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Career and Non-Career Jobs: Dangling the Carrot

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Career and Non-Career Jobs: Dangling the Carrot

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Abstract

We develop a model of the labor market with career and non-career jobs. Workers in career jobs start at the low rank and can be promoted to a higher rank. Non-career jobs have the typical single-rank structure. We show that it is optimal for career firms to incentivize their employees through the option value of a promotion. By increasing the wage spread between low and top positions they can elicit more effort from the mass of the workers in low ranks, while rewarding handsomely only the very few that get promoted. We explore the macroeconomic implications of this hierarchical payment structure. We show how our model can provide interesting insights into various puzzles such as the wage gap between men and women, the cyclicality of the labor wedge and the low volatility of the real wage relative to hours and output along the business cycle, without imposing ad-hoc nominal wage rigidities.

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1 Introduction

Standard theories in modern labor economics predict that workers' wages increase with their opportunity cost and productivity. However, numerous empirical studies document that wages often deviate from this prediction, as there is a clear hierarchical structure within firms: wages are closely related to job levels, rather than effort or productivity, and a significant

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fraction of wage growth occurs through promotions. Workers at lower-level jobs may be willing to exert high effort, without an immediate pay reward, to increase their chances of promotion, and will gain a wage increase only upon promotion.[1] In light of these evidence, a theoretical literature has emerged to demonstrate how firms can strategically use wage spreads across job levels and promotion tournaments to elicit more effort from their employees and enhance their efficiency. Previous work, however, analyzes the implications of such practices on wage dynamics of individuals within individual firms and in isolation from the rest of the labor market. To the best of our knowledge no previous paper analyzes the implications of such practices on aggregate labor market outcomes. Existing macroeconomic models of the labor market build around the standard assumptions that closely link wages to productivity (skills, human capital, experience) and outside options, and overlook incentive-based compensation schemes, and the existence of career paths within firms.

This paper fills this gap by developing a model of the labor market in which there are both "career" and "non-career" firms. Non-career firms have the typical one-type job structure and pay their employees according to their marginal product, which exactly compensates them for the marginal disutility of their last unit of effort. In career firms, on the other hand, all workers start at the low rank, but have the possibility of getting promoted to a higher rank as a reward to a relatively higher effort. It is optimal for career firms to create a payment structure where workers' effort is compensated disproportionally depending on their job level. They compensate more the few workers that get promoted to the higher rank, thereby creating incentives for the mass of the workers in lower ranks who are paid less. The latter are willing to put too high an effort for the compensation they receive as they also value the possibility of getting promoted to the higher rank that is compensated handsomely. Career firms may react to changes in economic conditions by adjusting not only hirings at

[1]See e.g Baker et al. (1993, 1994a,b), for evidence that wages are tied to job levels and on the existence of career ladders within firms. McCue (1996) shows that within-firm mobility is an important source of wage growth for the average full-time worker, accounting for roughly one-sixth of life-cycle wage growth. Most of these moves workers label as "promotions." DeVaro (2006) show that larger wage spreads across job levels within the firm are associated with higher levels of worker performance, supporting the notion of internal promotion competitions. Likewise, Seltzer & Frank (2007) find evidence that firms use large increases in wages through the job hierarchy to induce effort. More recently, Huitfeldt et al. (2022) confirm previous evidence, based mainly on case studies for single firms, on the existence of job ladders within firms, using detailed employer-employee data on a large number of private-sector firms in Norway. Their findings also support theories of incentive-based promotion schemers in which firms offer wage premiums to promoted workers to elicit effort. Finally, Medoff & Abraham (1980), Hutchens (1989) and more recently Flabbi & Ichino (2001) show that the positive effect of seniority on wages is not due to the higher productivity of more senior workers.

lower ranks, but also wage spreads and opportunities for internal promotions. We contribute to the literature by demonstrating that this hierarchical wage structure, which, despite its empirical relevance, has been largely overlooked in macroeconomic models, is of particular importance for our understanding of aggregate labor market outcomes. We show how our model can shed light on macroeconomic puzzles such as the gender wage gap, the cyclicality of the labor wedge, and the cyclicality of the labor income share. The extant literature on promotion tournaments and wage schemes that promote higher effort mainly focuses on the micro level, ignoring the effects on the aggregate labor market and the macroeconomy.[2]

In our model the unemployed can search for either career or non-career jobs. A worker getting a career job starts from the low rank and puts effort to reach the top. The promotion depends both on the effort and the quality of the worker. All workers are identical, but supply labor having in mind that the higher their effort relative to that of other employees, the higher their chances to get promoted to the top position in the following period. This is exploited by the firms that offer lower wages at the starting career positions and much higher compensation at the top positions. As workers compete with each other for the top position, they end up putting an extraordinary amount of effort for what they are compensated for in monetary terms, which increases firms profits. There are search frictions in the labor market and new hires in career and non-career firms are determined, respectively, by two matching functions. Firms choose how many vacancies to open and search for workers. Career firms choose not only how many low-rank positions to open, but also the number of top positions compared to low-level jobs, which eventually determines the probability to get promoted.

The supply of labor of workers in the low-rank positions of career firms deviates from the standard neoclassical labor supply, as the wage in these positions is lower than the marginal rate of substitution (while for the top positions the wage matches the MRS). The reason is that workers in these positions are willing to exert effort, without an immediate pay reward, simply to increase their chances to be promoted. Hence, the competition between workers to reach the top position raises their effort to levels above the socially optimal level (MRS). The demand for labor responds in a similar manner. For the workers in the low rank, the wage is significantly lower than their marginal product of labor (MPL), as they are partly

[2]See seminal paper by Lazear & Rosen (1981). Other studies include Green & Stokey (1983), Nalebuff & Stiglitz (1983), Carmichael (1983), Malcomson (1984), Mookherjee (1984), Rosen (1986), Baker et al. (1988), McLaughlin (1991), Prendergast (1993), Gibbons & Waldman (1999), Zabojnik & Bernhardt (2001) and Gibbons & Waldman (2006). Some more recent studies are Zabojnik (2012), DeVaro & Waldman (2012), Moallemi et al. (2017), Ekinci et al. (2018), DeVaro et al. (2019) and Gürtler & Gürtler (2019).

compensated for their contribution to production with the opportunity of a promotion to the top rank. The workers in top ranks, by contrast, are paid more than their marginal product, which makes the opportunity of a promotion even more valuable and allows even lower wages at the bottom.[3]

This feature is key to generating realistic business cycle variation in the labor wedge. The labor wedge is particularly volatile along the business cycle and countercyclical to output (see Chari et al. (2007) and Shimer (2009)). To deviate from the neoclassical prediction that the labor wedge does not exist, there are basically two approaches according to Karabarbounis (2014b). A successful model must include either a wage gap in the labor supply (i.e. $wage \neq MRS$) or a gap in labor demand (i.e. $wage \neq MPL$) or both. Gali et al. (2007) report that the gap between wage and MPL explains only 2% of the variation in the wedge while the gap between the wage and the MRS explains 80% of the variation in the labor wedge. Our model implies that most of the variation in the wedge comes from the gap between the wage and MRS which arises due to the expected value of getting promoted which is volatile along the business cycle. Nonetheless, the gap between the wage and MPL is smaller because on one hand the wage is below MPL for the low-rank workers, but on the other hand the wage is larger than the MPL for top-rank workers and the effects are always opposite along the business cycle which makes the aggregate effect of the wage-MPL gap on the wedge very small.

For career firms the strategy of increasing the value of a promotion to the workers is optimal, as it induces more effort from the mass of the workers at the low rank, while paying them an even lower wage. This explains why career firms respond to an increase in aggregate productivity by increasing wages at the top and opportunities for promotions, while reducing new hires from unemployment and wages at the lower ranks. This forms

[3] In our model it is optimal for firms to backload wages through incentive-based promotion schemes to elicit higher effort at lower cost. This differs from wage- or contract-posting models such as Burdett & Mortensen (1998), Postel-Vinay & Robin (2002), Moscarini & Postel-Vinay (2013, 2016b), Menzio & Shi (2011) and Robin (2011), as in these models the key mechanism driving within-firm wage dynamics is the outside competition from other firms. That is, it is optimal for firms to backload wages over the worker's tenure to reduce quitting rates, provided they can fully commit (see e.g. Burdett & Coles (2003) and Stevens (2004)), or react to workers' outside wage offers by raising their wage (e.g. Postel-Vinay & Robin (2002)). While this generates wage dynamics within the firm, those are unrelated to worker performance and effort incentives. In the other canonical model of the labor market, the Diamond-Mortensen-Pissarides framework (Diamond (1982); Pissarides (1985); Mortensen & Pissarides (1994)), wages are the outcome of bargaining between the worker and the firm. In all these environments wages reflect outside options and performance-related attributes such as worker abilities or firm/worker productivity, are typically exogenous and independent of labour demand or supply.

the basis for another key model prediction which we use to validate the model. The labor share of income is countercyclical for the low-rank positions and procyclical for the high-rank positions. This deviates from the standard predictions of a neoclassical model where firms respond to an increase in profitability by increasing wages and new hires, implying a procyclical share of labor income. We test this prediction using annual data on labor share by income percentiles from Saez & Zucman (2016) covering the period from 1962 to 2012. We consider all workers in top-career jobs in our model as belonging to the top 10% in the wage distribution as workers in this income category are unlikely to be in low positions trying to reach the top. In line with the model predictions we find that the share of labor income for the bottom 90% is countercyclical while for the top 10% and above is procyclical.

Finally, our model can provide another explanation for the gender wage gap and enrich the still-growing literature on why females have a lower success in the labor market, especially at senior levels. The explanation our model offers is that the wage structure of career firms attracts more men than women because of differences in their confidence levels. In our model, career firms, which offer promotion tournaments to the high-paying top positions, attract the overconfident, especially those that overestimate their ability to climb the ladder. Hence, provided that men are more confident than women, our model can generate a wage gap between men and women who are identical in all other respects, as more men than women will self-select into career firms and opt for promotions to high-paying jobs. While an abundance of evidence point to men being more confident and more prone to competitions when choosing their payment scheme than women, we test whether indeed there are more men than women in those professions with a larger disparity in wages across their different ranks. We confirm that there are significantly fewer women than men in professions with greater disparity in pay across various ranks.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 presents simulations of the model's long run equilibrium to describe the workings of its main mechanisms, and provides intuition for its main results. Section 4 describes the parameterization of the full dynamic model and presents impulse responses of key labor market outcomes such as wages, effort, vacancies, employment and promotions to a shock that raises aggregate productivity, workers' perceived probability of a promotion and the cost to opening career jobs, respectively. Section 5 identifies some testable model predictions and demonstrates that the hierarchical wage structure, present in career firms, is of particular importance for our understanding of aggregate labor market outcomes, and in particular, the gender wage gap, the cyclicality of the labor wedge, and the cyclicality of the labor income share. Section 6 concludes.

2 The Model

There is a continuum of households, each with a unit measure of members. The members of each household can be either unemployed and searching for a job, or employed. Only unemployed workers search for jobs. There are "non-career" and "career" firms and members of each household may search for jobs in either type. Career firms differ from the non-career firms in that they offer the worker the chance to be promoted to a higher rank, based on merit. They have two ranks, the low and the top rank, and workers can move from the low to the top rank through a promotion. No worker can be hired into the top rank from unemployment. Both types of firms decide the number of job vacancies to open and search for unemployed workers, but career firms decide also the number of the workers to be promoted to the top rank each period, which eventually determines the probability for each low-rank employee to get promoted. The two ranks offer different compensation and both types of firms reward workers competitively. There are search frictions in the labor market and new hires in career and non-career firms are determined, respectively, by two matching functions.

2.1 Matching

At each point in time there are N_t^N active non-career jobs while new ones are created from the following constant returns to scale matching function

$$M_t^N = m\left(v_t^N\right)^{\gamma}\left(s_t^N u_t\right)^{1-\gamma}$$

where m is a constant matching efficiency, v_t^N the vacancies for non-career jobs, u_t the number of unemployed and s_t^N the portion of the household's unemployed that are searching for non-career jobs so that $s_t^N u_t$ gives the number of unemployed searching for non-career jobs. The probability for the unemployed to secure one is:

$$\rho_t^N = \frac{m\left(v_t^N\right)^{\gamma}\left(s_t^N u_t\right)^{1-\gamma}}{s_t^N u_t} \tag{1}$$

The law of motion for non-career jobs is

$$N_{t+1}^{N} = \left(1-\lambda^{N}\right) N_{t}^{N} + s_{t}^{N}\rho_{t}^{N} u_{t}$$

where λ^N is an exogenous separation rate and $s_t^N u_t \rho_t^N = M_t^N$ the number of new matches.

New career jobs always start at the lower rank which is denoted by the letter C. For example, at any given point in time, there are N_t^C low-rank workers in the representative career firm and N_t^H high-ranked workers. New career jobs arise from an identical matching function as in the non-career-job case. That is

$$M_{t}^{C} = m\left(v_{t}^{C}\right)^{\gamma}\left(s_{t}^{C} u_{t}\right)^{1-\gamma}$$

where v_t^C is the number of vacancies and $s_t^C u_t$ the number of workers where $s_t^C = 1 - s_t^N$ is the portion of the u_t unemployed in the economy that search for (starting) career jobs. The probability an unemployed searching for a career job to succeed is:

$$\rho_{t}^{C} = \frac{m\left(v_{t}^{C}\right)^{\gamma}\left(s_{t}^{C} u_{t}\right)^{1-\gamma}}{s_{t}^{C} u_{t}} \tag{2}$$

Workers in low-rank positions can get promoted to top positions as each period N_t^{CH} workers move from low ranks to the top and each worker has an average probability ρ_t^H to be promoted. This implies that the law of motion for low-rank positions in career firms is

$$N_{t+1}^{C} = \left(1-\lambda^{C}\right)\left(1-\rho_{t}^{H}\right) N_{t}^{C} + s_{t}^{C} u_{t} \rho_{t}^{C} \tag{3}$$

where λ^C is the exogenous destruction rate, ρ_t^H the probability to get promoted and switch ranks, and $s_t^C \rho_t^C u_t = M_t^C$ the number of new matches. The number of workers that move to the top position at the end of the period are $\left(1-\lambda^C\right)\rho_t^H N_t^C$ and thus the evolution of top jobs follows

$$N_{t+1}^{H} = \left(1-\lambda^{H}\right) N_{t}^{H} + \left(1-\lambda^{C}\right)\rho_{t}^{H} N_{t}^{C}$$

where λ^H is the exogenous destruction rate and $\left(1-\lambda^C\right)\rho_t^H N_t^C = N_t^{CH}$ the new top positions at the end of the period.

2.2 The Household Problem

To avoid complicating the framework we assume that employed and unemployed workers make their decisions according to their current state and once those decisions for the labor supply and job searching are determined, they pool their income together and decide on consumption and bond holdings to avoid shifting to a heterogenous agent framework.

2.2.1 Unemployed Decision Problem

As jobs can be kept for many periods it is easier to use Bellman equations to determine the optimal decisions. The unemployed within the household u_t are searching for either a non-career or career job. The household optimally assigns s_t^C and $s_t^N = 1 - s_t^C$ of its unemployed to search for career and non-career jobs respectively. Let U_t denote the value of unemployment for a single household worker before she is assigned to search for either type of job:

$$U_t = b_t + s_t^C \rho_t^C E_t Q_{t,t+1} W_{t+1}^C + s_t^N \rho_t^N E_t Q_{t,t+1} W_{t+1}^N + \left(1 - s_t^C \rho_t^C - s_t^N \rho_t^N\right) E_t Q_{t,t+1} U_{t+1} \quad (4)$$

The worker has a probability of s_t^C (s_t^N) to be assigned to a career job (non-career-job). Stated otherwise, we assume that unemployed workers do not know exante to which kind of firm they are going to be assigned to apply to by the household. Thus, the worker has a probability of s_t^C to be assigned to a career job and s_t^N probability to be assigned to a non-career job. The term $Q_{t,t+1}$ is the stochastic discount factor:

$$Q_{t,t+1} \equiv \beta \frac{\mu_{t+1}}{\mu_t}$$

where $\mu_t \equiv u_c(C_t)$ is the marginal utility of consumption (C_t). The first term in the value of unemployment is the unemployment benefit b_t. The second term is the expected value of getting a career job $E_t W_{t+1}^C$ and the third term the expected value of a non-career job $E_t W_{t+1}^N$. The probabilities of each of those states have been already defined in (2) and (1) respectively.

Optimality requires that the expected value of assigning an extra worker to search for a career job should be the same as the expected value from assigning the worker to a non-career job. That is maximizing (4) with respect to s_t^C implies:

$$\rho_t^C E_t Q_{t,t+1} \left(W_{t+1}^C - U_{t+1}\right) = \rho_t^N E_t Q_{t,t+1} \left(W_{t+1}^N - U_{t+1}\right) \quad (5)$$

Stated otherwise, suppose the career jobs constitute a better employment option than non career ones i.e. $W_{t+1}^{C} > W_{t+1}^{N}$, then unemployed workers are going to switch their attention to career jobs (s^{C} increases) and the probability to match with such a firm is going to be relatively lower, i.e. $\rho_t^C < \rho_t^N$, such that in equilibrium eq. (5) holds. Eq. (5) implies that the search is determined by free mobility as the marginal worker is indifferent between searching for a career or a non-career job.

2.2.2 Employed Decision Problem

The probability for a low-rank worker to get promoted depends on various parameters such as the worker's skills A compared to the average $\bar{A}$, the effort L_t^C compared to the average $\bar{L}_t^C$ and the number of top positions opened N_t^{CH} relative to the total number of low-rank employees N_t^C at the end of the period. We set

$$\rho_t^H = A^e \left(\frac{L_t^C}{\bar{L}_t^C}\right)^{\gamma_L} \left(\frac{N_t^{CH}}{(1-\lambda^C)\, N_t^C}\right)^{\gamma_N} \tag{6}$$

where $A^e \equiv \frac{A}{\bar{A}}$ is the "perceived" skill premium of the individual worker relative to the average worker.[4] The actual value of A^e equals 1 as everyone is identical, however, the possibility for workers to overestimate their quality is also explored below.[5]

The workers maximize the value of a career job by choosing the amount of labor effort:

$$W_t^C = \max_{L_t^C} \left\{ \begin{array}{c} w_t^C L_t^C - \frac{G\left(L_t^C\right)}{\mu_t} + \left(1-\lambda^C\right) \rho_t^H \left(L_t^C\right) E_t Q_{t,t+1} W_{t+1}^H \\ + \left(1-\lambda^C\right) \left(1-\rho_t^H\left(L_t^C\right)\right) E_t Q_{t,t+1} W_{t+1}^C + \lambda^C E_t Q_{t,t+1} U_{t+1} \end{array} \right\} \tag{7}$$

The value of a career job involves the wage compensation $w_t^C L_t^C$ minus the disutility from labor $G\left(L_t^C\right)$ which is divided by the marginal utility μ_t to transform the units of measure to units of the real good. In the case the job is not terminated, the worker either gets promoted to the top position with probability ρ_t^H (corresponding to a value of W_{t+1}^H next period) or remains at the same rank. If the job is destroyed the worker returns to the unemployment pool.

Maximize the value of the low career job (7) subject to the probability to get promoted

[4] In a simple model γ_N should be equal to 1, especially when workers are identical. However, $\gamma_N \neq 1$ could proxy for a more complicated framework where seniority in a position plays a role. Seniority is actually the best predictor of a promotion, ahead of effort and skill as documented by Hospido et al. (2019).

[5] Since all workers are identical, in equilibrium $L_t^C = \bar{L}_t^C$ and $A^e \equiv \frac{A}{\bar{A}} = 1$.

(6) which is a function of effort as well. Solving for the wage this first order condition implies that:

$$w_t^C = \frac{G_L\left(L_t^C\right)}{\mu_t} - \frac{\partial \rho_t^H}{\partial L_t^C} E_t Q_{t,t+1}\left(W_{t+1}^H - W_{t+1}^C\right) \tag{8}$$

where $\frac{\partial \rho_t^H}{\partial L_t^C} = \gamma_L A^e \left(\frac{L_t^C}{\bar{L}_t^C}\right)^{\gamma_L - 1} \frac{1}{\bar{L}_t^C} \left(\frac{N_t^{CH}}{(1-\lambda^C)N_t^C}\right)^{\gamma_N} = \gamma_L \frac{\rho_t^H}{L_t^C}$. The Labor supply eq. (8) states that the wage does not only depend on the marginal disutility of labor. It is also affected by the expected surplus of the high position over the low position $E_t Q_{t,t+1}\left(W_{t+1}^H - W_{t+1}^C\right)$. It is possible, *ceteris paribus*, for a worker to put extraordinary amounts of effort and be compensated considerably lower just because either the top job is much more rewarding than the current position or it is perceived very likely to reach the top position, or both. The relationship between the wage and labor effort can be weakened by making the top job paying significantly higher and controlling the associated marginal probability to get promoted $\frac{\partial \rho_t^H}{\partial L_t^C}$. Especially if the worker significantly overstates the probability to be promoted ($A^e > 1$), the wage can be disproportionally lower than the corresponding effort in units of the real good $\frac{G_L\left(L_t^C\right)}{\mu_t}$.

The value of the top position is denoted by W_t^H and it includes the wage compensation and disutility as before. That is

$$W_t^H = \max_{L_t^H}\left\{w_t^H L_t^H - \frac{G\left(L_t^H\right)}{\mu_t} + \left(1-\lambda^H\right) E_t Q_{t,t+1} W_{t+1}^H + \lambda^H E_t Q_{t,t+1}\left(U_{t+1}\right)\right\} \tag{9}$$

As there is no possibility of getting demoted to the old position, termination of the job returns the worker to unemployment.

The first order condition deriving from eq. (9) is

$$w_t^H = \frac{G_L\left(L_t^H\right)}{\mu_t} \tag{10}$$

Eq. (10) is identical to what the standard theory predicts and states that the wage fully maximizes the surplus to the worker (producer surplus). The difference between the values to the workers for the top eq. (7) and lower positions eq. (9) is important in decision making and it evolves according to:

$$\begin{aligned} W_t^H - W_t^C &= w_t^H L_t^H - \frac{G\left(L_t^H\right)}{\mu_t} - w_t^C L_t^C + \frac{G\left(L_t^C\right)}{\mu_t} - \lambda^H E_t Q_{t,t+1}\left(W_{t+1}^H - U_{t+1}\right) \\ &+ \left(1 - \left(1-\lambda^C\right)\rho_t^H\right) E_t Q_{t,t+1}\left(W_{t+1}^H - W_{t+1}^C\right) + \lambda^C E_t Q_{t,t+1}\left(W_{t+1}^C - U_{t+1}\right) \end{aligned} \tag{11}$$

For the non-career jobs, the household seeks to maximize

$$W_t^N = \max_{L_t^N}\left\{ w_t^N L_t^N - \frac{G\left(L_t^N\right)}{\mu_t} + \left(1-\lambda^N\right) E_t Q_{t,t+1} W_{t+1}^N + \lambda^N E_t Q_{t,t+1}\left(U_{t+1}\right)\right\} \tag{12}$$

In a similar manner, maximization of (12) gives the labor supply for non-career jobs

$$w_t^N = \frac{G_L\left(L_t^N\right)}{\mu_t} \tag{13}$$

For the rest of the paper the functional form for $G(L)$ is

$$G(L) = \omega \frac{L^{1+\ell}}{1+\ell} \tag{14}$$

where ℓ is the Frisch elasticity of labor supply.

2.2.3 Pooling Income

The household is large, consisting of employed and unemployed people pooling their incomes together to choose consumption and bond holdings. Each period the household consumes C_t and purchases bonds that are in zero net supply. The household's income comes from the labor earnings of the its workers $N_t^C w_t^C L_t^C + N_t^H w_t^H L_t^H + N_t^N w_t^N L_t^N$, the previous period's bonds B_{t-1} times the gross real return R_{t-1}, the intermediate and final good firm profits Π_t^{IF} and other transfer payments T_t^v that is basically the vacancy costs for advertising while the taxes to the government are T_t. w_t^C, w_t^H and w_t^N are the per-unit-of-effort wage rates of workers in starting career jobs, top career jobs and non-career jobs, respectively, and L_t^C, L_t^H, L_t^N, the corresponding effort levels of workers in each of these three types of jobs. The household's budget constraint is

$$C_t + B_t = N_t^C w_t^C L_t^C + N_t^H w_t^H L_t^H + N_t^N w_t^N L_t^N + R_{t-1}B_{t-1} + \Pi_t^{IF} + T_t^v - T_t. \tag{15}$$

Households get utility denoted by $u(.)$ from consumption and disutility from labor denoted by $G(.)$. Hence, the common household, subject to the budget constraint in equation (15), chooses $\{C_t, B_t\}$ for all t to maximize its expected lifetime utility function defined as follows

$$\max_{\{C_t^C, B_t\}_{t=0}^{\infty}} E_t \sum_{t=0}^{\infty} \beta^t \left\{ u\left(C_t^C\right) - N_t^C G\left(L_t^C\right) - N_t^H G\left(L_t^H\right) - N_t^N G\left(L_t^N\right) \right\} \tag{16}$$

where $0 < \beta < 1$ is the discount rate. By denoting μ_t the Lagrange multiplier on the budget constraint, the common household's first-order condition (FOC) with respect to C_t is $u'(C_t) = \mu_t$ as usual. Combining this with the foc for bonds B_t produces the usual Euler equation which is:

$$1 = \beta E_t \left(\frac{u_c(C_{t+1})}{u_c(C_t)} R_t \right). \tag{17}$$

2.3 The Production Function

It is assumed that the productivity of career, and non career jobs is exactly the same and all produce according to a common production function which has the following form

$$X^j = A \left(\left(N^j\right)^{\xi - \frac{1}{\vartheta - 1}} \left[\int_{N^j} \left(L_t^i\right)^{\frac{\vartheta - 1}{\vartheta}} di \right]^{\frac{\vartheta}{\vartheta - 1}} \right)^{\alpha} \tag{18}$$

which implies that the total production X depends on total productivity A, the number of workers N^j, the effort of each worker L_t^i and the elasticity of substitution between workers ϑ. Each worker is a differentiated product and thus the aggregate output depends on the degree of substitutability of those inputs. Also, there is a Love Of Variety (LOV) term $\left(N^j\right)^{\xi - \frac{1}{\vartheta - 1}}$, which determines how desirable it is to produce using more labor effort from each worker than using the same overall effort from many workers. It also implicitly controls the effect of the overall competition on the individual firm.

For a career firm that has N_t^C workers in the low ranks and N_t^H in the higher ranks, the production function is

$$X_t^{C,H} = A\left(\left(N_t^C + N_t^H\right)^{\xi - \frac{1}{\vartheta - 1}}\left[\int\limits_{N_t^C + N_t^H}\left(L_t^i\right)^{\frac{\vartheta-1}{\vartheta}}di\right]^{\frac{\vartheta}{\vartheta-1}}\right)^{\alpha} \tag{19}$$

and since workers within each groups provide the same labor effort, the above becomes

$$X_t^{C,H} = A\left(\left(N_t^C + N_t^H\right)^{\xi - \frac{1}{\vartheta - 1}}\left[N_t^C\left(L_t^C\right)^{\frac{\vartheta-1}{\vartheta}} + N_t^H\left(L_t^H\right)^{\frac{\vartheta-1}{\vartheta}}\right]^{\frac{\vartheta}{\vartheta-1}}\right)^{\alpha} \tag{20}$$

Similarly the production function for non-career jobs (18) becomes

$$X_t^N = A\left(\left(N_t^N\right)^{1+\xi} L_t^N\right)^{\alpha} \tag{21}$$

2.4 Intermediate Firms

The following section demonstrates that the division between career and non-career jobs stems from the way they approach the respective wage decisions. As the problem appears to be time inconsistent, the firms that can pre-commit to the wage and their promises in the past, are the ones designated as "career" firms and the ones that cannot commit as "non-career" firms even though every other aspect of those firms is identical. A firm announces different wages per effort unit in the previous period for the low and high ranks and plans to stick to it. The firms that can stick to this payment structure are the career firms offering two different job levels. The firms that cannot commit are possibly more lenient towards workers, and eventually end up paying all the workers the same. Meanwhile, workers are expecting this behavior and thus they all provide the same effort as in a standard model. Therefore, if a firm cannot fulfill its promises and more importantly the workers do not expect it to do so either, the firm does not discriminate between workers, pays the same wage to everyone and it is classified as a non-career firm. If the firm is able to commit because it has successfully committed in the past, it becomes a career firm which shares the concept of the "timeless perspective" in Woodford (1999).

2.4.1 Wage and Time-inconsistency

Career firms take into consideration the labor supply of the workers[6], eq. (8) and (10) and optimally set the wage for low and top position workers, w_t^C and w_t^H, respectively, which implies they are not price takers in the market for labor. Suppose firms sell their whole quantity at price P_t^x to the final-good firms and maximize the following objective function with respect to $w_{t+i}^C, w_{t+i}^H, v_{t+i}^C$ and N_{t+i}^{CH} :

$$\max_{\left\{w_{t+i}^C, w_{t+i}^H, v_{t+i}^C, N_{t+i}^{CH}\right\}_{i=0}^{\infty}} \sum_{i=0}^{\infty} Q_{t,t+i} \left\{ P_{t+i}^x X_{t+i}^{C,H} - N_{t+i}^C w_{t+i}^C L_{t+i}^C - N_{t+i}^H w_{t+i}^H L_{t+i}^H - \kappa_{t+i}^C v_{t+i}^C - \kappa_{t+i}^{CH} N_{t+i}^{CH} \right\} \tag{22}$$

which includes the production from the N_t^C low-rank workers and the N_t^H top-rank workers minus the corresponding wage costs. The unit cost of opening vacancies (v_t^C) is κ_t^C and it is assumed that promoting N_t^{CH} workers is associated with κ_t^{CH} cost per person. The maximization of (22) is subject to the supply of labor, eq. (8) which solved for L_t^C becomes

$$L_{t+i}^C = \left(\frac{1}{\omega} \mu_{t+i} w_{t+i}^C + \frac{\mu_{t+i}}{\omega} \frac{\partial \rho_{t+i}^H}{\partial L_{t+i}^C} E_{t+i} Q_{t+i,t+i+1} \left(W_{t+i+1}^H - W_{t+i+1}^C \right) \right)^{\frac{1}{\ell}}, \tag{23}$$

The problem with respect to w_t^C, w_t^H is time inconsistent.[7] In the next period the firm promises to offer a higher wage but when the time comes the firm may fail to deliver. This is evident after maximizing (22) with respect to w_t^C. In the current period, $i = 0$, the first order condition for w_t^C is:

$$\left(P_t^x \frac{dX_t^{C,H}}{dL_t^C} - N_t^C w_t^C \right) \frac{dL_t^C}{dw_t^C} = N_t^C L_t^C \tag{24}$$

Eq. (24) implies that the wage is maximized at the point where the cost of increasing the wage by one dollar (right hand side), equals the gain in profits (left hand side). The marginal gain in profits comes from an increase in labor effort due to an increase in the wage compensation $\frac{dL_t^C}{dw_t^C}$ from the chain rule.

[6] It is necessary for firms to be able to incentivize workers.

[7] The problem for the other variables is time inconsistent as well, but it is explored in the next section. However, if the firm cannot commit to the wage then the problem seises to be time inconsistent for the rest of the decision variables.

If the firm can commit, for $i \geq 1$, the first order condition after rearrangements is

$$E_t Q_{t,t+i} \left[\begin{array}{c} \left(P^x_{t+i} \frac{dX^{C,H}_{t+i}}{dL^C_{t+i}} - N^C_{t+i} w^C_{t+i} \right) \frac{dL^C_{t+i}}{dw^C_{t+i}} - N^C_{t+i} L^C_{t+i} \\ +E_{t+j-1} \left(P^x_{t+i-1} \frac{dX^{C,H}_{t+i-1}}{dL^C_{t+i-1}} - N^C_{t+i-1} w^C_{t+i-1} \right) \frac{dL^C_{t+i-1}}{dw^C_{t+i}} \end{array} \right] = 0 \tag{25}$$

which includes an extra term compare to eq. (24). In the case the firm is able to commit, then Eq. (25) holds from the first period and on (timeless perspective). That is,

$$\left(P^x_t \frac{dX^{C,H}_t}{dL^C_t} - N^C_t w^C_t \right) \frac{dL^C_t}{dw^C_t} + \left(P^x_{t-1} \frac{dX^{C,H}_{t-1}}{dL^C_{t-1}} - N^C_{t-1} w^C_{t-1} \right) \frac{dL^C_{t-1}}{dw^C_t} - N^C_t L^C_t = 0 \tag{26}$$

Eq. (26) states that by promising a higher wage in period t, the firm affects incentives to work in period $t-1$ which corresponds to the last term in equation (26). The first two terms have the same interpretation as in eq. (24). To pin down $\frac{dL^C_t}{dw^C_t}$ in eq. (26), use eq. (23) in $i = 0$. That is

$$\frac{dL^C_t}{dw^C_t} = \frac{\frac{\mu_t}{\omega}}{\ell \left(L^C_t \right)^{\ell - 1} - \frac{\mu_t}{\omega} \frac{\partial^2 \rho^H_t}{\partial L^C_t \partial L^C_t} E_t Q_{t,t+1} \left(W^H_{t+1} - W^C_{t+1} \right)} \tag{27}$$

The above states that an increase in the wage results in an increase in labor effort in the case $\frac{\partial^2 \rho^H_t}{\partial L^C_t \partial L^C_t} = 0$ which is a standard result coming from a labor supply. However, if for example $\frac{\partial^2 \rho^H_t}{\partial L^C_t \partial L^C_t} > 0$, then increasing the wage leads to an increase in the labor effort, that gives the worker the impression of gaining an edge over the rest of the workers and thus increases the marginal probability to be promoted further. In this case the increase in labor effort from an increase in the wage $\frac{dL^C_t}{dw^C_t}$ becomes even more pronounced.

Use eq. (23) for $i = 0$, after substituting in eq. (7) and (9) a period forward. This implies that the derivative of $\frac{dL^C_{t-1}}{dw^C_t}$ using also (27) is

$$\frac{dL^C_{t-1}}{dw^C_t} = \frac{\frac{\mu_{t-1}}{\omega} \frac{\partial \rho^H_{t-1}}{\partial L^C_{t-1}} E_{t-1} Q_{t-1,t} \partial \frac{W^H_t - W^C_t}{\partial w^C_t}}{\ell \left(L^C_{t-1} \right)^{\ell - 1} - \frac{\mu_{t-1}}{\omega} \partial \frac{\partial^2 \rho^H_{t-1}}{\partial L^C_{t-1} \partial L^C_{t-1}} E_{t-1} Q_{t-1,t} \left(W^H_t - W^C_t \right)} \tag{28}$$

The derivative in (28) is negative because by definition $\partial \frac{W^H_t - W^C_t}{\partial w^C_t} < 0$, which implies that when the firm announces that next period the wage in low-rank positions is going to be higher, the top positions become less enticing and thus low-rank workers are less willing to put effort to reach the top.

Maximizing objective (22) with respect to w_t^H and subject to (10) and (8) as the latter depends on w_t^H, leads to a similar time inconsistency problem. If the firm cannot commit then the first order condition for w_t^H is

$$\left(P_t^x \frac{dX_t^{C,H}}{dL_t^H} - N_t^H w_t^H\right) \frac{dL_t^H}{dw_t^H} = N_t^H L_t^H \tag{29}$$

The solution to the above problem of maximizing (22) for $i \geq 1$ is

$$E_t Q_{t,t+i} \left[\begin{array}{c} \left(P_{t+i}^x \frac{dX_{t+i}^{C,H}}{dL_{t+i}^H} - N_{t+i}^H w_{t+i}^H\right) \frac{dL_{t+i}^H}{dw_{t+i}^H} - N_{t+i}^H L_{t+i}^H \\ + \left(P_{t+i-1}^x \frac{dX_{t+i-1}^{C,H}}{dL_{t+i-1}^H} - N_{t+i-1}^C w_{t+i-1}^C\right) \frac{dL_{t+i-1}^C}{dw_{t+i}^H} \end{array} \right] = 0 \tag{30}$$

Under the *time-less perspective*, (30) holds for all periods and thus holds also from period t, which implies

$$\left(P_t^x \frac{dX_t^{C,H}}{dL_t^H} - N_t^H w_t^H\right) \frac{dL_t^H}{dw_t^H} + \left(P_{t-1}^x \frac{dX_{t-1}^{C,H}}{dL_{t-1}^C} - N_{t-1}^C w_{t-1}^C\right) \frac{dL_{t-1}^C}{dw_t^H} - N_t^H L_t^H = 0 \tag{31}$$

is the equation that determines the optimal wage for top positions. As before, the difference in the first order condition when the firm can and cannot commit (by comparing eq. 29 and 31) is the second term in (31), $\left(P_{t-1}^x \frac{dX_{t-1}^{C,H}}{dL_{t-1}^C} - N_{t-1}^C w_{t-1}^C\right) \frac{dL_{t-1}^C}{dw_t^H}$. Provided that in this case the firm is considered by workers credible, this term captures the benefit from the increase in labor effort of workers in lower ranks, when the firm increases the wage that it promises to pay in top positions next period (w_t^H).

To determine $\frac{dL_t^H}{dw_t^H}$ take the derivative of (10) to get

$$\frac{dL_t^H}{dw_t^H} = \frac{\mu_t}{\omega \ell} \left(L_t^H\right)^{1-\ell} \tag{32}$$

while for $\frac{dL_{t-1}^C}{dw_t^H}$ use (8) along with eq. (7) and (9) a period forward along with (32). That is

$$\frac{dL_{t-1}^C}{dw_t^H} = \frac{\frac{\mu_{t-1}}{\omega}\frac{\partial \rho_{t-1}^H}{\partial L_{t-1}^C}E_{t-1}Q_{t-1,t}\partial\frac{W_t^H - W_t^C}{\partial w_t^H}}{\ell\left(L_{t-1}^C\right)^{\ell-1} - \frac{\mu_{t-1}}{\omega}\partial\frac{\partial^2\rho_{t-1}^H}{\partial L_{t-1}^C \partial L_{t-1}^C}E_{t-1}Q_{t-1,t}\left(W_t^H - W_t^C\right)} \tag{33}$$

The above derivative in (33) is positive as $\partial\frac{W_t^H - W_t^C}{\partial w_t^H} > 0$ by definition, signifying that an announcement of a higher wage at the top position next period, induces higher effort from low-rank workers in this period, because the prize of getting a promotion becomes even more rewarding.

The following proposition highlights the distinction between career and non-career firms:

Proposition 1 *Suppose firms cannot commit. Also let $\lambda^C = \lambda^H$ and $\gamma_L = \gamma_N = 1$. In the long-run equilibrium, $w_t^C = w_t^H$ and $L_t^C = L_t^H$; all firms offer only one type of job, all workers supply labor equally and there are no promotions.*

The proof is in Appendix B. The above proposition suggests that if the firm cannot commit then even if it has the option to treat two different groups of workers differently, it is optimal to compensate everyone equally, leading to workers putting the exact same effort. Therefore, not being able to commit creates naturally the non-career and career firm distinction. The non-career firms are basically those that for whatever reason they cannot pre-commit to such a specific payment system.

In what follows we assume that the distinction between career and non-career firms exists. That is, some firms can pre-commit to a specific payment structure so that it is optimal for them to offer low and high-paying positions and the possibility of promotion to their employees.

2.4.2 Job Creation and Promotions in Career firms

We explain here how career firms make decisions about how many new vacancies to open and how many workers to promote to the top position.

Since in equilibrium all workers are the same, the number of starting career jobs evolves according to

$$N_{t+1}^C = \left(1 - \lambda^C\right)N_t^C - N_t^{CH} + q_t^C v_t^C \tag{34}$$

New career jobs arise next period by posting vacancies v_t^C while $q_t^C \equiv \rho_t^C \frac{s_t^c u_t}{v_t^C}$ is the probability for a vacancy to match. At the end of the period N_t^{CH} workers are promoted to the top position. The number of top jobs evolves according to

$$N_{t+1}^H = \left(1-\lambda^H\right) N_t^H + N_t^{CH} \tag{35}$$

where N_t^{CH} are the new promotions that are in the firm's control.

Career firms maximize the objective in (22) with respect to vacancies v_{t+i}^C and promotions N_{t+i}^{CH} subject to the labor supply (23) as before. The resulting first order conditions give the job creation condition for new career jobs and the worker promotion condition which are given, respectively, by:

$$\begin{aligned} \frac{\kappa_t^C}{q_t^C} &= \frac{1}{q_t^C}\left(P_{t-1}^x \frac{dX_{t-1}^{CH}}{dL_{t-1}^C} - N_{t-1}^C w_{t-1}^C\right)\frac{dL_{t-1}^C}{dv_t^C} \\ &+E_t Q_{t,t+1}\left[\begin{array}{c} P_{t+1}^x \frac{dX_{t+1}^{CH}}{dN_{t+1}^C} - w_{t+1}^C L_{t+1}^C + \\ \left(P_{t+1}^x \frac{dX_{t+1}^{CH}}{dL_{t+1}^C} - N_{t+1}^C w_{t+1}^C\right)\frac{dL_{t+1}^C}{dN_{t+1}^C} + \\ \left(1-\lambda^C\right)\left(\frac{\kappa_{t+1}^C}{q_{t+1}^C} - \frac{1}{q_{t+1}^C}\left(P_t^x \frac{dX_t^{CH}}{dL_t^C} - N_t^C w_t^C\right)\frac{dL_t^C}{dv_{t+1}^C}\right) \end{array}\right] \end{aligned} \tag{36}$$

$$\begin{aligned} \kappa_t^{CH} &= \left(P_t^x \frac{dX_t^{CH}}{dL_t^C} - N_t^C w_t^C\right)\frac{dL_t^C}{N_t^{CH}} + \left(P_{t-1}^x \frac{dX_{t-1}^{CH}}{dL_{t-1}^C} - N_{t-1}^C w_{t-1}^C\right)\frac{dL_{t-1}^C}{dN_t^{CH}} \\ &-\left(\frac{\kappa_t^C}{q_t^C} - \frac{1}{q_t^C}\left(P_{t-1}^x \frac{dX_{t-1}^{CH}}{dL_{t-1}^C} - N_{t-1}^C w_{t-1}^C\right)\frac{dL_{t-1}^C}{dv_t^C}\right) \\ &+E_t Q_{t,t+1}\left(P_{t+1}^x \frac{dX_{t+1}^{CH}}{dN_{t+1}^H} - w_{t+1}^H L_{t+1}^H\right) \\ &+E_t Q_{t,t+1}\left(1-\lambda^H\right)\left(\begin{array}{c} \kappa_{t+1}^{CH} - \left(P_{t+1}^x \frac{dX_{t+1}^{CH}}{dL_{t+1}^C} - N_{t+1}^C w_{t+1}^C\right)\frac{dL_{t+1}^C}{N_{t+1}^{CH}} \\ -\left(P_t^x \frac{dX_t^{CH}}{dL_t^C} - N_t^C w_t^C\right)\frac{dL_t^C}{dN_{t+1}^{CH}} \\ +\left(\frac{\kappa_{t+1}^C}{q_{t+1}^C} - \frac{1}{q_{t+1}^C}\left(P_t^x \frac{dX_t^{CH}}{dL_t^C} - N_t^C w_t^C\right)\frac{dL_t^C}{dv_{t+1}^C}\right) \end{array}\right) \end{aligned} \tag{37}$$

The proof is in Appendix C. In each case, the optimality condition is such that the benefit (right-hand-side) equals the cost (left-hand-side). Notice that in both cases the benefit depends also on how the firm's decision affects workers' incentives to supply effort in low-rank positions.

$$\frac{dL_t^C}{dN_t^{CH}} = \frac{1}{\ell}\left(L_t^C\right)^{1-\ell}\left(\frac{\mu_t}{\omega}\frac{\partial\left(\frac{\partial\rho_t^H}{\partial L_t^C}\right)}{\partial N_t^{CH}}E_tQ_{t,t+1}\left(W_{t+1}^H - W_{t+1}^C\right)\right) > 0 \tag{38}$$

The effect of more promotions on labor effort in low-rank positions is positive and comes from increasing directly the probability to be promoted.[8]

$$\frac{dL_t^C}{dN_t^C} = \frac{1}{\ell}\left(L_t^C\right)^{1-\ell}\left(\begin{array}{c}\frac{\mu_t}{\omega}\frac{\partial\left(\frac{\partial\rho_t^H}{\partial L_t^C}\right)}{\partial N_t^C}E_tQ_{t,t+1}\left(W_{t+1}^H - W_{t+1}^C\right)\\ +\frac{\mu_t}{\omega}\frac{\partial\rho_t^H}{\partial L_t^C}E_tQ_{t,t+1}\left(\partial\frac{W_{t+1}^H-W_{t+1}^C}{\partial N_{t+1}^C}\right)\frac{\partial N_{t+1}^C}{\partial N_t^C}\end{array}\right) \tag{39}$$

The effect of increasing the number of workers in low-rank jobs on effort in these jobs, given by (39) above, is twofold. First it negatively affects the probability to be promoted to a top position (first term in the bracket), with a negative impact on effort and second it affects the future payoff from a promotion $W_{t+1}^H - W_{t+1}^C$ (second term in the bracket).

The firm also anticipates that its decisions of how many workers to promote and how many new vacancies to open at time t will also affect workers' effort in period $t-1$, given that, as discussed above, the firm can credibly commit to a future plan. Thew relevant expressions are the following:

$$\frac{dL_{t-1}^C}{dN_t^{CH}} = \frac{1}{\ell}\left(L_{t-1}^C\right)^{1-\ell}\frac{\mu_{t-1}}{\omega}\frac{\partial\rho_{t-1}^H}{\partial L_{t-1}^C}E_{t-1}Q_{t-1,t}\left(\partial\frac{W_t^H - W_t^C}{\partial N_t^{CH}}\right)$$

$$\frac{dL_{t-1}^C}{dv_t^C} = \frac{1}{\ell}\left(L_{t-1}^C\right)^{1-\ell}\frac{\mu_{t-1}}{\omega}\frac{\partial\rho_{t-1}^H}{\partial L_{t-1}^C}E_{t-1}Q_{t-1,t}\left(\partial\frac{W_t^H - W_t^C}{\partial v_t^C}\right)$$

The impact of the firm's future plans on effort of workers in previous periods depend on how the firm's decisions affect the payoff from a promotion in the future, highlighting how firms may use promises for future promotion possibilities to elicit more effort from workers.

2.4.3 Job Creation in Non-Career Firms

We turn now to the job creation decision of non-career firms. The objective in this case is

$$\max_{w_t^N, v_t^N}\sum_{t=0}^{\infty}\beta^t\left\{P_t^x X_t^N - N_t^N w_t^N L_t^N - \kappa_t^N v_t^N\right\} \tag{40}$$

[8] Note that the firm understands that all workers in the same rank supply equal effort thus $L_t^C = \bar{L}_t^C$.

as non-career firms set the same wage for all workers and offer only one type of job. The optimization is subject to the usual law of motion for jobs which is

$$N_{t+1}^N = \left(1 - \lambda^N\right) N_t^N + q_t^N v_t^N \tag{41}$$

where $q_t^N \equiv \rho_t^N \frac{s_t^N u_t}{v_t^N}$ is the job filling probability of a non-career firm. Maximizing (40) subject to (41) gives the job creation condition

$$\frac{\kappa_t^N}{q_t^N} = E_t Q_{t,t+1} P_{t+1}^x \frac{dX_{t+1}^N}{dN_{t+1}^N} - E_t Q_{t,t+1} w_{t+1}^N L_{t+1}^N + \left(1 - \lambda^N\right) E_t Q_{t,t+1} \frac{\kappa_{t+1}^N}{q_{t+1}^N}$$

where, as above, the cost of a vacancy equals the benefit, and the labor demand for non-career firms

$$\left(P_t^x \frac{dX_t^N}{dL_t^N} - N_t^N w_t^C\right) \frac{dL_t^N}{dw_t^N} = N_t^N L_t^N \tag{42}$$

where

$$\frac{dL_t^N}{dw_t^N} = \frac{1}{\omega \ell} \left(L_t^N\right)^{1-\ell} \mu_t \tag{43}$$

The above condition stems from eq. (27) when $W_{t+1}^H - W_{t+1}^C = 0$ which holds in the case of firms that cannot commit (via Proposition 1).

2.5 Final Good Firms

Final good firms purchase the product of intermediate firms to produce the final good. There is only one good in this economy and thus its price is one. The production function of the i^{th} final good for firm is

$$y_{it} = z_t^f \left(X_{it}^{C,H} + X_{it}^N\right) \tag{44}$$

implying that it purchases the output of career and non-career firms at price P_t^x and produces the final good. The objective of the final goods-producing firms is to maximize profits subject the demand for each firm derived from a cost minimization problem of the household as defined below

$$y_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{-\theta} Y_t \tag{45}$$

where the aggregate price level is

$$P_t = \left[\int_0^1 (p_{jt})^{1-\theta} \, dj \right]^{\frac{1}{1-\theta}} \tag{46}$$

while the aggregate output is

$$Y_t = \left[\int_0^1 (y_{jt})^{\frac{\theta-1}{\theta}} \, dj \right]^{\frac{\theta}{\theta-1}} \tag{47}$$

with θ being the elasticity of substitution.

Profit maximization implies that $\frac{p_t}{P_t} = \frac{\theta}{\theta-1} \frac{P_t^x}{z_t^f}$ and since there are no price rigidities, $\frac{p_t}{P_t} = 1$ and thus

$$P_t^x = \frac{\theta - 1}{\theta} z_t^f \tag{48}$$

3 The Long-Run Equilibrium

In this section we provide some simulations of the model's long-run equilibrium for different values of the parameters to better visualize the main mechanism. Table 1 summarizes the calibration of the parameters necessary for the exercises. We keep the size of career and non-career firms equal to facilitate their comparison. Figure 1 depicts the labor supply (solid lines) and the labor demand (dash-and-dot lines) along with the equilibrium values for the wage and the labor effort for the non-career jobs (blue set of lines), the low career jobs (red set of lines) and the top career jobs (green set of lines) for various values of the parameters. The supply of labor for top jobs comes from the household problem which is eq. (8). The supply of labor for top jobs is eq. (10). The labor demand for starting positions comes from the firm problem and it is summarized in (26) after substituting in eq. (27) and (28). The labor demand for top jobs is summarized in (31) after substituting in both (32) and (33).

We are particularly interested in the parameters that affect the perceived marginal probability to be promoted for the extra unit of effort in the low position $\frac{\partial \rho_t^H}{\partial L_t^C}$ which is the term that differentiates the labor supply in (8) from the standard labor supply where the wage equals the MRS (Marginal Rate of Substitution). The top left panel characterizes the equilibrium when $A^e = 0$ and thus the probability to get promotes is perceived to be zero no matter how large is the relative effort of the worker. It is evident that all 3 equilibrium wages and labor efforts align to the same value, such that wage equals MRS, which is what a standard model of labor supply and labor demand would predict. However, on the top right panel where $A^e = 1$ and the marginal probability is no longer zero, the wage equals MRS in only the non-career jobs (blue lines). The equilibrium for the low career jobs implies a higher labor effort $L_c > L_n$ meaning that the wage is lower than MRS. The additional effort is mainly for the worker to be able increase his/her chances to be promoted. It comes with no direct compensation, but from a lottery ticket that promises a higher compensation in the future in case the worker gets promoted. The compensation mechanism that rewards the few (the "champions") disproportionably more, makes workers' effort larger than the socially optimal, because the reward (the promotion) depends on relative effort.

In the bottom left plot of 1, we set γ_N to a lower value (γ_N=0.9). In this case the effect of competition becomes less pronounced and thus low-rank workers are willing to put more effort than in the previous figure with even less compensation. Same results are also evident in the bottom right figure where agents are overconfident about their own skills, $A^e > 1$, which induces higher effort for much lower compensation again. When workers are overconfident, the firm is better able to manipulate the workers' desire to reach the top positions. It raises the wage for the top jobs and by doing so, the top-rank workers are putting higher effort, which benefits the firm, even though there are diminishing returns. However, this higher wage induces the low-rank workers to put an extraordinary amount of effort in exchange for even lower compensation than before.

Figures 2 to 4 present the long-run equilibrium values of various endogenous variables for different values of the productivity parameter A, the relative perceived productivity A^e and the destruction rate for top positions λ_H, respectively, when all other things remain constant. In each Figure, results are reported for wages, labor effort, wage per effort unit, output, firm profit, and workers' surplus from employment in all three types of jobs and from a promotion. We see in Figure 2 that higher productivity increases all long run wages albeit the wage for the low-rank workers is not growing as rapidly as the other two. The labor effort is almost identical for low-ranked workers and non-career workers even though the wages of the non-career workers are disproportionally higher. Therefore, the wage paid per effort unit to the low-rank workers is significantly lower than the other wage contracts and the gap widens as productivity increases.

Figure 3 shows how career firms can exploit workers' overconfidence to significantly increase their profits, by offering workers in low-rank positions significantly lower pay per unit of effort, while increasing the per-unit of effort wage of the few that get promoted. As workers become more confident, i.e., their perceived skill relative to the others increases, the wage in starting positions of career firms decreases, while workers' effort in these jobs increases. The more confident workers are willing to exert higher effort with not an immediate pay compensation, as they anticipate that their chances for a promotion are high. The firm takes advantage of this ambition by increasing wages at the top, while lowering those at the bottom, which in turn, induces even more effort from workers in low-rank positions.

The rate at which the top positions are destroyed, λ_H, is a parameter of a particular interest, since, based on our model, it has two opposite effects on the wage of starting positions in career firms. The job destruction rate must equal the top position creation rate in a stable equilibrium and thus higher job destruction rate inevitably increases the number of workers that get promoted in the long run. This decreases the compensation workers are willing to accept per unit of effort, since the probability to get promoted increases. In other words, workers' effort in low positions is now compensated by a higher probability of a promotion, which allows for the wage in these jobs to fall. On the other hand, a higher job destruction directly decreases the value of the top position. However, as shown in Figure 4, the increased probability to get promoted dominates and as the job destruction rate of top positions increases, workers' effort increases, while the wage for low positions decreases.

4 Dynamic Model

This section presents the full model in the dynamic setup introduced in the previous sections. The parameters of the dynamic model are estimated to match quarterly US data from 1948:Q2 to 2020:Q4. We rely on Dynare's Bayesian estimation techniques that employ the Metropolis-Hastings algorithm to determine the posterior distributions of the parameters to be estimated. The observed variables in the estimation are detrended data from the Fred database.[9] The variable and shock descriptions are in the Appendix A along with details about the estimation. We match the trajectory of aggregated variables in the data such as the real GDP, labor productivity, total wage, total hours and total employment with those in the model. For the task, we need to specify as many shocks as the matched empirical variables. The estimated parameter posterior modes are presented in Table 2 along with the

[9]The trend is removed by keeping the residuals of a regression of the log of the variable on a linear and quadratic deterministic trend. HP-filtered data produce similar results.

prior distribution's first 2 moments. The parameter descriptions are presented in Table 3. Figure 5 presents impulse responses after a positive productivity shock from an empirical VAR[10] using the data from Appendix A and from our benchmark model using the estimated parameters in Table 2. The model matches the empirical responses fairly well and creates responses for hours that are as volatile as output, even though wages are not as volatile, which is particularly challenging. Details of the VAR estimation is kept in the Appendix A.

Figure 6 presents plots of the impulse responses (log deviations from steady state) after a positive intermediate-good productivity shock that increases A_t that follows an AR(1) when linearized. The increased productivity gives the opportunity to both types of firms to expand their production albeit in a different way. The non-career firms expand both the number of jobs (N^N) and the labor effort from each worker (L^N), by posting more vacancies (v^N). However, the strategy of the career jobs is different. They increase the compensation at the top (w^H) and also increase the number of promotions (N^{CH}). At the same time they decrease the number of vacancies at the bottom (v^C) implying a higher probability of promotion for low-ranked workers as their pool shrinks while opportunities for promotions increase. As the promotion probability (ρ^H) increases, the wage of the low-ranked workers (w^C) decreases. Despite the decrease in their wage, low-ranked workers increase their effort (L^C), to take advantage of the better prospects of a promotion. As the wage at the bottom drops and at the top increases, the gap between the values of top and low jobs ($W^H - W^C$) increases, which decreases the wage of the low positions (w^C) even more, making the value of a promotion even higher and inducing even more effort from low-ranked workers. Consequently, even though the profits of both types of firms, (Π^C, Π^N), increase, career firms benefit the most from the increase in productivity, by capitalizing on their wage structure. They are able to induce higher effort from the mass of the workers by rewarding them less and rewarding more only the few that get promoted.

Figure 7 presents the impulse responses for the same variables for a shock that increases the perceived gap between the workers own skill level and the average skill level of the competition $A_t^e = \frac{A_t}{\bar{A}_t}$ which in the long run approaches 1 as all workers are identical. The results are similar with the ones before only more exaggerated. If each low-rank worker all of a sudden perceives a higher probability to be promoted, he/she will provide a larger amount of effort (L^C) for an even lower wage (w^C). The higher effort for lower compensation makes

[10]The empirical VAR has no effect in the estimation of the model parameters and its sole purpose is to visualize somehow the structure of the data.

non-career jobs more attractive ($s^N > s^C$) and thus non-career firms find optimal to increase output by hiring more workers while reducing the hours of each worker and the wage.

Figure 8 depicts the impulse responses after a shock that increases the cost of opening career-jobs κ_t^C. This indirectly replicates similar dynamics as before even though, as expected, it induces a recession. The increased cost induces a decrease in vacancies for low-rank positions (v^C) and a decrease in promotions (N^{CH}). This induces a drop in the wages of low-ranked workers and an increase in their effort, as the effect of the decrease in hiring is more pronounced and the probability to get promoted increases. The gap $W^H - W^C$ also increases, which also lowers the wage of low-rank workers, despite their increased effort. Similar dynamics can be deciphered by also having firms endogenously destroy jobs. The fear of the job getting terminated can also boost effort in a similar way.[11]

An important takeaway from the results presented here is that it is optimal to create winners and losers even if workers are identical. A situation where the possibility of a promotion enables the firm to distinguish skilled from less skilled workers is important of course, but having a champion is sometimes more important than simply rewarding the best. It is irrelevant in this framework who is better than whom. All that matters is that only a few are going to be compensated handsomely and well above their marginal product. The rest are going to be paid partly with a lottery ticket that can move them to the higher ranks. Increasing the value of this lottery ticket to the mass of the workers can induce effort from the mass at lower cost to the firm.

5 Testing Model Predictions

In this section we identify some testable model predictions and discuss how the incorporation of career firms in our model can shed light on some important empirical puzzles in macroeconomics. First, the model can provide an explanation to the gender wage gap. As we demonstrate below, differences in confidence between men and women can drive different allocations of men and women across career and non-career firms. We also provide a small empirical exercise to demonstrate this. Second, the model can make realistic predictions for the share of labor income along the business cycle. Third, the model can provide realistic predictions for the labor wedge. The last two rest on the key prediction of our model: that

[11] We have also experimented with a version of the model with endogenous job destruction with very similar results.

effort does not only respond to compensation, but also to a varying value of promotion opportunities along the business cycle, hence effort does not move hand in hand with the wage along the business cycle.

5.1 Gender Wage Gap

There is a still-growing literature on why females have a lower success in the labor market especially at senior levels, even though women tend to have higher education on average than men. Various studies document that to a large part, the gender wage gap arises because either women are less likely to be employed in high-paying firms or women receive worse wage bargains than men (see e.g. Card et al. (2016), Fortin et al. (2017)). In our model the high paying jobs are offered by career firms with strong hierarchical structures and competitive environments, where only the few get promoted. Based on experimental evidence, a number of studies conclude that women tend to be less competitive and less confident than men. Niederle & Vesterlund (2007), for instance, find that men select tournaments twice as much as women when choosing their compensation scheme and that this gender gap in tournament entry is not explained by performance or risk aversion. Instead, it is driven by men being more overconfident and by gender differences in preferences for performing in a competition.[12] In our model, career firms, which offer promotion tournaments to the high-paying top positions, attract the overconfident, especially those that overestimate their ability to climb the ladder. Hence, provided that men are more confident than women, our model can generate a wage gap between men and women who are identical in all other respects, as more men than women will self-select into career firms and opt for promotions to high-paying jobs.

[12] Results similar in spirit are found in Gneezy et al. (2003), Croson & Gneezy (2009) and Buser et al. (2014). Fortin et al. (2017) ask what are the consequences of the under-representation of women in top jobs for the overall gender pay gap. They find that it accounts for a substantial share of the gender pay gap in annual earnings in the three countries they study (the UK, Canada and Sweden). Gupta & Bhawe (2007) show that men are more concerned about their status. There is also evidence that men tend to be more confident than women about their own skills as Lichtenstein et al. (1977) report and even more so when the task is considered masculin according to Moore & Small (2007). McCarty (1986) documents that women were more likely to express lower levels of self-confidence even when achievement situations are experienced. Lundeberg et al. (1994) report in an experiment that although both men and women were overconfident in their ability to answer various questions, men were especially overconfident when they were in fact incorrect. Since confidence in this model affects the perceived probability to be promoted, more relevant to this distinction is the work by Kay & Shipman (2014) reporting that women applied for a promotion only when they met 100 percent of the qualifications while men applied only when they met just 50%.

Proposition 2 *The more overconfident a workers is about his/her own skills relative to average skills (higher A^e), the more likely the worker is to search for a job in a career firm.*

The proof is in Appendix D. Even though confidence levels are not idiosyncratic in the model, the above proposition suggests that if there are two types of workers that differ in their confidence levels, the more confident will search more intensively for career jobs while their less confident counterparts are targeting the non-career jobs more actively.

This model prediction could be ideally tested by examining the proportion of men and women in career and non-career jobs, provided that data availability allowed for an accurate classification of workers' jobs into career and non-career jobs. However an accurate such classification is difficult. Below we rely on a distinct characteristic of career firms/paths in order to test this prediction. In particular, career firms have a strong hierarchical structure according to which wages increase substantially with job levels. The model prediction that can therefore be tested empirically is:

Prediction *The higher the earnings spread across ranks of the same career type, the fewer women relative to men should follow this particular career option.*

The following sections describe the steps to estimate the above model prediction.

5.1.1 The Data

We use a dataset from Data.gov that holds US government open data. Our dataset includes hourly earnings for different ranks in various job categories in Seattle in 2015. There are 829 observations that include male and female earnings and also the number of males and females in each job category. Table 4 presents a few rows and columns from the dataset. Early Ed Spec (early education specialist) has 2 ranks, the higher of the two is Early Ed Spec, Sr (senior). The career of an economist has 3 ranks, Economist, Senior, and Principal, each paying a higher salary than the previous rank. The dummy variable c_dum receives the value of 1 when a position is part of a career with multiple levels/ranks and zero if there is only a single earnings category. Std_earn is the standard deviation of total earnings within each career group. For example the value 6.24 is the standard deviation of earnings across the 3 levels of the career path of an Economist, (Economist, Senior, and Principal).

Our interest is to investigate whether there are relatively more women than men in those careers that the volatility of earnings (Std_earn) is lower. According to Table 4 for example, The volatility in the "Economist" career option (6.24) is larger compared to "Early Education Specialist" (1.65). Our interest is investigating if there are relatively more women in "Early Education Specialist" career path as our theory predicts that the less confident will search more actively for jobs whose wage depends less on the rank. We expect to find that there are relatively less women in career paths whose standard deviation of earnings across ranks is higher. In the first exercise we run regressions on the full sample and in the second we average each career group into a single observation, which reduces the sample significantly. For instance, for the 3 ranks of the economist career path are merged to 1 by averaging across ranks. All variables used in the estimation are in logarithms.

5.1.2 Estimation

In the first exercise we use the full sample while excluding all the single rank careers as there can be an overlap between those and some career groups. The estimated model is:

$$Women/Men_i = \alpha + \beta_1 Std\_ranks_i + \beta_2^{'} \mathbf{X}_i + \varepsilon_i \tag{49}$$

where the depended variable is the ratio of women to men, Std_ranks is the volatility of earnings across the various ranks of each career group and $\mathbf{X}_i$ are different controls. Table 5 presents the estimated coefficients and t-statistics in parenthesis from the model in (49). The coefficient of the volatility of earnings along each career path is negative and statistically significant when controls are added. For example, in model 3 of Table 5, if the earning's standard deviation increases by 1%, the ratio of women to men decreases by nearly half a percentage point. This implies that in careers where earnings depend more heavily on the rank, there are relatively fewer women than men.

We control for the relative number of women to men in the lowest ranks of each group to account for possible wage discrimination as this is an indication of whether the profession is male or female dominated (model 2). As discrimination might prevent women to reach the top ranks, the relative number of women in the lowest ranks is a good proxy to account for preferences of males and females on certain professions. We also control for total employment in each career group (model 3), as broader career groups should be expected to have more variation in ranks and earnings. We also control for the relative earnings of women to men (model 4) to account for possible wage discrimination. Furthermore, controlling for job position longevity (model 5) seems to add little to the estimation. Table 6 presents the estimated coefficients and t-statistics from estimating the same model (49) by averaging each career group into a single observation as explained above. The results are unchanged and the coefficient estimates are very close to the previous exercise.

5.2 Share of Labor Income

In the impulse responses derived from the model, an interesting result is that the labor share of income is countercyclical for the low-rank positions and procyclical for the high-rank positions of career firms. This can provide another testable prediction to validate the model, as this results seems to be, at first, in contradiction with empirical findings. For example, evidence by Saez & Zucman (2016) report that wealth inequality increases during recessions, which seems opposite, at first, with our findings. If we assume that workers in the top rank of career firms belong to the top 10%, while all other workers with lower-rank jobs belong to the bottom 90% of the wage distribution, then evidence suggest that the wealth of the low-ranked workers should be procyclical. But an important difference, however, is that our model predicts a countercyclical share of labor income (not wealth) of workers in low-rank positions. To be more specific, if LI_t is the labor income in a period which in our model is

$$LI_t = N_t^C w_t^C L_t^C + N_t^H w_t^H L_t^H + N_t^N w_t^N L_t^N$$

then the labor share of income for the bottom 90% is either

$$S_t^{0.90} = \left\{ \begin{array}{cc} N_t^C w_t^C L_t^C / LI_t & \textit{if } N \textit{ in top 10\%} \\ \left(N_t^C w_t^C L_t^C + N_t^N w_t^N L_t^N\right) / LI_t & \textit{if } N \textit{ not in top 10\%} \end{array} \right\}$$

Both measures are countercyclical in the model.

The Saez & Zucman (2016) database allows us to test the cyclicality of the labor share as it reports the share of labor income to total income for all major percentiles (sums to 100%). The data are annual from $1962 - 2012$. Given that it is hard to distinguish empirically the low-rank from the high-rank career jobs without resting on earning differences, we can use the information on labor share by income percentile in this data set to test the model's predictions. We consider that all workers in top-career jobs in our model belong to the top

10% of the wage distribution as workers in this income category are unlikely to be found in low positions trying to reach the top. We are interested in examining whether the share of labor income for the bottom 90% is countercyclical, in line with our model's predictions. Note that this prediction stems mainly from the presence of career firms in the model. As explained above, their strategy for increasing workers' effort after a positive shock is to increase the option value of a promotion by increasing the spread in wages between low and top positions, while reducing new hires (at the low ranks) and increasing opportunities for promotions to the top ranks. Non-career firms, on the other hand, follow the standard strategy of increasing wages and hirings to take advantage of the increased productivity, implying a procyclical labor share of income.

Table 7 shows the results from simple regressions of the labor share for each percentile on a linear trend and either unemployment or detrended GDP.[13] For the bottom 90%, the labor share is indeed countercyclical while for the top 10% and above (top 5%, top 1%, top 0.1% and top 0.01%) it is strongly procyclical as predicted by our model for all income percentiles including and below 10%. This matches the findings in Figure 6 where the responses after a productivity shock (holds for any expansionary shock) of the share of the labor income is actually countercyclical as either the solid-red or the dashed-black lines are both below the steady state when income is above the steady state. The solid-red line corresponds to the low-rank career jobs and the dashed-black line if both low-rank career jobs and non-career jobs correspond to the bottom 90% of the income distribution.

5.3 Labor Wedge

A key prediction of our model is that creating a career ranking implies a labor supply where the wage deviates from the MRS (marginal rate of substitution) and a labor demand where the wage deviates from the MPL (marginal product of labor). Thus our model generates a labor wedge ($MPL_t - MRS_t$) and offers predictions regarding its behavior over the business cycle, which we can test empirically.

The literature on the labor wedge documents that it varies substantially along the business cycle and it is countercyclical to output.[14] To explain the business cycle dynamics of the labor wedge, there are 2 commonly used approaches. The first is to modify the labor demand

[13] The unemployment and GDP data come form the s.t Louis FRED database

[14] The measure of the wedge is commonly constructed using standard aggregate production function and utility function.

side by focusing on the "firm problem."[15] The second focuses on the "household problem" in order to modify the labor supply.[16] Gali et al. (2007), decompose the labor wedge LW_t into these two components, which implies:

$$LW_t = \log\left(\frac{MPL_t}{MRS_t}\right) = [\log(MPL_t) - \log(w_t)] + [\log(w_t) - \log(MRS_t)] \tag{50}$$

They document that the "price markup" which is the deviation of the marginal product from the marginal cost (first bracket) explains only 2% of the cyclical variation of the labor wedge for the US economy. Nonetheless, the "wage markup" (second bracket) explains around 80% of the labor-wedge deviation along the business cycle and this is not an empirical fact for the US economy only, but also characterizes the behavior of the labor wedge for most OECD countries.

Figure 9 presents the predictions of our estimated model for the labor wedge. The left (right) panel is the response of the labor wedge after a positive productivity shock for the low (top) positions, decomposed according to eq. (50) into price markup and wage markup. The green dashed line is the labor wedge while the blue and red solid lines are the price and wage markups respectively. The responses of the markups for the non-career jobs are not reported as they are zero.[17] The aggregate response of the markup is countercyclical according to the black dash-and-dot line in either plot which is supported by the evidence. What is interesting also is that the price markup for the low career jobs is procyclical while for the top is countercyclical, which implies that the overall price markup remains low over the business cycle and stable. Depending on the relative number of low and top positions the price markup can become even smaller and less volatile.[18] On the other end, the wage markup of low positions is significantly larger than their price markup, while the wage markup of the top positions is zero. Given that the majority of positions are low and only the very few are top, in our model the wage markup overall is significantly larger than the price markup.

[15] Rotemberg & Woodford (1999) focus on price markups, Jermann & Quadrini (2012) and Arellano et al. (2019) use financial friction to modify the firm problem to create realistic labor wedges and Bigio et al. (2016) use pleadgeability constraints.

[16] Cole & Ohanian (2004) modify the bargaining problem to affect labor supply. Hall (2009) modifies the MRS by introducing complimentarity between consumption and labor. Karabarbounis (2014a) introduces firm production for the wage to deviate from the MRS. Chang & Kim (2007) demonstrate that agent heterogeneity can replicate the observable labor wedge.

[17] As explained above, the non-career jobs correspond to the standard neo-classical framework where the wage is such that $MPL = MRS$.

[18] Our estimation was not targeting the labor wedge.

It follows that most of the volatility of the wedge in our model is attributed to the wage markup, consistent with the evidence, since the price markups move in opposite directions for low and top positions.

Moreover, the wage markup in our model, which drives the volatility of the labor wedge, is countercyclcal in line with the evidence that the labor wedge is countercyclical. To understand why notice that according to eq. (8) the gap between the wage and the MRS is:

$$wm_t^C = w_t^C - \frac{G_L\left(L_t^C\right)}{\mu_t} = -\frac{\partial \rho_t^H}{\partial L_t^C} E_t Q_{t,t+1}\left(W_{t+1}^H - W_{t+1}^C\right) \tag{51}$$

In Figure 9 the red line on the left graph is the log-linearized version of eq. (51) which is the response of the wage markup. As the marginal probability to get promoted $\frac{\partial \rho_t^H}{\partial L_t^C}$ increases in an expansion and also the gap between the value of the top over the low position $W_{t+1}^H - W_{t+1}^C$ (see Figure 6), the wage markup decreases during an expansion. Along the business cycle the labor effort does not simply adjust to reflect changes in the wage compensation. It also adjusts to changes in the prospects of a better offer or position. As discussed above, firms respond to an increase in productivity by increasing opportunities for promotions and the wages at higher ranks thereby also eliciting more effort from the mass of their workers who hold low positions, while keeping the wage in these positions low.

To derive the price markup for the low position simply take the difference between the first order conditions for the career and non-career firms, eq. (24) and (26), as their common terms is the MPL minus the markup.[19] Then plug in eq. (28) to eliminate $\frac{dL_{t-1}^C}{dw_t^C}$. The price markup for the low position is:

$$pm_t^C = \frac{\left(P_{t-1}^x \frac{dX_{t-1}^{C,H}}{dL_{t-1}^C} - N_{t-1}^C w_{t-1}^C\right) \frac{\mu_{t-1}}{\omega} \frac{\partial \rho_{t-1}^H}{\partial L_{t-1}^C}}{\ell\left(L_{t-1}^C\right)^{\ell-1} - \frac{\mu_{t-1}}{\omega} \partial \frac{\partial^2 \rho_{t-1}^H}{\partial L_{t-1}^C \partial L_{t-1}^C} E_{t-1} Q_{t-1,t}\left(W_t^H - W_t^C\right)} E_{t-1} Q_{t-1,t} \partial \frac{W_t^H - W_t^C}{\partial w_t^C} \tag{52}$$

while following the same formula, the price markup for the top position is:

$$pm_t^H = \frac{\left(P_{t-1}^x \frac{dX_{t-1}^{C,H}}{dL_{t-1}^C} - N_{t-1}^C w_{t-1}^C\right) \frac{\mu_{t-1}}{\omega} \frac{\partial \rho_{t-1}^H}{\partial L_{t-1}^C}}{\ell\left(L_{t-1}^C\right)^{\ell-1} - \frac{\mu_{t-1}}{\omega} \partial \frac{\partial^2 \rho_{t-1}^H}{\partial L_{t-1}^C \partial L_{t-1}^C} E_{t-1} Q_{t-1,t}\left(W_t^H - W_t^C\right)} E_{t-1} Q_{t-1,t} \partial \frac{W_t^H - W_t^C}{\partial w_t^H} \tag{53}$$

[19] Keep in mind that the firm is not a price taker in the labor market and also sets the wage instead of the labor effort.

The signs of the price markups in (52) and (53) depend on the terms $\partial \frac{W_t^H - W_t^C}{\partial w_t^C}$ and $\partial \frac{W_t^H - W_t^C}{\partial w_t^H}$. However, those terms are going to have an opposite sign as $W_t^H - W_t^C$ is increasing in w_t^H and decreasing in w_t^C. This implies that the price markups for low and top positions are always going to move in the opposite direction over the business cycle, thus the overall price markup is small as documented by the empirical evidence.

6 Conclusion

We have developed a theoretical model in which some firms offer career jobs. In career jobs workers start at the low rank and can be promoted to a higher rank. We demonstrated that even if all workers are identical in all respects, it is optimal for those firms that can commit to a wage rule to offer career jobs by creating a hierarchy in the payment structure. They set the per unit of effort wage much higher for the few workers that get promoted to the top rank, while keeping it low for the mass of the workers who remain in the lower rank. This payment structure is optimal as it induces higher effort at lower pay from the mass of the workers who opt for a promotion, while rewarding with a pay increase only the few that get promoted. The key mechanism we highlight is that some firms can compensate workers' effort through the option value of a promotion, a feature missing from canonical models of the labor market.

While there is plenty of evidence that wages are closely related to job levels, the macroeconomic implications of this hierarchical wage structure that links wages to job levels, instead of effort or productivity and outside options, has been largely overlooked in the literature. We demonstrated that this framework can potentially explain various puzzles in economics. A key prediction of the model is that wages in career firms deviate both from the marginal product of labor (standard labor demand) and the Marginal Rate of Substitution (standard labor supply). Hence, the model predicts a labor wedge. Moreover career firms respond differently to changes in the economic environment by adjusting not only hirings but also opportunities for promotion and the wage spread between low- and high-rank positions. Our model therefore deviates from the standard predictions of a neoclassical model where firms respond to an increase in profitability by increasing wages and new hires. Career firms respond differently by increasing wages at the top and opportunities for promotions, while reducing new hires and wages at the lower ranks.

We have demonstrated that accounting for the presence of career jobs helps generate realistic business cycle variation in the labor wedge. The model can also explain why the labor share of income is countercylical at the bottom 90% and procyclical at the top 10% of the wage distribution, as evident using data from Saez & Zucman (2016). Finally, the model can explain part of the wage gap between men and women. Experimental evidence point to men having a stronger preference for tournaments and competitions as they tend to be more overconfident about their own skills relative to others. In our model the more confident individuals will search more intensively for career jobs and consequently allocate more frequently into career firms that offer the high-paying top positions. We run an empirical exercise that confirms that there are fewer women in professions that exhibit greater variation in their payment across ranks.

Our analysis highlights the individual wage growth that occurs through incentive-based promotion schemes. While there is evidence that the earnings changes of workers staying with the same firm are of primary importance in explaining average wage dynamics (see e.g.Joyce et al. (2021)), we cannot rule out the importance of employer-to employer transitions in explaining individual wage dynamics (see e.g.Moscarini & Postel-Vinay (2016a), Moscarini & Postel-Vinay (2017)). Our model can be extended in interesting ways to account for also employer-to-employer transitions and flesh out much more comprehensively the implications for wage dynamics. To the extent that information about a worker's quality can be also inferred by outside firms, promotions could also occur through transitions to other firms. Another possibility is to allow for on-the-job search and an arrival rate of outside employment offers, as is standard in job ladder models. While this creates avenues for interesting results on wage dynamics, it might also strengthen the mechanism emphasized here, since the risk of loosing workers to outside competition provides stronger incentive for firms to offer jobs with higher option values by backloading their employees compensations through the possibility of more generous promotions. While this reduces the probability of quits, it also provides incentives for higher effort at lower pay, making loosing a worker to outside competition less painful.

Table 1: The parametrization of the long-run model.

Var.	Value	Description	Var.	Value	Description
ℓ	2	Elasticity of labor supply	γ_L	2	Elasticity of ρ^H to labor effort
N^H	0.25	Number of top positions	γ_N	1	Elasticity of ρ^H to other candidates
N^C	0.25	Number of career jobs	ω	3	Disutility of labor parameter
N^N	0.5	Number of non-career firms	σ	0.1	Risk aversion coefficient
β	0.98	Discounting parameter	θ	10	Elasticity of substitution
A^e	1	Expected productivity gap	m	0.4	Matching efficiency
α	0.6	Elasticity of production fun.	$\bar{u}$	0.1	Steady state unemployment
λ	0.05	Destruction rate	γ	0.5	Elasticity of matching to vacancies

Table 2: Initial prior and posterior estimated distribution moments

Par.	*Prior*		*Posterior*		*Par.*	*Prior*		*Posterior*	
	μ	σ	μ	σ		μ	σ	μ	σ
ℓ	2	0.5	1.9926	0.00066	a_c	1.687	0.5	1.3632	0.00056
α	0.6	0.2	0.6852	0.00010	av_c	1	0.5	1.2417	0.00051
γ_L	1.5	1.5	0.98	0.00015	av_n	1	0.5	0.7963	0.00163
γ_N	0.9	0.2	0.8372	0.00015	av_{ch}	0.2	0.7	0.2038	0.00046
χ	0.8	0.1	0.7459	0.00036	a_{zf}	0.6	0.2	0.4654	0.00036
σ	0.1	0.15	0.1396	0.00010	a_A	0.6	0.2	0.6398	0.00015
$\bar{k}^C$	1.286	0.5	1.0611	0.00082	a_{Ae}	0.6	0.2	0.6305	0.00015
$\bar{k}^{CH}$	0.704	0.25	0.5882	0.00010	a_G	0.6	0.2	0.6659	0.00010
ω	3	2	1.188	0.00561	ξ	0	0.5	0.0864	0.00020
λ_C	0.05	0.02	0.0488	0.00002	θ_c	11	5	11.4332	0.01816
λ_H	0.05	0.02	0.0426	0.00005	$\bar{z}^f$	1	0.4	0.839	0.00046
λ_N	0.05	0.02	0.0462	0.00005	$\bar{A}^e$	1.5	0.6	1.5073	0.00179
m	0.4	0.15	0.3619	0.00010	$\bar{A}$	1	0.4	0.6473	0.00061

Notes: The estimates rely on a Metropolis-Hastings algorithm (MH) which uses a Markov Chain Monte Carlo (MCMC) method to create random samples for the parameters to approximate their distribution. All prior distributions are normal. The results are very similar when a Log-Likelihood method is utilized instead which may suggest that the prior assumptions are not particularly restrictive.

Table 3: The description of the estimated parameters

Par.	Description	Par.	Description
ℓ	Frisch Elasticity of Substitution	a_c	Utility scale parameter
α	Returns to scale coefficient	av_c	Adjustment cost of vacancies C
γ_L	Effect of competition on ρ_t	av_n	Adjustment cost of vacancies N
γ_N	Effect of effort on ρ_t	av_{ch}	Adjustment cost promotions
χ	Habit persistence	a_{zf}	Persistence of final good prod.
σ	Risk aversion coefficient	a_A	Persistence int. good prod.
$\bar{k}^C$	Vacancy cost C	a_{Ae}	Persistence perceived skill gap
$\bar{k}^{CH}$	Cost of promoting	a_G	Government purchases persistence
ω	Disutility of labor scale	ξ	Love for variety
λ_C	Job destruction rate C	θ_c	Elasticity of Substitution
λ_H	Job destruction rate N	$\bar{z}^f$	Productivity
λ_N	Job destruction rate H	$\bar{A}^e$	Perceived skill gap
m	Matching efficiency	$\bar{A}$	Productivity intermediate goods

Notes: The descriptions of the parameters estimated in Table 2.

Table 4: Data Example

Jobtitle	***F_Rate***	***No. Fem***	***M_Rate***	***No. Male***	***Tot_Rate***	***c_dum***	***c_label***	***Std_earn***
Disposal CC I	33.93	2	33.93	2	33.93	0	0	0
Dispute Res. Med.	38.48	1	-	0	38.48	0	0	0
Early Ed Spec	33.89	6	34.32	1	33.95	1	42	1.65
Early Ed Spec,Sr	36.28	2	-	0	36.28	1	42	1.65
Economist	34.32	1	35.66	2	35.21	1	43	6.24
Economist,Prin	47.26	1	47.26	3	47.26	1	43	6.24
Economist,Sr	44.28	7	43.14	2	44.03	1	43	6.24

Notes: A few rows and columns from the dataset to better visualize the structure of the data.

Table 5: Estimation on the full sample, variation within career groups possible

Depended Variable:	***Number of women to men***				
Models	**1**	**2**	**3**	**4**	**5**
Constant	1.204*** (6.125)	1.629*** (10.145)	1.226*** (7.756)	1.196*** (4.210)	0.812 (0.474)
Earnings_Std_ranks	-0.134 (-1.134)	-0.321*** (-3.339)	-0.400*** (-4.406)	-0.618*** (-4.549)	-0.617*** (-4.527)
No_WtoM_low_ranks		0.766*** (17.075)	0.752*** (17.836)	0.860*** (13.650)	0.861*** (13.629)
Total_Employment			0.341*** (8.551)	0.434*** (6.407)	0.435*** (6.400)
W_m_Job_longevity				0.010 (0.636)	0.010 (0.617)
Women_rate_to_men					0.382 (0.228)
Total Observations	829	829	829	829	829
Restrictions on Sample	*No single rank*	*No single rank*	*No single rank*	*No single rank*	*No single rank*
Included Observations	546	546	546	292	292
Log Likelihood	-988.539	-871.206	-836.655	-451.484	-451.458

Notes: Earnings_Std_ranks is the standard deviation of the average earnings between men and women along a specific job career path, No_WtoM_low_ranks is the ratio of the number of women to men at the lowest rank of each specific career group, Total_Employment is the total number of employees, W_m_Job_longevity is the ratio of female to male job longevity in months at the specific position Women_rate_to_men is the ratio of women earnings to men's and No_WtoM_high_ranks is the ratio of the number of women to men at the highest rank of each specific career group. The t-statistics are reported in parentheses below the coefficient. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table 6: Estimation on a restricted sample, career groups are averaged to a single observation

Depended Variable:	***Number of women to men***			
Models	**1**	**2**	**3**	**4**
Constant	1.714***	2.457***	2.04***	3.529
	(4.573)	(8.395)	(4.753)	(0.828)
Earnings_Std_ranks	-0.164	-0.463**	-0.539**	-0.950***
	(-0.603)	(-2.224)	(-2.500)	(-2.793)
No_WtoM_low_ranks		1.413***	1.399***	1.897***
		(11.271)	(11.144)	(9.168)
Total_Employment			0.181	0.263
			(1.325)	(1.098)
No_WtoM_high_ranks				-1.167
				(-0.278)
Total Observations	829	829	829	829
Restrictions on Sample	*No single rank Sectors merged*	*No single rank Sectors merged*	*No single rank Sectors merged*	*No single rank Sectors merged*
Included Observations	175	175	175	98
Log Likelihood	-412.210	-363.817	-362.923	-213.323

Notes: Earnings_Std_ranks is the standard deviation of the average earnings between men and women along a specific job career path, No_WtoM_low_ranks is the ratio of the number of women to men at the lowest rank of each specific career group, Total_Employment is the total number of employees, W_m_Job_longevity is the ratio of female to male job longevity in months at the specific position Women_rate_to_men is the ratio of women earnings to men's and No_WtoM_high_ranks is the ratio of the number of women to men at the highest rank of each specific career group. The t-statistics are reported in parentheses below the coefficient. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table 7: The cyclicality of different percentiles of labor income

Labor Income Share	*constant*	*trend*	*un_rate*	*GDP*
Bottom 90%	0.829*** (0.0053)	-0.002*** (0.0001)	0.367*** (0.0853)	
Bottom 90%	0.8497*** (0.0030)	-0.0027*** (0.0001)		-0.0791** (0.0395)
Top 10%	0.1702*** (0.0053)	0.0028*** (0.0001)	-0.3667*** (0.0853)	
Top 10%	0.1503*** (0.0030)	0.0027*** (0.0001)		0.0791** (0.0395)
Top 5%	0.1062*** (0.0041)	0.0022*** (0.0001)	-0.3261*** (0.0660)	
Top 5%	0.0885*** (0.0023)	0.0021*** (0.0001)		0.0866*** (0.0308)
Top 1%	0.0383*** (0.0028)	0.0014*** (0.0000)	-0.2060*** (0.0440)	
Top 1%	0.0271*** (0.0015)	0.0013*** (0.0001)		0.0572*** (0.0201)
Top 0.1%	0.0070*** (0.0015)	0.0007*** (0.0000)	-0.1003*** (0.0235)	
Top 0.1%	0.0016*** (0.0008)	0.0006*** (0.0000)		0.0278*** (0.0106)
Top 0.01%	0.0070*** (0.0015)	0.0007*** (0.0000)	-0.1003*** (0.0235)	
Top 0.01%	-0.0003*** (0.0003)	0.0002*** (0.0000)		0.0110** (0.0044)

Notes: The data for the labor income percentiles come from Saez & Zucman (2016). The uemployment rate and GDP data come from s.t Louis FRED database. The left column corresponds to the depended variables and the rows the regressors.The standard deviations are reported in parentheses below the coefficient. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

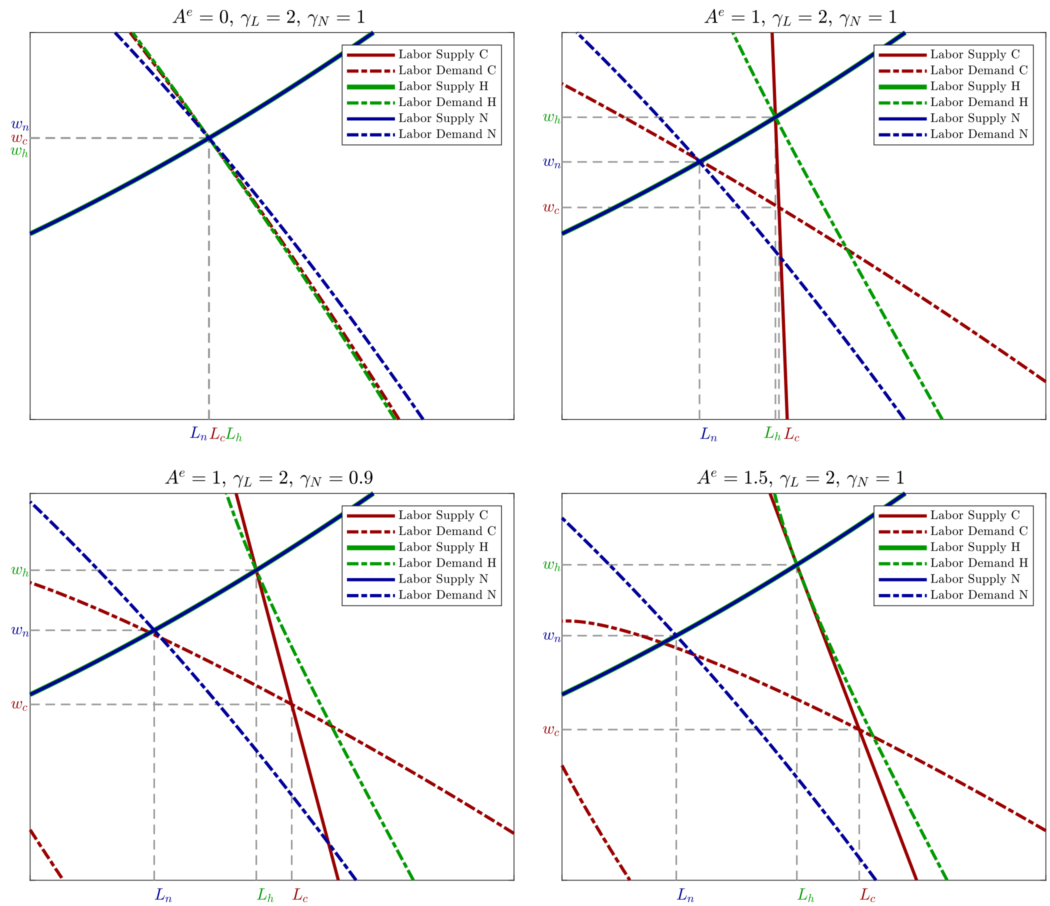

Figure 1: The equilibrium in the market for the wage and effort for low-level career jobs (red lines), top-level career jobs (Green lines) and non-career jobs (Blue lines). The labor supply curves are depicted with dashed lines and are the same color as the associated demand curve.

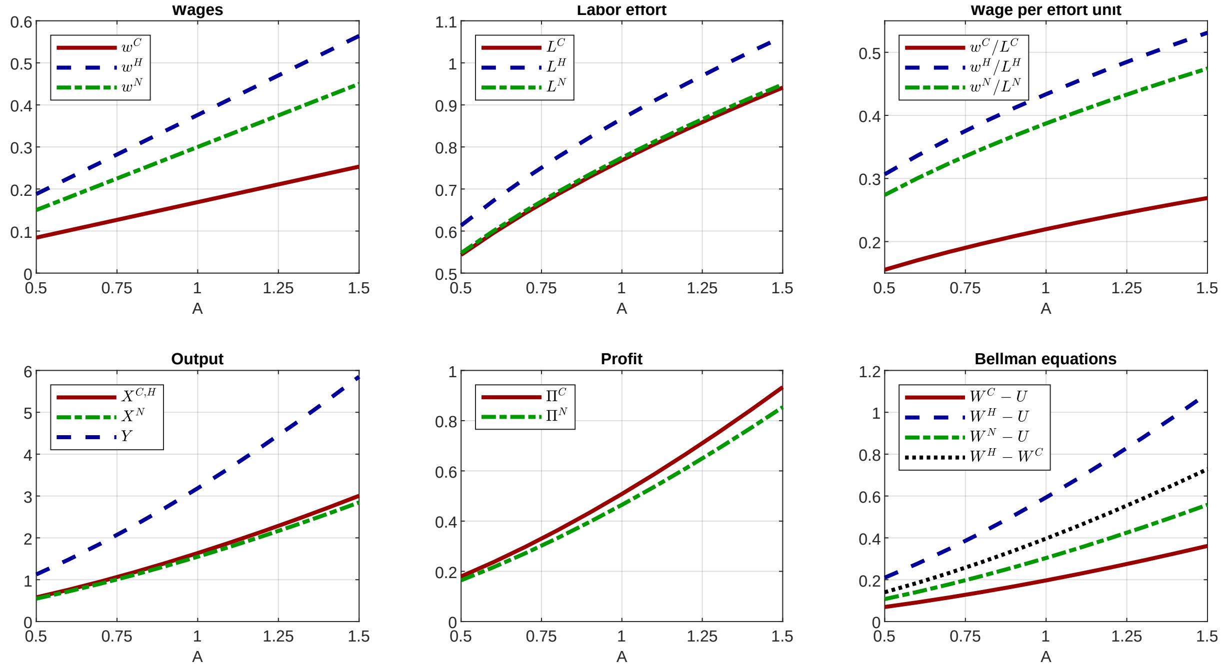

Figure 2: The change in the long-run equilibrium (steady state) for different values of the productivity parameter A.

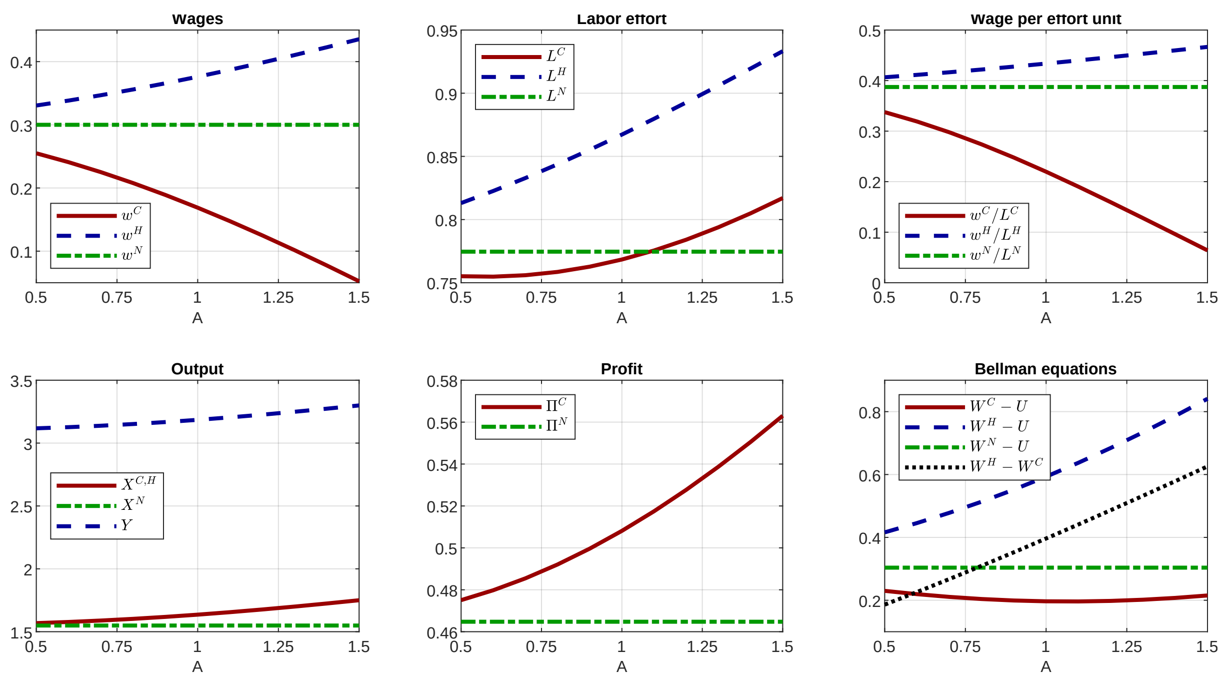

Figure 3: The change in the long-run equilibrium (steady state) for different values of the relative productivity parameter A^e.

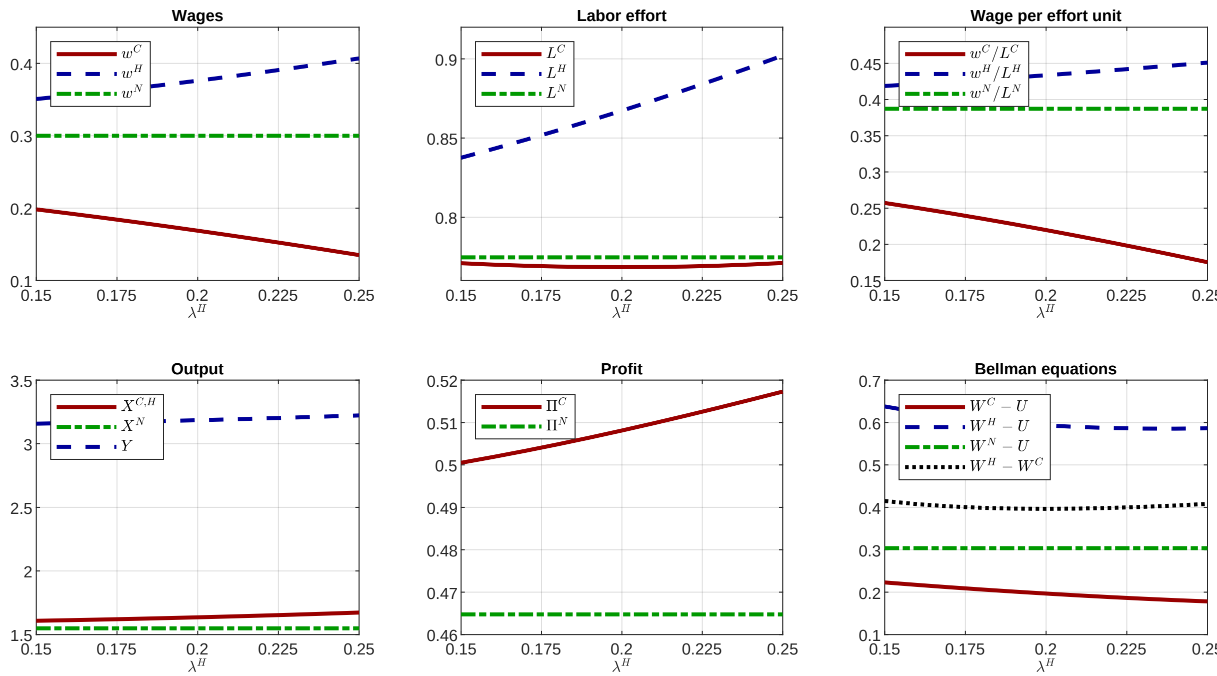

Figure 4: The change in the long-run equilibrium (steady state) for different values of the job destruction rate for top positions λ^H.

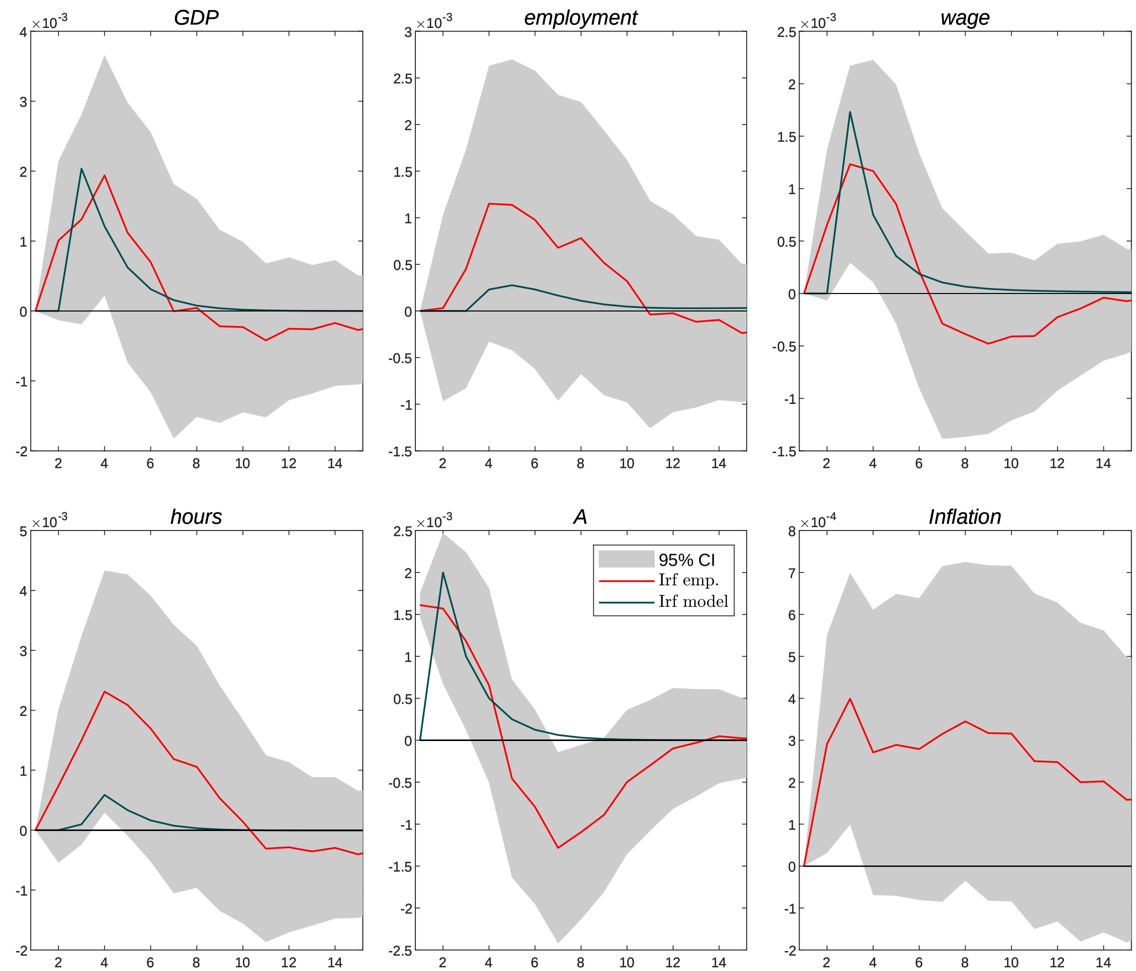

Figure 5: The impulse responses after a 1 standard deviation shock from both the model and from quarterly US data 1947:Q1 to 2020:Q4. The red solid lines correspond to the empirical impulse responses and the shaded areas are 90% confidence intervals. The dark green lines are the model implied impulse response using the parameters obtained from the Bayesian estimation.

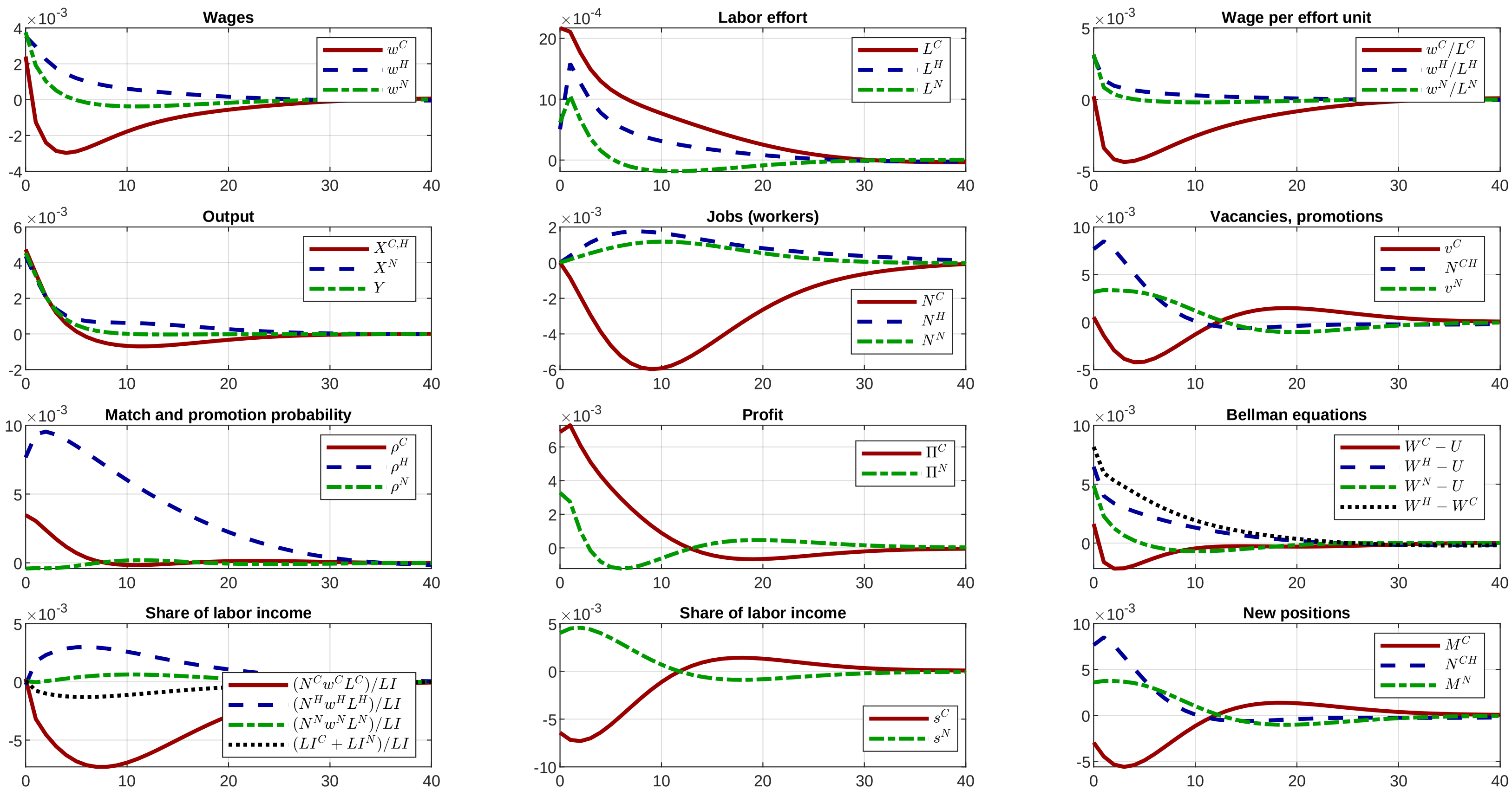

Figure 6: Impulse responses after a unit shock that raises the productivity level of the economy A_t.

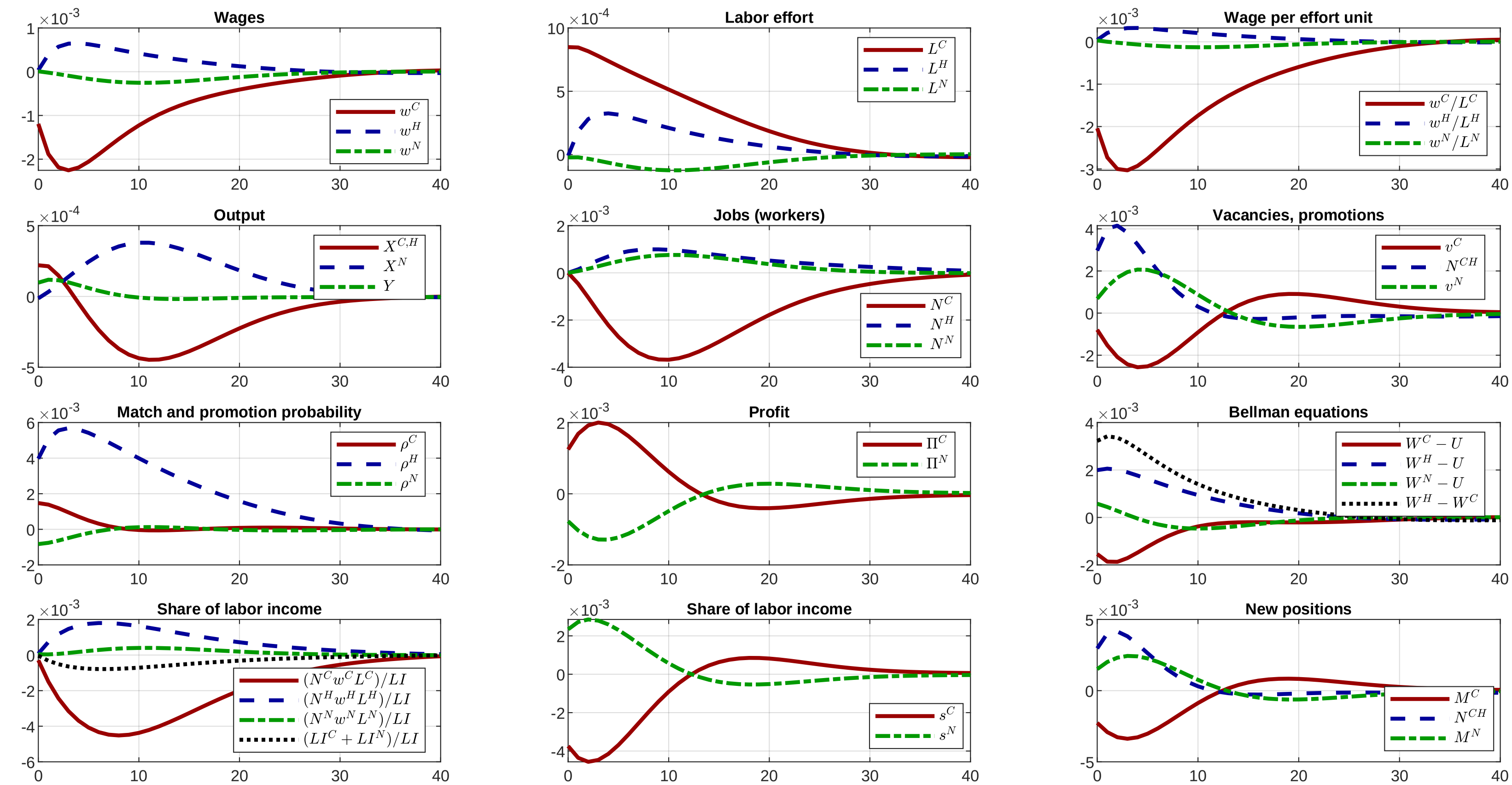

Figure 7: Impulse responses after a unit shock that raises the perceived productivity of workers A^e above the "rational" level which is A.

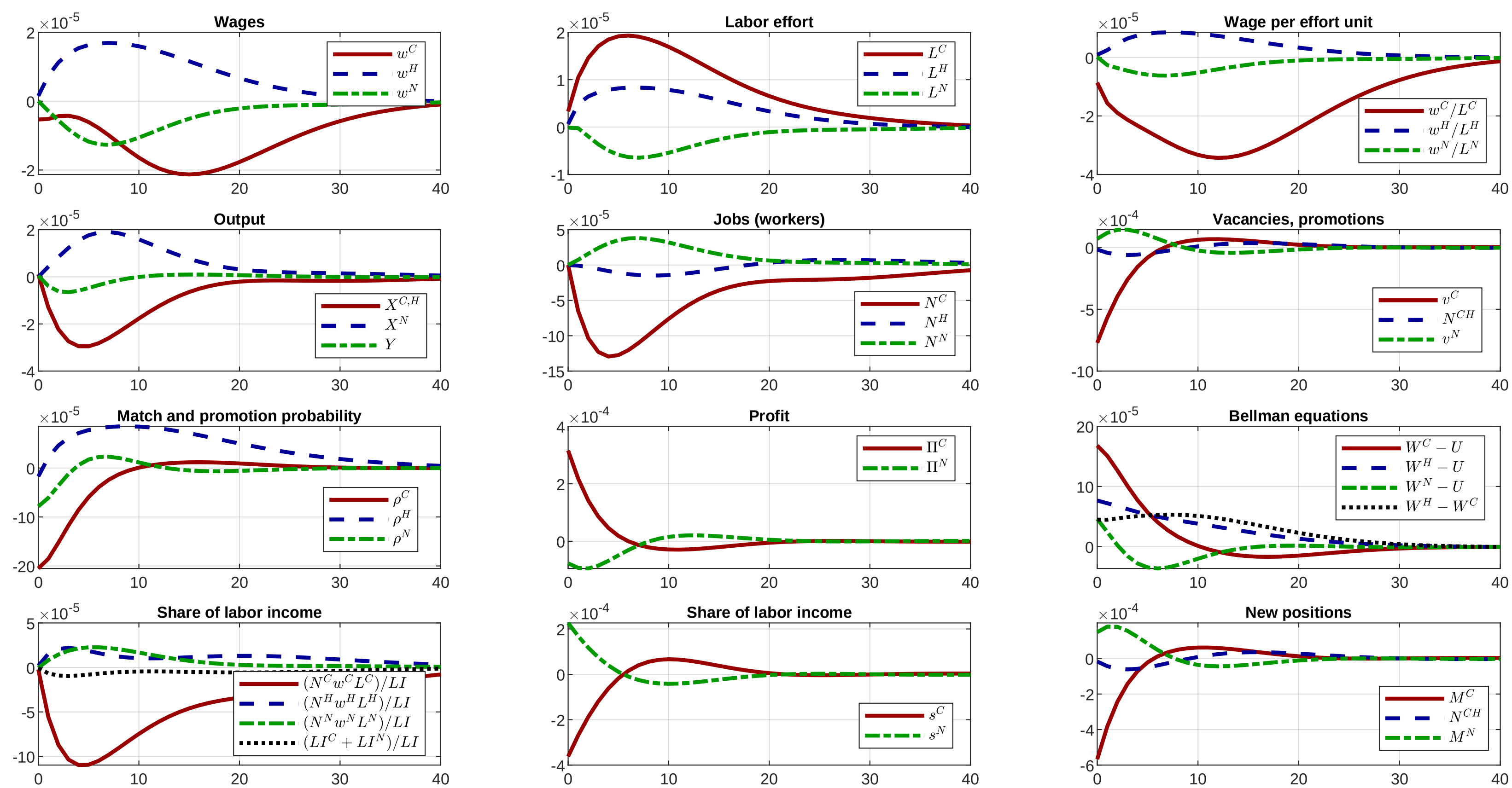

Figure 8: Impulse responses after a shock that raises the cost to open career jobs κ_t^C.

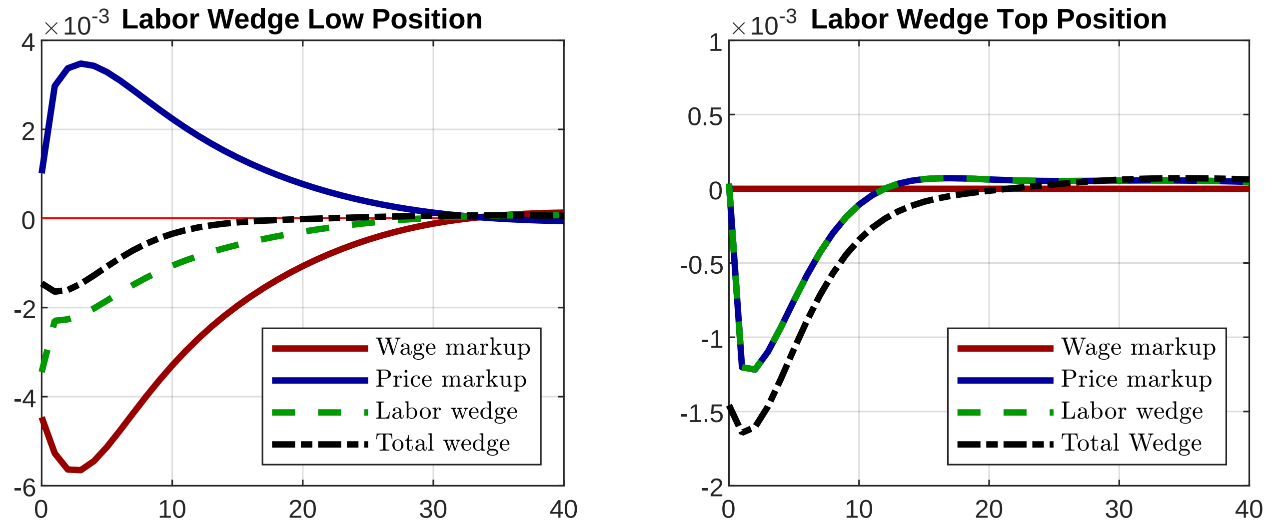

Figure 9: Labor wedge and its decomposition into price and wage markups for both low and top positions. The total labor wedge in the ecomomy is displayed on both plots.

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A Data and Estimation

Variable	Description
Wage	Average Weekly Hours of Production and Nonsupervisory Employees
GDP	Real gross domestic product
Inflation	Implicit Price Deflator
Hours	Non farm Business Sector: Hours of All Persons
A	Non farm Business Sector: Real Output Per Hour of All Persons
Employment	Total Nonfarm, commonly known as Total Nonfarm Payroll

The estimation of the model parameters is based on Bayesian estimation. Specifically, we use a Monte-Carlo based optimization routine for the mode computation, the number of replications for Metropolis-Hastings algorithm is set to 2000 , the number of parallel chains for Metropolis-Hastings algorithm is set to the default value, 2, and half the starting parameter vectors are discarded before the posterior is estimated. The prior distributions are normal for all parameters except for the shocks and parameters that are close to zero where an inverse Gamma distribution is employed to ensure non-negativity. We need to specify at least as many shocks as the matched empirical variables to ensure identification and thus there should be at least 5 shocks in the model. We assume AR(1) processes for the following shocks: Labor productivity A_t, perceived productivity gap A_t^e, vacancy cost k_t^C, cost of promoting k_t^{CH} and final good productivity z_t^f. The inclusion of 5 shocks implies exact identification as there are 5 time series variables to be matched.

The data in the table above are also used for the empirical VAR in Table 5. We identify the VAR using Cholesky decomposition even though there are better ways to identify productivity shocks. The reason is because the empirical VAR is simply for visualization purposes and does not affect the parameter estimation in any other way. Better results can be achieved using the TFP series by Fernald (2014) or identify the VAR using long-run restrictions as in Gali (1999) or Beaudry & Portier (2006).

B Proposition 1: Career vs Non-Career Firms

The labor supply for the top jobs is summarized in eq. (10) which is repeated below

$$w_t^H = \frac{G_L\left(L_t^H\right)}{\mu_t} \tag{54}$$

and it states that the wage is equal to the marginal disutility of labor in terms of the real good. The respective labor supply of low career jobs is eq. (8) and is also repeated below

$$w_t^C = \frac{G_L\left(L_t^C\right)}{\mu_t} - \frac{\partial \rho_t^H}{\partial L_t^C} E_t Q_{t,t+1}\left(W_{t+1}^H - W_{t+1}^C\right) \tag{55}$$

The wage from the labor supply in (55) deviates from the marginal disutility $\frac{G_L\left(L_t^C\right)}{\mu_t}$ due to the second term that depends on the magnitude of both the marginal probability to get promoted $\frac{\partial \rho_t^H}{\partial L_t^C}$ and the future gap between the payoffs of the low and top positions $E_t Q_{t,t+1}\left(W_{t+1}^H - W_{t+1}^C\right)$. The evolution of the latter is summarized in the Bellman equation (80) repeated below (when $\lambda^C = \lambda^H$)

$$\begin{aligned} W_t^H - W_t^C \quad = \quad & w_t^H L_t^H - w_t^C L_t^C - \frac{G\left(L_t^H\right)}{\mu_t} + \frac{G\left(L_t^C\right)}{\mu_t} \\ & + \left(1 - \lambda^H - \left(1 - \lambda^C\right)\rho_t^H\right) E_t Q_{t,t+1}\left(W_{t+1}^H - W_{t+1}^C\right) \end{aligned} \tag{56}$$

If the firm cannot commit then the labor demand for the low career jobs is as in eq. (24) which shares the same functional form as the labor demand for top career jobs in eq. (29).

Without loss of generality suppose that $w_t^H > w_t^C$. From the labor demands (24) and (29), if $w_t^H > w_t^C$ it means that the marginal product of top jobs is higher and due to diminishing returns $L_t^C > L_t^H$. Now Plug the labor supply equations (54) and (55) in (56) and use the functional form $G\left(L_t^i\right) = \omega \frac{\left(L_t^i\right)^{1+\ell}}{1+\ell}$ for $i = \{C, H\}$. That is

$$\begin{aligned} W_t^H - W_t^C \quad = \quad & \frac{\omega}{\mu_t}\left(L_t^H\right)^{\ell} L_t^H - \frac{\omega}{\mu_t}\left(L_t^C\right)^{\ell} L_t^C + \frac{\partial \rho_t^H}{\partial L_t^C} E_t Q_{t,t+1}\left(W_{t+1}^H - W_{t+1}^C\right) \\ & - \frac{\omega}{\mu_t}\frac{\left(L_t^H\right)^{1+\ell}}{1+\ell} + \frac{\omega}{\mu_t}\frac{\left(L_t^C\right)^{1+\ell}}{1+\ell} \\ & + \left(1 - \lambda^H - \left(1 - \lambda^C\right)\rho_t^H\right) E_t Q_{t,t+1}\left(W_{t+1}^H - W_{t+1}^C\right) \end{aligned} \tag{57}$$

The above implies that

$$W_t^H - W_t^C = \frac{\omega}{\mu_t}\frac{\ell}{1+\ell}\left(L_t^H\right)^{1+\ell} - \frac{\omega}{\mu_t}\frac{\ell}{1+\ell}\left(L_t^C\right)^{1+\ell} + \frac{\partial \rho_t^H}{\partial L_t^C} E_t Q_{t,t+1}\left(W_{t+1}^H - W_{t+1}^C\right) + \left(1-\lambda^H - \left(1-\lambda^C\right)\rho_t^H\right) E_t Q_{t,t+1}\left(W_{t+1}^H - W_{t+1}^C\right) \quad (58)$$

In steady state the above is proportional

$$W_t^H - W_t^C = \frac{\frac{\omega}{\mu_t}\frac{\ell}{1+\ell}\left(L_t^H\right)^{1+\ell} - \frac{\omega}{\mu_t}\frac{\ell}{1+\ell}\left(L_t^C\right)^{1+\ell}}{1 - \frac{\partial \rho_t^H}{\partial L_t^C} - \beta\left(1-\lambda^H - \left(1-\lambda^C\right)\rho_t^H\right)} \quad (59)$$

Since $L_t^C > L_t^H$ and $1 - \frac{\partial \rho_t^H}{\partial L_t^C} - \beta\left(1-\lambda^H - \left(1-\lambda^C\right)\rho_t^H\right) > 0$, $W_t^H - W_t^C$ must be negative. If $W_t^H - W_t^C < 0$ (in steady state) and $L_t^C > L_t^H$, then $w_t^C > w_t^H$. This is because from 55 in steady state,

$$w_t^C = \frac{G_L\left(L_t^C\right)}{\mu_t} - \frac{\partial \rho_t^H}{\partial L_t^C}\beta\left(W_t^H - W_t^C\right) \overset{W_t^H - W_t^C < 0}{>} \frac{G_L\left(L_t^C\right)}{\mu_t} \overset{L_t^C > L_t^H}{>} \frac{G_L\left(L_t^H\right)}{\mu_t} = w_t^H$$

Which contradicts the initial assumption that $w_t^H > w_t^C$. The same holds when $w_t^H < w_t^C$ and thus $w_t^H = w_t^C$ in this case which implies $L_t^C = L_t^H$ and there is no distinction between high and low paying jobs if the firm cannot commit.

C Proposition 2: Job Creation Conditions

The firms maximize the following objective with respect to v_{t+i}^C and N_{t+i}^{CH}

$$J_t\left(N_t^C, N_t^H\right) = \max_{v_{t+i}^C, N_{t+i}^{CH}} E_t \sum_{i=0}^{\infty} Q_{t,t+i}\left\{\begin{array}{c} P_{t+i}^x X_{t+i}^{C,H} - N_{t+i}^C w_{t+i}^C L_{t+i}^C \\ -N_{t+i}^H w_{t+i}^H L_t^H - \kappa_{t+i}^C v_{t+i}^C - \kappa_{t+i}^{CH} N_{t+i}^{CH} \end{array}\right\} \quad (60)$$

subject to the laws of motion of low and high career jobs and the labor supply, eq. (34), (35) and (23) respectively. The problem is once again time inconsistent as maximizing for $i = 0$

is different than maximizing for the rest of the periods $i > 0$.

$$\begin{aligned} J_t\left(N_t^C, N_t^H\right) &= \max P_t^x X_t^{C,H} - N_t^C w_t^C L_t^C - N_t^H w_t^H L_t^H - \kappa_t^C v_t^C - \kappa_t^{CH} N_t^{CH} \\ &\quad + E_t Q_{t,t+1} \left\{ \begin{array}{c} P_{t+1}^x X_{t+1}^{C,H} - N_{t+1}^C w_{t+1}^C L_{t+1}^C - N_{t+1}^H w_{t+1}^H L_{t+1}^H \\ -\kappa_{t+1}^C v_{t+1}^C - \kappa_{t+1}^{CH} N_{t+1}^{CH} \end{array} \right\} \\ &\quad E_t Q_{t,t+2} J_{t+2}\left(N_{t+2}^C, N_{t+2}^H\right) \end{aligned} \tag{61}$$

Maximizing with respect to v_{t+1}^C implies

$$-\kappa_{t+1}^C + \left(P_t^x \frac{dX_t^{CH}}{dL_t^C} - N_t^C w_t^C\right) \frac{dL_t^C}{dv_{t+1}^C} + q_{t+1}^C E_{t+1} Q_{t+1,t+2} \frac{dJ_{t+2}\left(N_{t+2}^C, N_{t+2}^H\right)}{dN_{t+2}^C} = 0$$

The second term is the one responsible for the time inconsistency which makes the first order conditions from $i = 1$ and on different from $i = 0$. Under a time-less perspective, the foc becomes

$$\frac{\kappa_t^C}{q_t^C} - \frac{1}{q_t^C}\left(P_{t-1}^x \frac{dX_{t-1}^{CH}}{dL_{t-1}^C} - N_{t-1}^C w_{t-1}^C\right) \frac{dL_{t-1}^C}{dv_t^C} = E_t Q_{t,t+1} \frac{dJ_{t+1}\left(N_{t+1}^C, N_{t+1}^H\right)}{dN_{t+1}^C} \tag{62}$$

Take the envelope condition with respect to N_t^C. That is

$$\begin{aligned} \frac{dJ_t\left(N_t^C, N_t^H\right)}{dN_t^C} &= P_t^x \frac{dX_t^{CH}}{dN_t^C} - w_t^C L_t^C + \left(P_t^x \frac{dX_t^{CH}}{dL_t^C} - N_t^C w_t^C\right) \frac{dL_t^C}{dN_t^C} \\ &\quad + \left(1 - \lambda^C\right) E_t Q_{t,t+1} \frac{dJ_{t+1}\left(N_{t+1}^C, N_{t+1}^H\right)}{dN_{t+1}^C} \end{aligned} \tag{63}$$

Lead the above a period in advance and substitute inside eq. (62) after moving the latter a period in advance as well. This leads to the job creation condition for the low career jobs:

$$\begin{aligned} & \frac{\kappa_t^C}{q_t^C} - \frac{1}{q_t^C}\left(P_{t-1}^x \frac{dX_{t-1}^{CH}}{dL_{t-1}^C} - N_{t-1}^C w_{t-1}^C\right) \frac{dL_{t-1}^C}{dv_t^C} \\ &= E_t Q_{t,t+1} \left[\begin{array}{c} P_{t+1}^x \frac{dX_{t+1}^{CH}}{dN_{t+1}^C} - w_{t+1}^C L_{t+1}^C \\ + \left(P_{t+1}^x \frac{dX_{t+1}^{CH}}{dL_{t+1}^C} - N_{t+1}^C w_{t+1}^C\right) \frac{dL_{t+1}^C}{dN_{t+1}^C} \\ + \left(1 - \lambda^C\right) \left(\frac{\kappa_{t+1}^C}{q_{t+1}^C} - \frac{1}{q_{t+1}^C}\left(P_t^x \frac{dX_t^{CH}}{dL_t^C} - N_t^C w_t^C\right) \frac{dL_t^C}{dv_{t+1}^C}\right) \end{array} \right] \end{aligned} \tag{64}$$

Maximize with respect to the promotions N_{t+1}^{CH} the objective in (61) to get

$$\begin{aligned}
\frac{dJ_t\left(N_t^C, N_t^H\right)}{dN_{t+1}^{CH}} &= -\kappa_{t+1}^{CH} + \left(P_{t+1}^x \frac{dX_{t+1}^{CH}}{dL_{t+1}^C} - N_{t+1}^C w_{t+1}^C\right) \frac{dL_{t+1}^C}{N_{t+1}^{CH}} \\
&\quad + \left(P_t^x \frac{dX_t^{CH}}{dL_t^C} - N_t^C w_t^C\right) \frac{dL_t^C}{dN_{t+1}^{CH}} \\
&\quad + E_{t+1} Q_{t+1,t+2} \left(\frac{dJ_{t+2}\left(N_{t+2}^C, N_{t+2}^H\right)}{dN_{t+2}^C} + \frac{dJ_{t+2}\left(N_{t+2}^C, N_{t+2}^H\right)}{dN_{t+2}^H}\right) \\
&= 0
\end{aligned} \tag{65}$$

Since (63) implies that

$$\frac{\kappa_{t+1}^C}{q_{t+1}^C} - \frac{1}{q_{t+1}^C}\left(P_t^x \frac{dX_t^{CH}}{dL_t^C} - N_t^C w_t^C\right) \frac{dL_t^C}{dv_{t+1}^C} = E_{t+1} Q_{t+1,t+2} \frac{dJ_{t+2}\left(N_{t+2}^C, N_{t+2}^H\right)}{dN_{t+2}^C}$$

then (62) if the firm can commit[20] becomes

$$\begin{aligned}
\kappa_t^{CH} &= \left(P_t^x \frac{dX_t^{CH}}{dL_t^C} - N_t^C w_t^C\right) \frac{dL_t^C}{N_t^{CH}} + \left(P_{t-1}^x \frac{dX_{t-1}^{CH}}{dL_{t-1}^C} - N_{t-1}^C w_{t-1}^C\right) \frac{dL_{t-1}^C}{dN_t^{CH}} \\
&\quad - \left(\frac{\kappa_t^C}{q_t^C} - \frac{1}{q_t^C}\left(P_{t-1}^x \frac{dX_{t-1}^{CH}}{dL_{t-1}^C} - N_{t-1}^C w_{t-1}^C\right) \frac{dL_{t-1}^C}{dv_t^C}\right) \\
&\quad + E_t Q_{t,t+1} \frac{dJ_{t+1}\left(N_{t+1}^C, N_{t+1}^H\right)}{dN_{t+1}^H}
\end{aligned} \tag{66}$$

To pin down $\frac{dJ_{t+1}\left(N_{t+1}^C, N_{t+1}^H\right)}{dN_{t+1}^H}$ we take an envelope condition with respect to N_t^H. That is

$$\frac{dJ_t\left(N_t^C, N_t^H\right)}{dN_t^H} = P_t^x \frac{dX_t^{CH}}{dN_t^H} - w_t^H L_t^H + \left(1 - \lambda^H\right) \beta E_t \frac{dJ_{t+1}\left(N_{t+1}^C, N_{t+1}^H\right)}{dN_{t+1}^H}$$

Take the above a period in advance and also use (66) a period ahead as well. Assuming the

[20] If it can commit then this holds for all periods and thus from the first.

underlying equation holds for the current period as well due to the time-less perspective:

$$\begin{aligned}
\kappa_t^{CH} = & \left(P_t^x \frac{dX_t^{CH}}{dL_t^C} - N_t^C w_t^C\right) \frac{dL_t^C}{N_t^{CH}} + \left(P_{t-1}^x \frac{dX_{t-1}^{CH}}{dL_{t-1}^C} - N_{t-1}^C w_{t-1}^C\right) \frac{dL_{t-1}^C}{dN_t^{CH}} \\
& - \left(\frac{\kappa_t^C}{q_t^C} - \frac{1}{q_t^C}\left(P_{t-1}^x \frac{dX_{t-1}^{CH}}{dL_{t-1}^C} - N_{t-1}^C w_{t-1}^C\right) \frac{dL_{t-1}^C}{dv_t^C}\right) \\
& + E_t Q_{t,t+1} \left(P_{t+1}^x \frac{dX_{t+1}^{CH}}{dN_{t+1}^H} - w_{t+1}^H L_{t+1}^H\right) \\
& + E_t Q_{t,t+1} \left(1-\lambda^H\right) \left(\begin{array}{c} \kappa_{t+1}^{CH} - \left(P_{t+1}^x \frac{dX_{t+1}^{CH}}{dL_{t+1}^C} - N_{t+1}^C w_{t+1}^C\right) \frac{dL_{t+1}^C}{N_{t+1}^{CH}} \\ - \left(P_t^x \frac{dX_t^{CH}}{dL_t^C} - N_t^C w_t^C\right) \frac{dL_t^C}{dN_{t+1}^{CH}} \\ + \left(\frac{\kappa_{t+1}^C}{q_{t+1}^C} - \frac{1}{q_{t+1}^C}\left(P_t^x \frac{dX_t^{CH}}{dL_t^C} - N_t^C w_t^C\right) \frac{dL_t^C}{dv_{t+1}^C}\right) \end{array}\right)
\end{aligned} \tag{67}$$

The above constitutes the top job creation condition for the top jobs in the case the firm can commit.

D Proposition 3: Overconfidence

From the first order condition that determines the share of the workers that search towards career jobs s_t^C is eq. (5) that is repeated below in steady state

$$\rho^C \left(W^C - U\right) = \rho^N \left(W^N - U\right) \tag{68}$$

Using eq. (2) and (1) in to substitute away ρ^C and ρ^N respectively implies

$$\left(\frac{s^C}{1-s^C}\right)^\gamma = \left(\frac{v^C}{v^N}\right)^\gamma \left(\frac{W^C - U}{W^N - U}\right) \tag{69}$$

Implicitly differentiate eq. (69) to get

$$\frac{ds^C}{dA^e} = \frac{1}{\gamma} \frac{\left(1-\bar{s}_t^C\right)^{1+\gamma}}{\left(\bar{s}_t^C\right)^{\gamma-1}} \left(\frac{v^C}{v^N}\right)^\gamma \frac{\partial \frac{W^N-U}{\partial \rho^H} \frac{\partial \rho^H}{\partial A^e}\left(W^C-U\right) - \partial \frac{W^C-U}{\partial \rho^H} \frac{\partial \rho^H}{\partial A^e}\left(W^N-U\right)}{\left(W^N-U\right)^2} \tag{70}$$

As using the chain rule $\partial \frac{\left(W^N-U\right)}{\partial A^e} = \partial \frac{\left(W^N-U\right)}{\partial \rho^H} \frac{\partial \rho^H}{\partial A^e}$ and $\frac{\partial \rho^H}{\partial A^e} > 0,$ then it is important to pin down the signs of $\partial \frac{\left(W^N-U\right)}{\partial \rho^H}$ and $\partial \frac{\left(W^C-U\right)}{\partial \rho^H}$.

For this part, use the Bellman equations for the value of each of the jobs in the steady state eq. (77) to (80) and as each equation depends on at least two of the others, put them in a matrix form, substitute the wage w^C using (8) and also use $\lambda^C = \lambda^H = \lambda^N$. That is

$$\mathbf{W} = \mathbf{A}^{-1}\mathbf{B} \tag{71}$$

where

$$\mathbf{W} = \begin{bmatrix} W^C - U \\ W^N - U \\ W^H - W^C \end{bmatrix}, \tag{72}$$

$$\mathbf{A} = \begin{bmatrix} 1-\beta\left(1-\lambda^C - s^C\rho^C\right) & \beta s^N\rho^N & \beta\lambda^C \\ \beta s^C\rho^C & 1-\beta\left(1-\lambda^N - s_t^N\rho_t^N\right) & 0 \\ 0 & 0 & 1-\beta\left(1-\lambda^H - \lambda^C\rho^H\right) \end{bmatrix} \tag{73}$$

and

$$\mathbf{B} = \begin{bmatrix} \frac{G\left(L^C\right)}{\mu}L^C - b - \frac{G\left(L^C\right)}{\mu} \\ w^N L^N - b - \frac{G\left(L^N\right)}{\mu} \\ w^H L^H - \frac{G_L\left(L^C\right)}{\mu}L^C + \frac{G\left(L^C\right)}{\mu} - \frac{G\left(L^H\right)}{\mu} \end{bmatrix} \tag{74}$$

As the wage has been substituted away from (8), then $\frac{\partial \mathbf{B}}{\partial \rho^H} = 0$. Therefore, $\frac{\partial \mathbf{W}}{\partial \rho^H} = \frac{\partial \mathbf{A}^{-1}\mathbf{B}}{\partial \rho^H} = \frac{\partial \mathbf{A}^{-1}}{\partial \rho^H}\mathbf{B}$. Using the

$$\frac{\partial \mathbf{W}}{\partial \rho^H} = -\mathbf{A}^{-1}\frac{\partial \mathbf{A}}{\partial \rho^H}\mathbf{A}^{-1}\mathbf{B} = -\,\mathbf{A}^{-1}\frac{\partial \mathbf{A}}{\partial \rho^H}\mathbf{W}$$

The above implies that

$$\frac{\partial \mathbf{W}}{\partial \rho^H} = -\beta\lambda^C\mathbf{A}^{-1}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}\mathbf{W}$$

Using the method of using the minor matrices for inverting a 3×3 matrix the above implies

$$\begin{bmatrix} \partial\frac{W^C-U}{\partial\rho^H} \\ \partial\frac{W^N-U}{\partial\rho^H} \\ \partial\frac{W^H-W^C}{\partial\rho^H} \end{bmatrix} = -\frac{\beta\lambda^C}{|\mathbf{A}|} \begin{bmatrix} 0 & 0 & \det\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} & -\det\begin{bmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{bmatrix} \\ 0 & 0 & \det\begin{bmatrix} a_{23} & a_{21} \\ a_{33} & a_{31} \end{bmatrix} & -\det\begin{bmatrix} a_{13} & a_{11} \\ a_{23} & a_{21} \end{bmatrix} \\ 0 & 0 & \det\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} & -\det\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \end{bmatrix} \mathbf{W}$$

where $a_{i,j}$ for $i = 1, 2, 3$ and $j = 1, 2, 3$ are the corresponding elements of the matrix $\mathbf{A}$ as defined in (73). Using the zero elements of $\mathbf{A}$ it reduces to

$$\frac{\partial \mathbf{W}}{\partial\rho^H} = \begin{bmatrix} \partial\frac{W^C-U}{\partial\rho^H} \\ \partial\frac{W^N-U}{\partial\rho^H} \\ \partial\frac{W^H-W^C}{\partial\rho^H} \end{bmatrix} = -\frac{\beta\lambda^C}{|\mathbf{A}|} \begin{bmatrix} (a_{22}a_{33} + a_{13}a_{22})\left(W^H - W^C\right) \\ (-a_{21}a_{33} - a_{13}a_{21})\left(W^H - W^C\right) \\ (-a_{11}a_{22} + a_{12}a_{21})\left(W^H - W^C\right) \end{bmatrix}$$

The derivatives of interest from the above are thus

$$\partial\frac{W^C - U}{\partial\rho^H} = -\frac{\beta\lambda^C}{|\mathbf{A}|}\left[1 - \beta\left(1 - \lambda^N - s_t^N\rho_t^N\right)\right]\left[1 - \beta\left(1 + \lambda^C\rho^H\right)\right]\left(W^H - W^C\right) > 0 \quad (75)$$

and

$$\partial\frac{W^N - U}{\partial\rho^H} = \frac{\beta\lambda^C}{|\mathbf{A}|}\beta s^C\rho^C\left[1 - \beta\left(1 + \lambda^C\rho^H\right)\right]\left(W^H - W^C\right) < 0 \quad (76)$$

since $\left[1 - \beta\left(1 + \lambda^C\rho^H\right)\right] < 0$. Use the signs of (75) and (76) in eq. (70) to get the following result

$$\frac{ds^C}{dA^e} = \frac{1}{\gamma}\frac{\partial\rho^H}{\partial A^e}\frac{\left(1 - \bar{s}_t^C\right)^{1+\gamma}}{\left(\bar{s}_t^C\right)^{\gamma-1}}\left(\frac{v^C}{v^N}\right)^\gamma \frac{\partial\frac{W^N-U}{\partial\rho^H}\left(W^C - U\right) - \partial\frac{W^C-U}{\partial\rho^H}\left(W^N - U\right)}{\left(W^N - U\right)^2} > 0$$

since $\frac{\partial\rho^H}{\partial A^e} > 0$ and $|\mathbf{A}| > 0$.

E The Surplus from Employment and Equilibrium

The value of employment for the different job opportunities in the economy are important for the determination of the equilibrium as it enters the surplus of the top over the low position

$W_t^H - W_t^C$. The value of the starting career job is:

$$\begin{aligned} W_t^C - U_t &= w_t^C L_t^C - b_t - \frac{G\left(L_t^C\right)}{\mu_t} - s_t^N \rho_t^N E_t Q_{t,t+1}\left(W_{t+1}^N - U_{t+1}\right) \\ &+ \left(1 - \lambda^C - s_t^C \rho_t^C\right) E_t Q_{t,t+1}\left(W_{t+1}^C - U_{t+1}\right) \\ &+ \left(1 - \lambda^C\right) \rho_t^H E_t Q_{t,t+1}\left(W_{t+1}^H - W_{t+1}^C\right) \end{aligned} \tag{77}$$

and the value of the the top position in career jobs is

$$\begin{aligned} W_t^N - U_t &= w_t^N L_t^N - b_t - \frac{G\left(L_t^N\right)}{\mu_t} - s_t^C \rho_t^C E_t Q_{t,t+1}\left(W_{t+1}^C - U_{t+1}\right) \\ &+ \left(1 - \lambda^N - s_t^N \rho_t^N\right) E_t Q_{t,t+1}\left(W_{t+1}^N - U_{t+1}\right) \end{aligned} \tag{78}$$

The value of the high paying position is

$$\begin{aligned} W_t^H - U_t &= w_t^H L_t^H - b_t - \frac{G\left(L_t^H\right)}{\mu_t} - s_t^C \rho_t^C E_t Q_{t,t+1}\left(W_{t+1}^C - U_{t+1}\right) \\ &+ \left(1 - \lambda^H\right) E_t Q_{t,t+1}\left(W_{t+1}^H - U_{t+1}\right) - s_t^N \rho_t^N E_t Q_{t,t+1}\left(W_{t+1}^N - U_{t+1}\right) \end{aligned} \tag{79}$$

The gap between low and top career jobs, an important ingredient for the wage is:

$$\begin{aligned} W_t^H - W_t^C &= w_t^H L_t^H - w_t^C L_t^C - \frac{G\left(L_t^H\right)}{\mu_t} + \frac{G\left(L_t^C\right)}{\mu_t} \\ &+ \left(\lambda^C - \lambda^H\right) E_t Q_{t,t+1}\left(W_{t+1}^C - U_{t+1}\right) \\ &+ \left(1 - \lambda^H - \left(1 - \lambda^C\right) \rho_t^H\right) E_t Q_{t,t+1}\left(W_{t+1}^H - W_{t+1}^C\right) \end{aligned} \tag{80}$$

Eq. (77) to (80) are important in deriving the optimal share of the unemployed that are searching for career and non-career jobs through eq. (5).

For the job creation conditions, eq. (36) and (37), the following derivatives of the job welfare gap $W_t^H - W_t^C$ are important.

With respect to N_t^C :

$$
\begin{aligned}
\partial \frac{W_t^H - W_t^C}{\partial N_t^C} \quad &= \quad -\left(1-\lambda^C\right) \frac{\partial \rho_t^H}{\partial N_t^C} E_t Q_{t,t+1} \left(W_{t+1}^H - W_{t+1}^C\right) \\
&\quad + \left(\lambda^C - \lambda^H\right) E_t Q_{t,t+1} \left(\partial \frac{W_{t+1}^C - U_{t+1}}{\partial N_{t+1}^C}\right) \frac{\partial N_{t+1}^C}{\partial N_t^C} \\
&\quad + \left(1 - \lambda^H - \left(1-\lambda^C\right) \rho_t^H\right) E_t Q_{t,t+1} \left(\partial \frac{W_{t+1}^H - W_{t+1}^C}{\partial N_{t+1}^C}\right) \frac{\partial N_{t+1}^C}{\partial N_t^C}
\end{aligned}
$$

With respect to ∂N_t^{CH} :

$$
\begin{aligned}
\partial \frac{W_t^H - W_t^C}{\partial N_t^{CH}} \quad &= \quad \left(\lambda^C - \lambda^H\right) E_t Q_{t,t+1} \left(\partial \frac{W_{t+1}^C - U_{t+1}}{\partial N_{t+1}^C}\right) \frac{\partial N_{t+1}^C}{N_t^{CH}} \\
&\quad - \left(1-\lambda^C\right) \frac{\partial \rho_t^H}{\partial N_t^{CH}} E_t Q_{t,t+1} \left(W_{t+1}^H - W_{t+1}^C\right) \\
&\quad + \left(1 - \lambda^H - \left(1-\lambda^C\right) \rho_t^H\right) E_t Q_{t,t+1} \left(\partial \frac{W_{t+1}^H - W_{t+1}^C}{\partial N_{t+1}^C}\right) \frac{\partial N_{t+1}^C}{\partial N_t^{CH}} \\
&\quad + \left(1 - \lambda^H - \left(1-\lambda^C\right) \rho_t^H\right) E_t Q_{t,t+1} \left(\partial \frac{W_{t+1}^H - W_{t+1}^C}{\partial N_{t+1}^H}\right) \frac{\partial N_{t+1}^H}{\partial N_t^{CH}}
\end{aligned}
$$

and with respect to v_t^C :

$$
\begin{aligned}
\partial \frac{W_t^H - W_t^C}{\partial v_t^C} \quad &= \quad \left(\lambda^C - \lambda^H\right) E_t Q_{t,t+1} \left(\partial \frac{W_{t+1}^C - U_{t+1}}{\partial N_{t+1}^C}\right) \frac{\partial N_{t+1}^C}{\partial v_t^C} \\
&\quad + \left(1 - \lambda^H - \left(1-\lambda^C\right) \rho_t^H\right) E_t Q_{t,t+1} \left(\partial \frac{W_{t+1}^H - W_{t+1}^C}{\partial N_{t+1}^C}\right) \frac{\partial N_{t+1}^C}{\partial v_t^C}
\end{aligned}
$$

and with respect to N_t^H :

$$
\partial \frac{W_t^H - W_t^C}{\partial N_t^H} = 0
$$

For w_t^C :

$$
\partial \frac{W_t^H - W_t^C}{\partial w_t^C} = -L_t^C + \left(\frac{G_L\left(L_t^C\right)}{\mu_t} - w_t^C - \left(1-\lambda^C\right) \frac{\partial \rho_t^H}{\partial L_t^C}\right) \frac{\partial L_t^C}{\partial w_t^C}
$$

For w_t^H :

$$W_t^H - W_t^C = w_t^H L_t^H - w_t^C L_t^C - \frac{G\left(L_t^H\right)}{\mu_t} + \frac{G\left(L_t^C\right)}{\mu_t} + \left(\lambda^C - \lambda^H\right) E_t Q_{t,t+1}\left(W_{t+1}^C - U_{t+1}\right) + \left(1 - \lambda^H - \left(1-\lambda^C\right)\rho_t^H\right) E_t Q_{t,t+1}\left(W_{t+1}^H - W_{t+1}^C\right) \quad (81)$$

$$\partial \frac{W_t^H - W_t^C}{\partial w_t^H} = L_t^H + \left(w_t^H - \frac{G_L\left(L_t^H\right)}{\mu_t}\right)\frac{\partial L_t^H}{\partial w_t^H}$$

For $W_t^C - U_t$:

$$\partial \frac{W_t^C - U_t}{\partial N_t^C} = \left(1-\lambda^C\right)\frac{\partial \rho_t^H}{\partial N_t^C} E_t Q_{t,t+1}\left(W_{t+1}^H - W_{t+1}^C\right) + \left(1-\lambda^C\right)\rho_t^H E_t Q_{t,t+1}\left(\partial \frac{W_{t+1}^H - W_{t+1}^C}{\partial N_{t+1}^C}\right)\frac{\partial N_{t+1}^C}{\partial N_t^C}$$

We assume that the vacancy costs are distributed to the households. The budget constraint is:

$$P_t C_t = N_t^C w_t^C L_t^C + N_t^H w_t^H L_t^H + N_t^N w_t^N L_t^N + \kappa_t^C v_t^C + \kappa_t^{CH} N_t^{CH} + \kappa_t^N v_t^N + \Pi_t^{CH} + \Pi_t^N$$

where the last term is the profit from final good firms. Therefore $Y_t = C_t$

The derivatives of the probability to be promoted are with respect to L_t^C :

$$\frac{\partial \rho_t^H}{\partial L_t^C} = \gamma_L A_t^e \left(\frac{L_t^C}{\bar{L}_t^C}\right)^{\gamma_L - 1}\frac{1}{\bar{L}_t^C}\left(\frac{N_t^{CH}}{\left(1-\lambda^C\right)N_t^C}\right)^{\gamma_N} = \gamma_L \frac{\rho_t^H}{L_t^C}$$

with respect to N_t^C :

$$\frac{\partial \rho_t^H}{\partial N_t^C} = -\gamma_N A_t^e \left(\frac{L_t^C}{\bar{L}_t^C}\right)^{\gamma_L}\left(\frac{N_t^{CH}}{\left(1-\lambda^C\right)}\right)^{\gamma_N}\left(N_t^C\right)^{-\gamma_N - 1} = -\gamma_N \frac{\rho_t^H}{N_t^C}$$

with respect to N_t^{CH} :

$$\frac{\partial \rho_t^H}{\partial N_t^{CH}} = \gamma_N \frac{\rho_t^H}{N_t^{CH}}$$

The cross derivative w.r.t N_t^C :

$$\partial \frac{\frac{\partial \rho_t^H}{\partial L_t^C}}{\partial N_t^C} = \gamma_L \frac{\frac{\partial \rho_t^H}{\partial N_t^C}}{L_t^C} = -\gamma_N \gamma_L \frac{\rho_t^H}{N_t^C L_t^C}$$

The second derivative w.r.t L_t^C :

$$\begin{aligned} \partial \frac{\frac{\partial \rho_t^H}{\partial L_t^C}}{\partial L_t^C} &= \gamma_L \left(\gamma_L - 1\right) A_t^e \left(\frac{L_t^C}{\bar{L}_t^C}\right)^{\gamma_L - 2} \frac{1}{\left(\bar{L}_t^C\right)^2} \left(\frac{N_t^{CH}}{\left(1-\lambda^C\right) N_t^C}\right)^{\gamma_N} \\ &= \gamma_L \left(\gamma_L - 1\right) \frac{\rho_t^H}{\left(L_t^C\right)^2} \end{aligned}$$

The cross derivative w.r.t N_t^{CH}:

$$\partial \frac{\frac{\partial \rho_t^H}{\partial L_t^C}}{\partial N_t^{CH}} = \gamma_L \frac{\frac{\partial \rho_t^H}{\partial N_t^{CH}}}{L_t^C} = \gamma_L \gamma_N \frac{\rho_t^H}{N_t^{CH} L_t^C}$$