beta = 1$ to $\alpha\rho > 1$ that cannot hold.
5. $q_1 > 0$, $q_2 = 0$ and $q_3 = 0 \Rightarrow q_1 = 1, \rho q_1 \geq \alpha$ and $kq_1 \geq \beta$ by Equality 2 and Inequalities 3 and 5 respectively. But the last inequality cannot hold for $\beta = 1$ and $k \in (-1, 1)$.
6. $q_1 > 0$, $q_2 = 0$ and $q_3 > 0 \Rightarrow q_1 + kq_3 = 1, kq_1 + q_3 = \beta$ and $\rho q_1 + \rho q_3 \geq \alpha$ by Equalities 2, 6 and Inequality 3. The two equalitiies lead to a linear system with unique solution for all $k \in (-1, 1)$, which is $q_1 = \frac{1-\beta k}{1-k^2}$ and $q_= \frac{\beta - k}{1-k^2}$ that for $\beta = 1$ leads to $q_1 = q_3 = \frac{1}{1+k}$. For this to be a solution it has to hold that $\rho q_1 + \rho q_3 \geq \alpha \Rightarrow \alpha \leq \frac{2\rho}{1+k}$.
7. $q_1 > 0$, $q_2 > 0$ and $q_3 = 0 \Rightarrow q_1 + \rho q_2 = 1, \rho q_1 + q_2 = \alpha$ and $kq_1 + \rho q_2 \geq \beta$ by Equalities 2, 4 and Inequality 5. The two equalities lead to a linear system with unique solution for all $\rho \in (0, 1)$, which is $q_1 = \frac{1-\alpha\rho}{1-\rho^2}$ and $q_2 = \frac{\alpha-\rho}{1-\rho^2}$, which requires $\alpha > \rho$. The solution needs to satisfy $kq_1 + \rho q_2 \geq \beta \Leftrightarrow k(1 - \alpha\rho) + \rho(\alpha - \rho) \geq \beta(1 - \rho^2)$, hence for $\beta = 1$ we need $\alpha\rho \geq 1$ which cannot hold.
8. $q_1 > 0$, $q_2 > 0$ and $q_3 > 0 \Rightarrow q_1 + \rho q_2 + kq_3 = 1, \rho q_1 + q_2 + \rho q_3 = \alpha$ and $kq_1 + \rho q_2 + q_3 = \beta$ by Equalities 2, 4 and 6 respectively. The three equalities form a linear system with unique and fully positive solution when $\alpha > \frac{2\rho}{1+k}$ and $1 + k - 2\rho^2 > 0$. Notice that for the condition on α to be feasible (i.e. $\alpha < 1$) the second condition is necessary –not sufficient–. Under these

conditions, the solution is $q_1 = q_3 = \frac{1-\alpha\rho}{1+k-2\rho^2}$ and $q_2 = \frac{(1+k)\alpha-2\rho}{1+k-2\rho^2}$

Overall the solution to the problem is the following:

- For $\alpha \le \frac{2\rho}{1+k} \Rightarrow q_1 = e_1 = q_3 = e_3 = \frac{1}{1+k}$ and $q_2 = e_2 = 0$.
- For $\alpha > \frac{2\rho}{1+k}$ and $1+k-2\rho^2 > 0 \Rightarrow q_1 = e_1 = q_3 = e_3 = \frac{1-\alpha\rho}{1+k-2\rho^2}$ and $q_2 = e_2 = \frac{(1+k)\alpha-2\rho}{1+k-2\rho^2}$.

And for the special case of $\alpha = 1$:

- For $\phi \ge 1-\rho \Rightarrow q_1 = e_1 = q_3 = e_3 = \frac{1}{1+k}$ and $q_2 = e_2 = 0$
- For $\phi < 1-\rho \Rightarrow q_1 = e_1 = q_3 = e_3 = \frac{1-\rho}{1+k-2\rho^2}$ and $q_2 = e_2 = \frac{1+k-2\rho}{1+k-2\rho^2}$.

A.3. Profits for optimal production levels

Given that the only corner solutions we consider are those that impose zero production for some of the firms, the problem admits a fully interior solution of the form $\boldsymbol{q} = (\boldsymbol{I} + \rho\boldsymbol{B} - \phi\boldsymbol{A})^{-1}\boldsymbol{\mu}$, where the given matrix is invertible, in which case König et.al (2014) have shown that $\pi = \frac{1}{2}q_i^2$. In fact, it is enough to notice that:

$$\rho q_i \sum_{j=1}^{n} b_{ij}q_j - \phi q_i \sum_{j=1}^{n} a_{ij}q_j = q_i((\rho\boldsymbol{B} - \phi\boldsymbol{A})\boldsymbol{q})_i = q_i((\boldsymbol{I}_n + \rho\boldsymbol{B} - \phi\boldsymbol{A})\boldsymbol{q} - \boldsymbol{q})_i = q_i(\mu_i - q_i) \Rightarrow$$

$$\Rightarrow \pi_i = \mu_i q_i - q_i(\mu_i - q_i) - \frac{1}{2}q_i^2 = \frac{1}{2}q_i^2$$

B. Equilibria: Symmetric Case – $\alpha = 1$

Before starting the analysis under potential entry, recall that with no entry the two incumbent firms always want to establish a collaboration, as long as they are equally efficient ($\alpha = 1$), since $q_i^{D,C} = \frac{1}{1+k} > \frac{1}{1+\rho} = q_i^{D,E}$.

For the remaining of this section, we exclude from our analysis the values of ϕ, ρ for which $k \le -1/2$. The only additional condition imposed by the previous analysis is whether $\phi \ge 1-\rho$ or not, which determines whether the isolated firm produces in the unconnected network. We provide the complete set of comparisons for all pairs of (ρ, ϕ) to give a complete picture of the problem, despite the fact that the equilibrium could be computed using only some of them.

For $\phi \ge 1-\rho$ the isolated firm does not produce in the unconnected network. Observing that, it becomes apparent that no firm would want to end up in this position, hence the two incumbents can always prevent that by establishing a collaboration before entry occurs. The only reason for not doing so is if being peripheral in a star network yields them the highest profits among all possible equilibria, but this will never be the case. Overall, to obtain the equilibrium outcome we need to make a few payoff comparisons. Recall that $\pi_i = \frac{1}{2}q_i^2$, so it is enough to compare produced quantities.

$q_i^E > q_i^{SP} \Leftrightarrow \frac{1}{1+2\rho} > \frac{1-k}{1+\rho-2k^2} \Leftrightarrow \underline{k > 1/2}$

$q_i^C > q_i^E \Leftrightarrow \frac{1}{1+2k} > \frac{1}{1+2\rho} \Leftrightarrow \rho > k$
$q_i^C > q_i^{SP} \Leftrightarrow \frac{1}{1+2k} > \frac{1-k}{1+\rho-2k^2} \Leftrightarrow \rho > k$
$q_i^{SC} > q_i^E \Leftrightarrow \frac{1+\rho-2k}{1+\rho-2k^2} > \frac{1}{1+2\rho} \Leftrightarrow 1+\rho-k > 0$
$q_i^{SC} > q_i^{SP} \Leftrightarrow \frac{1+\rho-2k}{1+\rho-2k^2} > \frac{1-k}{1+\rho-2k^2} \Leftrightarrow \rho > k$
$q_i^{UI} > q_i^E \Leftrightarrow \frac{1}{1+k} > \frac{1}{1+2\rho} \Leftrightarrow 2\rho > k$
$q_i^{UI} > q_i^{SP} \Leftrightarrow \frac{1}{1+k} > \frac{1-k}{1+\rho-2k^2} \Leftrightarrow \underline{\rho > k^2}$
$q_i^{SC} > q_i^{UI} \Leftrightarrow \frac{1+\rho-2k}{1+\rho-2k^2} > \frac{1}{1+k} \Leftrightarrow \underline{k < 0}$
$q_i^C > q_i^{UI} \Leftrightarrow \frac{1}{1+2k} > \frac{1}{1+k} \Leftrightarrow \underline{k < 0}$
$q_i^C > q_i^{SC} \Leftrightarrow \frac{1}{1+2k} > \frac{1+\rho-2k}{1+\rho-2k^2} \Leftrightarrow \underline{k < 0}$
where $\rho < k^2$ for $\phi > 1-\rho$ requires $k < -1/2$, therefore it is never satisfied.

These inequalities are already sufficient to conclude that for $k < 0$ (i.e. $\phi > \rho$) the unique equilibrium network is the complete network and for $k > 0$ the unique equilibrium is the unconnected network $U12$. This is because, for $k < 0$, the incumbent firms prefer the complete from the unconnected network, therefore upon establishment of a collaboration between the two incumbents, the first incumbent that is chosen to discuss a link with the entrant, will agree to its establishment, as will do the entrant in order to produce positive quantity. After this, the remaining link will also be created, since the firms prefer to be part of a complete network, rather than in the periphery of a star. Analogously, for $k > 0$, the unconnected network is the most preferred option for both incumbents, therefore having no incentives to collaborate with the entrant, they reject any proposed link, leading firm 3 not to produce, thus effectively preventing entry. Note, that it is not even necessary to compare all potential payoffs.

We proceed analogously for $\phi < 1-\rho$. In this case, entry cannot be prevented since even an isolated firm in an unconnected network produces positive quantity. The following inequalities are sufficient to find the equilibirum networks.
$q_i^E > q_i^{UO} \Leftrightarrow \frac{1}{1+2\rho} > \frac{1+k-2\rho}{1+k-2\rho^2} \Leftrightarrow \rho > k$
$q_i^{SP} > q_i^{UO} \Leftrightarrow \frac{1-k}{1+\rho-2k^2} > \frac{1+k-2\rho}{1+k-2\rho^2} \Leftrightarrow (\rho-k)(1-k+2k\rho-2k^2) > 0 \Leftrightarrow 2k > -\frac{1-k}{\rho-k}$
which holds always for $k > -1/2$, since $2k > -1$ and $\frac{1-k}{\rho-k} > 1 \Leftrightarrow -\frac{1-k}{\rho-k} < -1$.
$q_i^{UI} > q_i^E \Leftrightarrow \frac{1-\rho}{1+k-2\rho^2} > \frac{1}{1+2\rho} \Leftrightarrow \rho > k$
$q_i^{UI} > q_i^{SP} \Leftrightarrow \frac{1-\rho}{1+k-2\rho^2} > \frac{1-k}{1+\rho-2k^2} \Leftrightarrow k+\rho-2k\rho > 0 \Leftrightarrow \begin{cases} \phi < 2\rho\frac{1-\rho}{1-2\rho} & \text{if } \rho < 1/2 \\ \phi < 1-\rho & \text{if } \rho \geq 1/2 \end{cases}$
$q_i^{SC} > q_i^{UI} \Leftrightarrow \frac{1+\rho-2k}{1+\rho-2k^2} > \frac{1-\rho}{1+k-2\rho^2} \Leftrightarrow (\rho-k)[1-\rho-2\rho(\rho-k)] > 0 \Leftrightarrow \phi < \frac{1-\rho}{2\rho}$
$q_i^C > q_i^{UI} \Leftrightarrow \frac{1}{1+2k} > \frac{1-\rho}{1+k-2\rho^2} \Leftrightarrow (2\rho-1)(\rho-k) < 0 \Leftrightarrow \rho < 1/2$

The most crucial of these results is that $q_i^C > q_i^{SP}$ always. This means that even if a firm tries to establish a star network, this is never stable, since the other two firms will always form a collaboration, leading to a complete network. Having observed that, the comparison $q_i^C > q_i^{UI} \Leftrightarrow \rho < 1/2$ means that for these low values of ρ the complete network is preferred to the unconnected, therefore as long as the entrant is willing to form collaborations (which is true since $q_i^{SP} > q_i^{UO}$) then this will be the unique equilibrium network. To the contrary, if $\rho > 1/2$, the complete network is no longer preferred, therefore the two incumbents prefer to keep the network unconnected by rejecting potential collaborations with the entrant. This occurs despite the fact that $q_i^{SC} > q_i^{UI}$, given that the star would not be stable. This behavior arises as a result of the farsightedness of the firms that are able to predict future changes in the network, as a result of newly established links.

Overall for $\alpha = 1$ and $k > -1/2$ there is always a unique equilibrium network whose form depends on ϕ, ρ as follows:

- For $\phi \geq 1 - \rho$ and $\phi > \rho \Rightarrow$ Complete.
- For $\phi \geq 1 - \rho$ and $\phi < \rho \Rightarrow$ Unconnected U12, with $q_1 = q_2 = \frac{1}{1+k}$ and $q_3 = 0$.
- For $\phi < 1 - \rho$ and $\rho < 1/2 \Rightarrow$ Complete.
- For $\phi < 1 - \rho$ and $\rho > 1/2 \Rightarrow$ Unconnected U12, with $q_1 = q_2 = \frac{1-\rho}{1+k-2\rho^2}$ and $q_3 = \frac{1+k-2\rho}{1+k-2\rho^2}$.

C. Equilibria: Asymmetric Case – $\alpha < 1$

C.1. Proof of Proposition 4.1

In this first case, we analyze a scenario where firm 2 is sufficiently efficient to ensure that under no entry firm 1 would prefer to establish a collaboration. However, the presence of a more efficient potential entrant (firm 3), changes the incentives for firm 1, because ending at U13 is the most preferred outcome for it. Therefore, it refrains from establishing a link with firm 2, waits until entry occurs and then establishes a collaboration with firm 3. Incentives are equivalent for firm 3, therefore after establishing a link, they both reject any offer from firm 2. In the considered scenario, all three firms produce in equilibrium, despite the network being unconnected. The aim of our analysis is to find conditions on the parameters (α, ρ, ϕ) for which the above mentioned scenario is the unique equilibrium. The following conditions induce a fully interior solution in all possible networks, as well as that both incumbent firms would prefer to collaborate in the absence of the entrant.

i. $k > 0$ and $\alpha > \frac{k+\rho}{1+k\rho}$ (Duop C) ii. $\alpha > \frac{2\rho}{1+\rho}$ (Tr E) iii. $k > -1/2$ and $\alpha > \frac{2k}{1+k}$ (Tr C)

iv. $\alpha > \frac{k+\rho}{1+k}$ and $1 + \rho - 2k^2 > 0$ (Tr S1) v. $\alpha > \frac{2k}{1+\rho}$ and $1 + \rho - 2k^2 > 0$ (Tr S1)

vi. $\alpha > \frac{2\rho}{1+k}$ and $1 + k - 2\rho^2 > 0$ (Tr U13) vii. $\frac{k+\rho}{1+\rho} < \alpha < \frac{1+k-\rho}{\rho}$ and $1 + k - 2\rho^2 > 0$ (Tr U12)

Let us first simplify the conditions for k. Notice that condition (i) implies both (iii), (iv) and (v).

Moreover, some of the conditions regarding α are also redundant.

- $\frac{2\rho}{1+k} > \frac{k+\rho}{1+k} > \frac{2k}{1+k} \Rightarrow$ (vi) implies (iv) that implies (iii).
- $\frac{2\rho}{1+\rho} > \frac{k+\rho}{1+\rho} > \frac{2k}{1+\rho} \Rightarrow$ (ii) implies lhs of (vii) that implies (v).
- $\frac{2\rho}{1+k} > \frac{2\rho}{1+\rho} \Rightarrow$ (vi) implies (ii).

We also need to check whether the values of the restrictions satisfy $\alpha < 1$.

- For (i): $\frac{k+\rho}{1+k\rho} < 1 \Leftrightarrow (1-k)(1-\rho) > 0$
- For (vi): $\frac{2\rho}{1+k} < 1 \Leftrightarrow \rho + \phi < 1$.
- For rhs (vii): $\frac{1+k-\rho}{\rho} < 1 \Leftrightarrow \rho + \phi > 1$, which cannot hold together with the above, so rhs (vii) is redundant.

Moreover, $\rho + \phi < 1 \Rightarrow 1 + k - 2\rho > 0 \Rightarrow 1 + k - 2\rho^2 > 1 + k - 2\rho > 0$ so the conditions on k for (vi) and (vii) are also redundant. Overall, there are four conditions that need to be satisfied:

I. $k > 0$ $(\rho > \phi)$ II. $\phi + \rho < 1$ III. $\alpha > \frac{k+\rho}{1+k\rho}$ IV. $\alpha > \frac{2\rho}{1+k}$

After that, we need to compare the equilibrium quanitities for different networks:

- $q_1^{U13} > q_1^{U12} \Leftrightarrow \frac{1-\alpha\rho}{1+k-2\rho^2} > \frac{1-\rho+k\rho-\rho^2-(k-\rho^2)\alpha}{(1-k)(1+k-2\rho^2)} \Leftrightarrow \alpha < 1$.
- $q_1^{U13} > q_1^{E} \Leftrightarrow \frac{1-\alpha\rho}{1+k-2\rho^2} > \frac{1-\alpha\rho}{1+\rho-2\rho^2} \Leftrightarrow \rho > k$.
- $q_1^{U13} > q_1^{C} \Leftrightarrow \frac{1-\alpha\rho}{1+k-2\rho^2} > \frac{1-\alpha k}{1+k-2k^2} \Leftrightarrow \alpha < \frac{2(\rho+k)}{1+k+2k\rho}$.
- $q_1^{C} > q_1^{S2} \Leftrightarrow \frac{1-\alpha k}{1+k-2k^2} > \frac{1-\alpha k}{1+\rho-2k^2} \Leftrightarrow \rho > k$.
- $q_1^{S1} > q_1^{S2} \Leftrightarrow \frac{1+\rho-k-\alpha k}{1+\rho-2k^2} > \frac{1-\alpha k}{1+\rho-2k^2} \Leftrightarrow \rho > k$
- $q_1^{U13} > q_1^{S1} \Leftrightarrow \frac{1-\alpha\rho}{1+k-2\rho^2} > \frac{1+\rho-k-\alpha k}{1+\rho-2k^2} \Leftrightarrow \alpha < \frac{2\rho+k+2\rho^2}{1+k+\rho+2k\rho}$.

If all conditions are satisfied simultaneously, this means that for firm 1 (hence also for firm 3) U13 is the most preferred network structure, therefore it will be the unique equilibrium network.[26]

$$k > 0 \ , \ \phi + \rho < 1 \text{ and } \max\left\{\frac{k+\rho}{1+k\rho}, \frac{2\rho}{1+k}\right\} < \alpha < \min\left\{\frac{2(\rho+k)}{1+k+2k\rho}, \frac{2\rho+k+2\rho^2}{1+k+\rho+2k\rho}\right\}$$

We still need to find whether there exists α that satisfies these conditions, as well as whether any of the bounds is redundant.

[26] Both firms 1 and 3 reject any collaboration with firm 2 and agree to connect between them the first time their link is chosen to be discussed.

- $$\frac{k+\rho}{1+k\rho} < \frac{2\rho}{1+k} \Leftrightarrow k^2 + k(1+\rho-2\rho^2) - \rho < 0 \Leftrightarrow k < \frac{-(1+\rho-2\rho^2)+\sqrt{(1+\rho-2\rho^2)^2+4\rho}}{2} \Leftrightarrow$$ $$\Leftrightarrow \phi > \frac{1+3\rho-2\rho^2-\sqrt{(1+\rho-2\rho^2)^2+4\rho}}{2}$$
- $\frac{2(\rho+k)}{1+k+2k\rho} > 1 \Leftrightarrow (1-2\rho)(1-k) < 0 \Leftrightarrow \rho > 1/2$
- $\frac{2\rho+k+2\rho^2}{1+k+\rho+2k\rho} > 1 \Leftrightarrow k < \frac{\rho+2\rho^2-1}{2\rho} \Leftrightarrow \phi > \rho - \frac{\rho+2\rho^2-1}{2\rho} \Leftrightarrow \phi > \frac{1-\rho}{2\rho}$
- $\frac{2(\rho+k)}{1+k+2k\rho} < \frac{2\rho+k+2\rho^2}{1+k+\rho+2k\rho} \Leftrightarrow k < \frac{4\rho^3+2\rho^2-2\rho-1}{1+2\rho} < 0 \Rightarrow \frac{2(\rho+k)}{1+k+2k\rho} \geq \frac{2\rho+k+2\rho^2}{1+k+\rho+2k\rho}$
- $\frac{2\rho}{1+k} < \frac{2\rho+k+2\rho^2}{1+k+\rho+2k\rho} \Leftrightarrow 1+k-2\rho^2 > 0$
- $\frac{k+\rho}{1+k\rho} < \frac{2\rho+k+2\rho^2}{1+k+\rho+2k\rho} \Leftrightarrow k^2 + 2k\rho(1-\rho) - \rho < 0$

But the latter condition is redundant since$k^2 + 2k\rho(1-\rho) - \rho < k^2 + k(1+\rho-2\rho^2) - \rho$, hence if $k^2 + k(1+\rho-2\rho^2) - \rho$ then also $k^2 + 2k\rho(1-\rho) - \rho < 0$.

Concluding whenever the following conditions are satisfied:

- $\rho \in (0,1)$, $\rho(2-\rho) - \sqrt{\rho^2(1-\rho)^2+\rho} < \phi < \min\{\rho, 1-\rho\}$ and
- $\max\left\{\frac{2\rho}{1+k}, \frac{k+\rho}{1+k\rho}\right\} < \alpha < \min\left\{\frac{2\rho+k+2\rho^2}{1+k+\rho+2k\rho}, 1\right\}$

C.2. Proof of Proposition 4.2

In the second case, we analyze a scenario where firm 2 is sufficiently inefficient to make firm 1 unwilling to form a collaboration under no entry. Moreover, α is within a range that under potential entry, if the network remains unconnected then the isolated firm does not produce. Having said that, the crucial observation is that if firm 1 can choose which firm to isolate, thus leading it out of the market, this would be firm 3. This is $q_1^{U12} > q_1^{U13}$. Therefore, in this scenario, firm 1 achieves its best outcome for network U12. This is also the case for firm 2, hence U12 is the unique equilibrium network arising in this case, preventing also the entry of firm 3 in the market.

i. $k > 0$ and $\rho \leq \alpha \leq \frac{k+\rho}{1+k\rho}$ (Duop E) ii. $\alpha \leq \frac{2\rho}{1+\rho}$ (Tr E) iii. $\alpha > \frac{2k}{1+k}$ (Tr C)

iv. $\alpha \leq \frac{k+\rho}{1+k}$ (Tr S1) v. $\alpha > \frac{2k}{1+\rho}$ (Tr S2) vi. $\alpha \leq \frac{2\rho}{1+k}$ (Tr U13) vii. $\alpha \geq \frac{1+k-\rho}{\rho}$ (Tr U12)

The last inequality for α is possible only for $\phi+\rho > 1$. Moreover, the above conditions ensure that firm 2 does not produce in either E, S1 or U13 and firm 3 does not produce in U13. The conditions can be summarized in the following two, always for $k > 0$:

I. $\alpha < \min\left\{\frac{k+\rho}{1+k\rho}, \frac{2\rho}{1+\rho}, \frac{k+\rho}{1+k}, \frac{2\rho}{1+k}\right\}$ II. $\alpha > \max\left\{\rho, \frac{2k}{1+k}, \frac{2k}{1+\rho}, \frac{1+k-\rho}{\rho}\right\}$

Notice that $\frac{2\rho}{1+k} > \frac{2\rho}{1+\rho} > \frac{1+\rho}{1+k}$ and also $\frac{k+\rho}{1+k\rho} > \frac{k+\rho}{1+k}$, hence condition (I) becomes $\alpha < \frac{k+\rho}{1+k}$. For the other condition it always holds $\frac{2k}{1+k} > \frac{2k}{1+\rho}$. For the other comparisons, it holds that $\rho > \frac{1+k-\rho}{\rho} \Leftrightarrow \phi > 1-\rho^2$ and $\rho > \frac{2k}{1+k} \Leftrightarrow \phi > \frac{\rho-\rho^2}{2-\rho}$, but $1-\rho^2 > \frac{\rho-\rho^2}{2-\rho}$ always, therefore $\phi > 1-\rho^2$ is a sufficient condition to ensure that the only relevant bound for condition (II) is $\alpha > \rho$. We should still check whether these two conditions could hold at the same time, which requires $\rho < \frac{k+\rho}{1+k}$ that is always true for $k > 0$. Finally, note that the imposed condition $\phi > 1-\rho^2$ implies $\phi + \rho > 1$. Overall, the conditions on (α, ρ, ϕ) are $1-\rho^2 < \phi < \rho$, $\rho > \frac{\sqrt{5}-1}{2}$ and $\rho < \alpha < \frac{k+\rho}{1+k}$, where the condition on ρ ensures that $\rho > 1-\rho^2$.

We then compare equilibrium quantities for firm 1 and find that U12 yields the highest profits for firm 1. Furthermore, contrary to the previous case, now we also need to check the incentives of firm 2, which might not be completely aligned with that of firm 1. Recall that we only need to consider networks U12, S2 and C, since in any other network firm 2 does not produce. We find that U12 is also the most preferred from the other two for firm 2:

- $q_1^{U13} = q_1^{S1} > q_1^{E} \Leftrightarrow \frac{1}{1+k} > \frac{1}{1+\rho} \Leftrightarrow \rho > k$.
- $q_1^{U12} > q_1^{U13} \Leftrightarrow \frac{1-\alpha k}{1-k^2} > \frac{1}{1+k} \Leftrightarrow k(1-\alpha) > 0$
- $q_1^{C} > q_1^{S2} \Leftrightarrow \frac{1-\alpha k}{1+k-2k^2} > \frac{1-\alpha k}{1+\rho-2k^2} \Leftrightarrow \rho > k$
- $q_1^{U12} > q_1^{C} \Leftrightarrow \frac{1-\alpha k}{1-k^2} > \frac{1-\alpha k}{1+k-2k^2} \Leftrightarrow k(1-k) > 0$
- $q_2^{U12} > q_2^{C} \Leftrightarrow \frac{\alpha-k}{1-k^2} > \frac{(1+k)\alpha-2k}{1+k-2k^2} \Leftrightarrow \alpha k < 1$
- $q_2^{U12} > q_2^{S2} \Leftrightarrow \frac{\alpha-k}{1-k^2} > \frac{(1+\rho)\alpha-2k}{1+\rho-2k^2} \Leftrightarrow \alpha k < 1$

The comparison of quantities does not impose any additional constraints therefore whenever $1-\rho^2 < \phi < \rho$, $\rho > \frac{\sqrt{5}-1}{2}$ and $\rho < \alpha < \frac{k+\rho}{1+k}$ the unique equilibrium network is U12, with firm 3 not entering in the market.

C.3. Proof of Proposition 4.3

In the third case, we present a scenario where the entry of more efficient firm leads to the exit of the inefficient incumbent. This happens because firms 1 and 3 find it mutually beneficial to end up in an unconnected U13 network, which leads firm 2 to exit the market. Absent of the entrant, firm 1 would still not prefer to establish a collaboration with firm 2 (as it is not sufficiently efficient), but firm 2 would still produce a positive quantity.

i. $k > 0$, $\rho \leq \alpha \leq \frac{k+\rho}{1+k\rho}$ (Du E) ii. $\alpha \leq \frac{2\rho}{1+\rho}$ (Tr E) iii. $\alpha > \frac{2k}{1+k}$ (Tr C) iv. $\alpha \leq \frac{k+\rho}{1+k}$ (Tr S1)

v. $\alpha > \frac{2k}{1+\rho}$ (Tr S2) vi. $\alpha \leq \frac{2\rho}{1+k}$ (Tr U13) vii. $\frac{k+\rho}{1+\rho} < \alpha < \frac{1+k-\rho}{\rho}$ and $1+k > 2\rho^2$ (Tr U12)

The only condition that is different from the previous proposition is that in U12 firm 3 also enters and produces. This turns out to alter the incentives for firm 1, leading to a different equilibrium.

It is easy to see that $\frac{k+\rho}{1+\rho} > \frac{2k}{1+k} > \frac{2k}{1+\rho}$ always for $k > 0$ and $\frac{k+\rho}{1+\rho} > \rho$ whenever $\phi < \rho - \rho^2$. Moreover, $\frac{k+\rho}{1+\rho} < \frac{2\rho}{1+\rho} < \frac{2\rho}{1+k}$ always and $\frac{k+\rho}{1+\rho} < \frac{k+\rho}{1+k\rho}$ for $k > 0$. Finally, $\frac{k+\rho}{1+k} < \frac{1+k-\rho}{\rho}$ holds always for $\rho < 1/2$ and whenever $\phi < 1 - \sqrt{\rho(2\rho-1)}$ for $\rho \geq 1/2$, where the last condition is implied by $\phi < \rho - \rho^2$. Overall, the above conditions can be summarized as $\phi < \rho - \rho^2$ and $\frac{k+\rho}{1+\rho} < \alpha < \frac{k+\rho}{1+k}$.

Still nees to be shown when U13 is the optimal for firms 1 and 3.

- $q_1^{U13} > q_1^C \Leftrightarrow \frac{1}{1+k} > \frac{1-\alpha k}{1+k-2k^2} \Leftrightarrow \alpha > \frac{2k}{1+k}$
- $q_1^{U13} > q_1^{U12} \Leftrightarrow \frac{1}{1+k} > \frac{1+k\rho-\rho-\rho^2-(k-\rho^2)\alpha}{(1-k)(1+k-2\rho^2)} \Leftrightarrow \alpha > \frac{k+k^2+\rho^2+k^2\rho-\rho-3k\rho^2}{(1+k)(k-\rho^2)}$
- $q_1^{U13} > q_1^{S2} \Leftrightarrow \frac{1}{1+k} > \frac{1-\alpha k}{1+\rho-2k^2} \Leftrightarrow \alpha k(1+k) > 2k^2 - k - \rho$
- $\frac{k+k^2+\rho^2+k^2\rho-\rho-3k\rho^2}{(1+k)(k-\rho^2)} < \frac{k+\rho}{1+\rho} \Leftrightarrow \phi < (1-\rho)(1+\sqrt{2(1+\rho)}$

and only the second inequality imposes an additional condition, which is shown to be redundant by the fourth inequality that is implied by $\phi < \rho - \rho^2$.

Overall, for $\phi < \rho - \rho^2$ and $\frac{k+\rho}{1+\rho} < \alpha < \frac{k+\rho}{1+k}$ the unique equilibrium network is U13, with firm 2 not producing, thus exiting the market.