Working Paper 06-2018

Non-Linearities in International Prices

Inkoo Lee, Sang Soo Park, and Marios Zachariadis

Department of Economics, University of Cyprus, P.O. Box 20537, 1678 Nicosia, Cyprus
Tel.: +357-22893700, Fax: +357-22895028, Web site: http://www.ucy.ac.cy/econ/en

Non-Linearities in International Prices

Inkoo Lee

Department of International Trade, Soongsil University, 369 Sangdo-Ro, Dongjak-Gu, Seoul 06978, Korea. Phone: 820-0573. E-mail: iklee1120@ssu.ac.kr

Sang Soo Park

Department of Economics, Korea University, 145 Anam-ro, Seongbuk-gu,Seoul 02841, Korea. Phone: +82 (2) 3290-2220. E-mail: starpac@korea.ac.kr

Marios Zachariadis

Department of Economics,University of Cyprus, 1678 Nicosia,Cyprus. Phone: 357-22893712. E-mail: zachariadis@ucy.ac.cy

---

Abstract

We consider multiple sources of non-linearity at the same time within a structural model that accounts for previously omitted variables and allows estimation of product-level convergence rates both within and outside the band of no trade. Accounting for the role of theoretically-implied variables and their non-linear interactions in the convergence process, we find that good-level convergence rates are systematically faster as compared to convergence estimates from reduced-form models. Contrary to conventional wisdom, we find that good-level price differentials exhibit mean-reverting behavior even within the bands of no trade, and that rates of mean-reversion within or outside the no-trade band are strongly related to goods' economics characteristics. Furthermore, while implied trade costs dramatically increase as we move from within country comparisons to comparisons across countries, inconsistent with our priors services have somewhat comparable trade costs to tradable goods. Finally, wage differentials are negatively associated with the speed of price adjustment and this effect is stronger for city pairs that are farther apart.

Keywords: law-of-one-price, threshold auto-regressive, structural estimation, convergence rates, trade costs
JEL Classification: F41

1. Introduction

We consider multiple sources of non-linearity at the same time within a structural threshold auto-regressive (TAR) model that accounts for previously omitted variables and allows estimation of product-level convergence rates within and outside the band of no trade, that do not suffer from the type of misspecification and omitted variables bias present in previous work based on reduced form TAR models. The null hypothesis in the latter non-linear models is that inside the no-trade band, price deviations from the LOP are persistent while above or below it arbitrage takes place inducing deviations from the LOP to mean-revert. Other possible reasons for nonlinearities were ignored in these previous studies.

Our model encompasses the main elements of conventional TAR models that allow the dynamics of relative prices to differ above and below the band of inaction. Relative to reduced form models, however, our model has two notable distinctions. First, although conventional models focused on transport costs as the major source of the inaction band, we show that the band is also generated by additional factors including differences in local distribution costs and wages across locations. Second, the behavior of price differentials within the band hinges upon differences in local distribution costs and wages, and hence does not necessarily follow a random walk process. The combination of the first and second features implies that local factors play an important role in the dynamics of relative prices through the channel of wages and distribution costs, consistent with a view that market segmentation is driven by local factors as well as international trade costs.

After all, assuming final goods are comprised of a traded and a non-traded input as per the retail pricing model in Crucini et.al (2005), the determinants of goods' prices should be related to their traded and non-traded components, influenced respectively by trade costs and by factors such as local input costs and productivity. It thus follows from basic economic theory that variables such us wages are relevant for examining whether poorer countries behind the technology frontier tend to exhibit faster price convergence leading to price convergence via the non-traded component of final prices, along the lines of the Balassa-Samuelson framework.[1] Basic economic theory, say gravity mod-

---

[1] Another possibility is that international movements of factors of production induce

els, also suggests variables like physical distance are relevant for examining the role of trade costs in price convergence, via the traded inputs channel.[2]

The specific theoretical framework within which we approach the empirical analysis of nonlinear price adjustment is an extension of the one-good, two-country endowment economy model of Sercu and Uppal (2003) that incorporates a nontradable good, local distribution costs and a labor input. This framework allows us to incorporate the role of additional factors and their non-linear interactions in the convergence process, in a theory-consistent manner.[3]

In line with the above, our empirical analysis differs from the existing empirical literature in that we estimate convergence speeds both outside and inside the thresholds for individual goods and services (both tradeable and non-tradeable) allowing for the theoretically-implied role of factors like wage differentials and distance, as well as for their non-linear interactions. By contrast, previous empirical studies had typically focused on price adjustment of tradables outside the band. Importantly, accounting for the role of theoretically-implied variables in the convergence process, we find that good-level convergence rates are systematically faster as compared to those estimated using reduced-form models. Our evidence suggests that the omission of variables which may affect price dynamics and resulting misspecification of econometric models, may lead to downward bias in reverting speeds of price differentials.

Contrary to conventional wisdom, we find that good-level price differentials exhibit mean-reverting behavior even within the bands of no trade.

---

convergence in wages as in Zachariadis (2012) where immigration is shown to matter for price convergence.

[2] These traded and non-traded components via which price convergence occurs can interact with each other, as shown in Glushenkova, Kourtellos and Zachariadis (2018). For example, lower trade costs appear to be conducive to price convergence only for countries that have the non-traded Balassa-Samuelson catch up process operating in full force given low initial incomes. We thus consider such non-linearities in addition to TAR-type ones.

[3] Several theory papers suggest international price processes are non-linear. Dumas (1992) and Sercu et.al (1995) argue threshold nonlinearities arise due to transactions costs in international arbitrage that create a “band of inaction” within which the marginal cost exceeds the marginal benefit of arbitrage, whereas outside this no-arbitrage band, arbitrage acts as a convergence force towards the LOP. These transaction costs have been interpreted by Dixit (1989) and Krugman (1989) as “market frictions” capturing sunk costs of international arbitrage where traders enter only if large enough opportunities arise.

Furthermore, rates of mean-reversion within or outside the no-trade band are strongly related to goods' economics characteristics. In addition, consistent with conventional wisdom, implied trade costs dramatically increase as we move from within country comparisons to comparisons across countries. Moreover, inconsistent with our priors, services have somewhat comparable trade costs to tradable goods. Finally, wage differentials are negatively associated with the speed of price adjustment suggesting a role for consumers' search intensity and firms' pricing-to-market producing persistent price deviations in line, e.g., with Alessandria and Kaboski (2011) where costly consumer search makes local wages matter for price-setting behavior. We also see that this effect of wages is stronger for city pairs that are farther apart.

The next section describes the theoretical framework from which our empirical specification used in the third section derives. The fourth section presents our empirical findings while the last section briefly concludes.

2. Empirical framework

2.1. Methodology

In this section, we provide an empirical framework with which we analyze nonlinear adjustment of relative prices. We extend the one-good, two-country endowment economy model of Sercu and Uppal (2003) to incorporate a nontradable good, local distribution costs and a labor input. As in the latter paper, we assume complete financial markets and focus on two types of goods market frictions, local and international transaction costs. Our approach is meant to provide a tractable framework to carry out explicit analysis of price adjustment, with emphasis on the interplay of different factors driving nonlinearity in international relative prices. Appendix A discusses the details of the approach that we use to derive the dynamics of relative prices, given by:

$$\Delta q_{i,j,t} = \begin{cases} -(q_{i,j,t-1} - (\eta_{i,j} + \tau)) + (\theta_1 - 1)(\eta_{i,j} + \tau) - \theta_2 w_{i,j,t} & \text{if } q_{i,j,t-1} > \eta_{i,j} + \tau \\ -q_{i,j,t-1} + \gamma\eta_{i,j} + \gamma w_{i,j,t} & \text{otherwise} \\ -(q_{i,j,t-1} - (\eta_{i,j} - \tau)) + (\theta_1 - 1)(\eta_{i,j} - \tau) - \theta_2 w_{i,j,t} & \text{if } q_{i,j,t-1} < \eta_{i,j} - \tau \end{cases} \tag{1}$$

The (logarithm of) international relative price of retail goods, $q_{i,j,t}$, is defined by the ratio of the retail price of country j to the retail price of country i in period t. $\theta_1 = \frac{\gamma\alpha-\alpha+\delta}{1-\alpha+\alpha\gamma}$ and $\theta_2 = \delta(\alpha+\gamma-\alpha\gamma)$. The parameter α is the expenditure share of the tradable good and γ is the inverse of the intertemporal elasticity of substitution. $w_{i,j,t} = w_{j,t} - w_{i,t}$ is the difference of the (logarithm of) real wages in country i relative to country j at time t. Parameter τ is the bilateral trade cost associated with bringing the tradable good from its point of loading abroad to the point of unloading in the importing country. $\eta_{ij} = \eta_j - \eta_i$ where η_i and η_j represent local costs that are entangled in the movement of the tradable good from its point of production or unloading to the point of retailers in countries i and j respectively.

Equation (1) encompasses the main elements of a conventional TAR model where the dynamics of relative prices differ above and below the band of inaction. However, whereas conventional models focused on transport costs as the major source of the inaction band, our model implies that the band is in part generated by additional factors such us cross-country differences in distribution costs and wages. Furthermore, in our model, the behavior of price differentials within the band hinges upon differences in local distribution costs and wages, and hence does not necessarily follow a random walk process. The combination of the first and second features implies that local factors play an important role in the dynamics of relative prices via the wages and distribution costs channels, consistent with the view that market segmentation is driven by both local factors and international trade costs. Explicitly deriving the determinants of price adjustment, we obtain the estimable equation shown below:

$$\Delta q_{i,j,t} = \begin{cases} \lambda_1^{out,u}(q_{i,j,t-1} - a^u_{i,j}) + \beta^u_a a^u_{i,j} + \beta_w^{out,u} w_{i,j,t} + e^{out}_{i,j,t} & \text{if } q_{i,j,t-1} > a^u_{i,j} \\ \lambda_1^{in} q_{i,j,t-1} + \beta_1 \eta_{i,j} + \beta_w^{in} w_{i,j,t} + e^{in}_{i,j,t} & \text{otherwise} \\ \lambda_1^{out,l}(q_{i,j,t-1} - a^l_{i,j}) + \beta^l_a a^l_{i,j} + \beta_w^{out,l} w_{i,j,t} + e^{out}_{i,j,t} & \text{if } q_{i,j,t-1} < a^l_{i,j} \end{cases} \tag{2}$$

where we introduced $a^u_{i,j} := \eta_{i,j} + \tau$ and $a^l_{i,j} := \eta_{i,j} - \tau$ for notational brevity and added $e^{out}_{i,j,t}$ and $e^{in}_{i,j,t}$ for the error terms for the (i, j) pair. Estimating the above-derived equation (2) allows us to examine theory-implied nonlinearities in international price reversion behavior. Parameters λ_1^{out}s and λ_1^{in} to be estimated are of particular interest. The first measures the speed at which price differentials between markets revert back to the band once they cross the thresholds. On the other hand, λ_1^{in}, relates to the speed of convergence within the band of no trade.

Model (2) must obey certain restrictions, such as $\beta^l_a = \beta^u_a$ and $\beta_w^{out,l} =$

$\beta_w^{out,u}$ due to the fact that party i's export is party j's import and vice versa.[4] With these restrictions imposed, the general TAR model that we consider is as follows:

$$\Delta q_{i,j,t} = \begin{cases} \lambda_1^{out}(q_{i,j,t-1} - a^u_{i,j}) + \beta_a^{out} a^u_{i,j} + \beta_0^{out} + w'_{i,j,t}\beta_w^{out} + \lambda_2^{out} q_{i,j,t-2} + e^{out}_{i,j,t} \\ \qquad \text{if } q_{i,j,t-1} \geq a^u_{i,j}, \\ \lambda_1^{in} q_{i,j,t-1} + \beta_\eta^{in}\eta_{i,j} + w'_{i,j,t}\beta_w^{in} + \lambda_2^{in} q_{i,j,t-2} + e^{in}_{i,j,t} \text{ if otherwise,} \\ \lambda_1^{out}(q_{i,j,t-1} - a^l_{i,j}) + \beta_a^{out} a^l_{i,j} - \beta_0^{out} + w'_{i,j,t}\beta_w^{out} + \lambda_2^{out} q_{i,j,t-2} + e^{out}_{i,j,t} \\ \qquad \text{if } q_{i,j,t-1} \leq a^l_{i,j}, \end{cases} \tag{3}$$

where $w_{i,j,t}$ can be a collection of variables such that $w_{i,j,t} = -w_{j,i,t}$. We consider price comparisons within the U.S. (UU), between the U.S. and the European Union (UE), and between the U.S. and other countries (UO), for goods and services separately, by modelling $\eta_i, \eta_j \in \left\{\eta^U, \eta^E, \eta^O\right\}$ i.e. $\eta_{i,j,t} = \sum_{k\in\{U,E,O\}} \eta^k \times 1_{\{j\in k\}} - \sum_{r\in\{U,E,O\}} \eta^r \times 1_{\{j\in r\}}$.

We estimate five variants of the model as follows:

(M0) $\tau = \delta_0$, no $w_{i,j,t}$, no $\lambda_2^{out} q_{i,j,t-2}$.

(M1) $\tau = \delta_0 + \delta_1 \ln(dist_{i,j})$, $w_{i,j,t}$ only, no $\lambda_2^{out} q_{i,j,t-2}$.

(M2) $\tau = \delta_0 + \delta_1 \ln(dist_{i,j})$, $w_{i,j,t}$ and $w_{i,j,t} \cdot \ln(dist_{i,j})$, no $\lambda_2^{out} q_{i,j,t-2}$.

(M3) $\tau = \delta_0 + \delta_1 \ln(dist_{i,j})$, $w_{i,j,t}$ only, $\lambda_2^{out} q_{i,j,t-2}$ included.

(M4) $\tau = \delta_0 + \delta_1 \ln(dist_{i,j})$, $w_{i,j,t}$ and $w_{i,j,t} \cdot \ln(dist_{i,j})$, $\lambda_2^{out} q_{i,j,t-2}$ included.

where $dist_{i,j}$ is the geographical distance between i and j. Specification (M0) is the simplest TAR model (M0) and excludes the terms for distribution costs and wages. Specification (M1) is a structural TAR model with neither interaction effects nor any higher order price adjustments terms included. Model (M2) includes interaction effects but excludes any higher order adjustment terms. Models (M3) and (M4) have a second order AR term in addition to (M1) and (M2) respectively.

2.2. Data

The source of our micro price data is the Worldwide Cost of Living Survey collected by the Economist Intelligence Unit (EIU). The survey covers 300 individual retail goods and services across 140 cities in 91 countries semi-annually over the period 1990-2013. Bergin, Glick, and Wu (2013), Andrade

---

[4] Detail of such restrictions and the derivation of the model is provided in appendix B.

and Zachariadis (2016), and Glushenkova, Kourtellos and Zachariadis (2018) also use these semi-annual EIU data or subsets of these, to study issues of price adjustment. The online appendix of Andrade and Zachariadis (2016) discusses issues related to sample selection and reliability of this dataset in great detail. As explained in detail there, this dataset is suitable to address the key questions at hand regarding price dispersion and price adjustment across countries. First, these survey prices are quite comparable across cities as they are usually specific in terms of both quality and quantity, e.g., aspirin (100 tablets), Coca Cola (1 liter), and tennis balls (six, Dunlop). Moreover, these price data are collected in a consistent manner by a single agency. Finally, since the data are absolute prices for goods and services rather than indexes, we are able to evaluate the absolute magnitude of cross-sectional LOP deviations and resulting price adjustment of each item.

Prices for most tradeable goods are sampled from two different outlets, a supermarket/chain store and mid-priced/branded store, and are separately reported in the survey. We examine the dynamics of relative prices for both types of outlets, but report results from the supermarket/chain store due to its higher comparability across locations. By doing so, we avoid the same goods appearing more than once in our analysis. Later, we compare the convergence speeds of these two types of outlets to check if prices from low-price outlets (supermarket/chain stores) exhibit different reverting behavior than prices in mid-priced stores.

3. Results

3.1. Main results

Our study differs from previous work in that we estimate convergence speeds for goods and services inside of the band, allowing for the effect of wage differentials, in addition to estimating rates of convergence outside of the band. Previous studies focused on price adjustment of tradables outside the band, in reduced-form settings. We outline the main results arising from our structural approach to the data below and present more details in the subsections that follow.

Accounting for theoretically-implied variables within a structural TAR model, we find that good-level convergence rates are systematically faster as compared to those implied by reduced-form TAR models previously considered. As expected, price shocks are relatively short lived for non-services and for city pairs within a country (the U.S. in particular). Contrary to conventional wisdom, the process of price differentials does not necessarily follow a random walk when trade does not occur.[5] Estimated trade costs vary widely across individual goods and services and across locations. Trade costs for services are comparable to those for tradeable goods and increasing in distance.[6] Finally, non-linearities in the form of interactions between the traded and non-traded channels play a role in the convergence process of price differences across the world. Accounting for the interaction between wages and physical distance, convergence rates become somewhat slower as compared to the case where this non-linearity is ignored, but still faster than rates of convergence from reduced-form TAR models.

3.2. Convergence rates

In this subsection, we describe the results arising from our structural estimation of convergence rates in more detail. The average (across goods or services) speed outside the band, λ_1^{out}, at which price differentials between markets revert back to the band once they cross the thresholds, and the mean convergence speed within the band, λ_1^{in}, along with the corresponding half-lives are reported respectively in Tables 1 and 2. In each case, we present separate results for comparisons of locations within the US (UU), between the US and European Union countries (UE), and between US and other country locations (UO), separately for goods and services.

The next few findings from Tables 1 and 2 constitute our main contribution in terms of novel empirical evidence. First, in all cases considered, the structural TAR models without (M1, M3) and with (M2, M4) interaction effects imply faster convergence speed $\lambda_1^{out,T}$ for tradeable goods than the standard reduced-form TAR model (M0) that has typically been estimated in previous work.[7] This can be inferred by comparing the first column of results in Table 1 with the second to fifth columns of results shown there. The latter finding suggests that the reverting patterns of price differentials

---

[5] This, however, is in line with Zachariadis (2012) where international movements of labor to initially high wage expensive countries induce price convergence even for non-tradeables.

[6] Consistent with recent views that the presence of non-traded inputs and the absence of a strict dichotomy between final goods and services in terms of tradeability, are important in order to understand international price dynamics.

[7] For services, only the benchmark structural model M1 always predicts faster convergence rates than the reduced form model M0.

are differently characterized by our models as compared to the typical model estimated in the literature. Our structural models (M1, M2, M3, M4) systematically predict faster reversion towards the thresholds than what the reduced form model (M0) suggests for tradables. As implied by the estimated values of λ_1^{out} shown in the first column of Table 1, the half-life for the average tradable good in the M0 model is roughly 1.8 years for UU comparisons, 6.9 years for UE comparisons, and 4.4 years for UO pairs. By contrast, the half-lives in our benchmark model, M1, implied by the λ_1^{out} estimates shown in the second column of Table 1, are only about 1.2 years for UU comparisons, 2.5 years for UE comparisons, and about 3 years for UO ones. These half-lives are also substantially shorter than the earlier consensus of 4 to 5 years suggested by estimating conventional linear models. Overall, the above evidence suggests that the omission of variables which may affect price dynamics and resulting misspecification of econometric models, may lead to downward bias in reverting speeds of price differentials.

Our next novel finding is that structural TAR models that account for interactions between wages and distance (M2 and M4) usually predict slower convergence speeds, $\lambda_1^{out,T}$ and $\lambda_1^{out,S}$, than those without interaction terms (M1 and M3) as can be seen by comparing column two with column three and column four with five in Table 1. This suggests that accounting for interactions between the traded and non-traded components as in Glushenkova, Kourtellos and Zachariadis (2018) provides us with lower estimates of price convergence as compared to our models which exclude these interactions.

We also find, somewhat surprisingly, that implied convergence speeds for services ($\lambda_1^{out,S}$) for comparisons within the US are usually comparable to those for goods ($\lambda_1^{out,T}$) for comparisons (UE and UO) across countries as can be seen in Table 1. In particular, models Mo, M1, M2, M3 show that convergence speeds for *services within the U.S.* are comparable to and sometimes faster than those for *goods (tradeables) across countries*, which tells us that price differentials of services within the U.S. are arbitraged away as quickly as those of tradables between the U.S. and other countries. This suggests that the role labor mobility across US cities plays for price convergence within the US is comparable in force to the role played by trade in final goods across international locations.

Our last potentially important finding is that, as shown in Table 2, convergence speeds inside the band implied for goods ($\lambda_1^{in,T}$) are faster than one would have expected based on the findings and assumptions in previous work. In particular, one would expect that price differentials follow a random walk process within the band where no adjustment takes place which is why a body of literature imposes the assumption that $\lambda_1^{in} = 0$. This view is not necessarily correct based on our estimates in Table 2. For example, in the case of comparisons between US and EU cities, our benchmark structural TAR model M1 suggests a half-life of 4.6 years within the band as shown in Table 2. This compares to a half-life of 2.5 years outside the band as shown in Table 1 for the same model and set of bilateral comparisons. As reported in Table 3, on average, the implied share of traded inputs for goods amounts to 70%. This means that price differentials within the band of inaction can be less persistent than expected, as price differentials of traded inputs contained in the final good tend to be arbitraged away.[8] It then comes as no surprise that the convergence speed $\lambda_1^{in,T}$ is estimated to be faster than $\lambda_1^{out,S}$ for international comparisons (UE and UO). The implied half-life for tradeable goods inside the band for our benchmark structural TAR model M1 shown in Table 2, is 4.6 years for UE comparisons and 3.3 years for UO comparisons as compared respectively to 5.5 years and 4.2 years for the half-life of services outside the band for model M1 in Table 1. We note that the share of traded inputs in services is only 33% and thus there is little error-correction force driven by traded-inputs. Instead, price differentials of services within the band will follow a process determined mostly by changes in local demand and supply so that half-lives for services within the band of inaction can be huge as shown in Table 2. For instance, our benchmark structural TAR model M1 implies a half-life of over 14 years within the band for price comparisons between the U.S. and EU countries for services.

Our last set of findings from Tables 1 and 2 serves to confirm previous standardized facts and in doing so to ensure the relevance of our data and methodology. First, as we can see in Table 1, implied convergence speeds for goods ($\lambda_1^{out,T}$) are faster than for services ($\lambda_1^{out,S}$) in all cases, irrespective of the statistical model or set of price comparisons considered. That is, it takes longer for price differentials of services as compared to those of tradeable

---

[8] Every individual retail good (for example, a car) encompasses both traded (steels, tyres, paints, robots etc) and non-traded inputs (e.g. labor), with goods having a higher share of traded inputs. We note that although we consider our items separately depending on whether they are goods (typically tradeable and outside the band) or services (mostly non-tradeables thus within the band), the structure of retail markets ensures that no individual item actually satisfies the strict definition tradeable or non-tradeable, due to the presence of intermediates.

goods to adjust. Second, as we can also see in Table 1, implied convergence speeds for goods ($\lambda_1^{out,T}$) and services ($\lambda_1^{out,S}$) within the U.S. (UU) are higher than those between countries (UE and UO) for all statistical models considered, implying a higher degree of market integration and arbitrage within a country. Furthermore, this holds for goods as well as services, suggesting that the mechanisms bringing prices closer faster within a country do not just relate to trade in goods but also perhaps relate to how fast factors of production move within a country as compared to across countries. Third, implied convergence speeds for goods ($\lambda_1^{out,T}$) and services ($\lambda_1^{out,S}$) outside the "bands of inaction" shown in Table 1 are faster than those inside those bands ($\lambda_1^{in,T}$ and $\lambda_1^{in,S}$) shown in Table 2 for the structural TAR models estimated here.[9] This means price differentials outside the band are relatively short lived as compared to those within the band, indicating the presence of TAR-type non-linear adjustment of price differentials.[10] Reassuringly, the slowest convergence speeds we find are within the bands of inaction for services, irrespective of the statistical model and the bilateral set of comparisons being considered. These findings square well with conventional wisdom, providing compelling evidence that our model and estimation methodology correctly capture reverting properties of Law-of-One-Price deviations.

Next, to help understand the role potentially played by the tradability of final goods but also by the share of non-traded inputs embodied in any final good, Table 4 reports correlations between (absolute values of) convergence speeds for individual goods and their characteristics; namely, degree of tradability and share of nontraded inputs. Because the tradability and nontraded input share variables are measured by industry and are more aggregated than the retail price data we have at hand, we assign each good-specific estimate of convergence speed to an industry and then choose the median for each industry to use as that industry's measure of convergence speed. Noting that the number of goods in each industry varies widely, we use these numbers as weights in computing the Pearson correlation coefficients. In Table 4, we show that convergence speeds are positively associated with tradability but negatively related to the nontraded input share both outside and inside the bands, which supports our assertions. The λs used are from the bench-

---

[9] The one exception relates to comparisons of services between the U.S. and other countries (UO) for the case of AR2 models (M3, M4.)

[10] The reduced-form TAR model M0 fails to reproduce this stylized fact as can be seen by comparing Tables 1 and 2.

mark model, M1. Although not reported here, convergence speeds estimated from other model specifications (M2, M3, M4) give similar results. Namely, substantially large and economically significant correlations. For price comparisons between the U.S. and the EU, the correlations equal 42% (65%) between tradeability and the estimated convergence rates outside (inside) the band, and minus 85.4% (90.3%) between the latter convergence rates and non-traded input shares. Overall, these strong statistically significant correlations suggest that the heterogeneity in convergence rates we estimate across individuals goods and services is meaningful in that it relates sensibly to their economic characteristics.

In order to further understand the role of goods characteristics for price convergence, we consider and contrast sub-categories of tradable goods. First, we consider perishable versus non-perishable goods. Perishable goods (for example, fresh chicken) decay more easily within a short period of time than non-perishable (frozen chicken) goods, and hence are less likely to be traded. However, if price differentials are large enough to induce trade occurrence, the nature of perishability makes arbitrage more active (i.e., urgent) and therefore leads to faster price reversion towards the band for perishable goods. This is exactly what we find in Table 5. Implied convergence speeds outside the band for perishable goods are faster than those for non-perishable ones.

Second, we consider goods sold at supermarkets versus goods sold at high-price or brand stores. Consumers who shop at supermarkets tend to price-shop for frequently purchased goods, while firms have more incentive to charge different markups across brand stores. Therefore, one would expect more persistent price differences across locations for goods sold at brand outlets, indicative of faster price reversion of goods at supermarkets. This is what we observe in Table 5. Convergence is faster for supermarkets compared to brand stores and other mid-priced outlets.

3.3. Implied trade costs

In addition to helping us obtain theory-consistent estimates of convergence, our structural estimation approach also provides us with additional meaningful parameter estimates we discuss in this and the next subsection. For instance, estimated parameter δ_0 of the structural models M1, M2, M3 and M4 specified in section 2, captures the component of trade costs not explained by distance. This parameter can then be related to things like the border, taxes, and pricing-to-market. Parameter δ_{dist}, on the other hand, relates to the impact of distance on trade and could be thought of as the component of trade costs related to distance. We can see several features in Tables 6 and 7 where we report respectively mean values of δ_0 and δ_{dist}.

First, there is variation in estimated trade costs both across different types of items and across different bilateral comparisons, e.g., within versus across countries. This points to the importance of product-specific and location-specific factors in characterizing international market frictions. Consistent with conventional wisdom, both δ_0 and δ_{dist} dramatically increase as we move from within country comparisons (UU) to comparisons across countries (UE and UO) for the structural models we consider M1, M2, M3 and M4.[11]

Inconsistent with our priors, we see in Table 6 that services have on average comparable trade costs to tradable goods. One would expect higher non-distance-related trade costs, δ_0, for services. However, the last feature we uncover poses a challenge to the traditional view that δ_0 is necessarily higher for services. We find that δ_0 is not systematically higher for services as compared to tradable goods. In fact, for bilateral comparisons between the US and Europe for our structural TAR models M1 to M4, δ_0 is always lower for services as compared to tradables.

In Table 7, we see that the impact of distance on trade, δ_{dist}, is not systematically higher for tradable goods as compared to services. A possible reason for this is that many service industries exhibit geographic concentration in production and therefore have trade costs similar to manufacturing industries, pertaining to the apparent role we estimate for δ^S_{dist}. A second reason could be that, considering there exists a significant degree of home bias in the consumption of services, it would be natural for trade costs of services to significantly depend upon distance, i.e., relatively high δ^S_{dist}. Finally, based on Table 7, we note that perishable goods exhibit larger δ_{dist} than non-perishable goods for within-country price comparisons (UU), but comparable δ_{dist} in an international context. This is probably because perishable goods are processed to be non-perishable when they are traded over long distances (internationally).

3.4. Wage differentials

High wage differentials are likely to hinder price adjustment by prohibiting price differentials from being arbitraged away. This is because higher income differentials are associated with larger differences in local costs, and

---

[11] For the reduced form model, M0, δ_0 also goes up but less dramatically.

with higher ability of firms to price-to-market. Our interest in wage differentials is motivated by basic features of consumer purchasing behavior. Consumers spend a considerable amount of time in search-related activities such as shopping. This search intensity is related to the opportunity cost of time so that high-income consumers tend to search less per purchase than low-income consumers. Thus, a change in the relative wage across locations changes the relative cost of consumer search so that consumers in a relatively high-income region search less intensively than consumers in a low-income region. This effectively makes firms vary their markups to these markets accordingly. This pricing-to-market leads to larger price dispersion across locations as in Alessandria and Kaboski (2011) or Alessandria (2009).[12] In this sense, if wages are greatly dispersed across cities in a particular region, the prices of a good will be widely dispersed as well.

Based on the above, our prior is that adjustment of price differentials will be lower the larger income differentials are. In this sense, β_w^{out} in equation (3) is expected to be positive. This is exactly what we see in Table 8. Thus, wage differentials are negatively associated with the speed of price adjustment, implying a potential role for consumers' search intensity and firms' pricing-to-market interacting to produce persistent price deviations. In the case of comparisons between the US and other countries (UO), the interaction terms show that this effect is stronger for city pairs that are farther apart. This implies that markets are more segmented for city pairs that are farther apart (i.e., $\beta_{(w*dist)}^{out} > 0$) the larger income differentials are. This becomes evident as we move from within country comparisons (UU) or US-Europe comparisons (UE) to UO. Evidently in Table 8, the effects of wage differentials are all positive once we incorporate the interaction terms. In the case of UO price comparisons, M4 predicts an average β_w^{out} and $\beta_{(w*dist)}^{out}$ of -0.014 and 0.005 respectively, indicating that the effect of wage differentials for two cities 2,000 kilometers apart will be -0.014+0.005×(log(2000))=0.024.

---

[12] In Alessandria and Kaboski (2011), costly consumer search makes local wages matter for the price-setting behavior of firms, as these endogenize the fact that consumers in low-income countries have a comparative advantage in producing search activities which makes them more price elastic than consumers in high-income countries. Alessandria (2009) builds a model where international price dispersion arises in the presence of costly search that leads firms to price-to-market based on the opportunity cost of their customers' time that in turn depends on local wages so that the distribution of prices differs across international locations.

4. Conclusion

We consider a structural threshold auto-regressive model to estimate product-level price convergence rates that do not suffer from misspecification and omitted variables bias present in previous work. Using a detailed dataset of retail prices, we estimate convergence rates both within and outside the no trade band for individual goods and services within and across countries. Accounting for the role of theoretically implied variables in the convergence process, we find that good-level convergence rates are systematically faster as compared to those implied by estimating reduced-form models. The heterogeneity of these convergence rates across product items relates strongly to their economic characteristics. Individual rates of convergence are positively associated with tradability but negatively related to the nontraded input share both outside and inside the bands, while goods that are more perishable exhibit faster reversion than non-perishables.

Furthermore, consistent with conventional wisdom, implied trade costs dramatically increase as we move from within country comparisons to comparisons across countries. Inconsistent with our priors, services have somewhat comparable trade costs to tradable goods.

In addition, we estimate in our context that non-linearities in the form of interactions between the traded and non-traded channels play a role in the convergence process of price differences across the world. Accounting for the interaction between wages and physical distance, we typically find that good-level convergence rates are somewhat slower than in the case where this form of non-linearity is ignored but still faster than estimates from reduced-form TAR models. Moreover, wage differentials are negatively associated with the speed of price adjustment, which suggests a role for consumers' search intensity and firms' pricing-to-market interacting to produce persistent price deviations, along the lines of Alessandria and Kaboski (2011) where costly consumer search makes local wages matter for the price-setting behavior of firms.

Appendix A. Derivation of Equation(1)

We consider a world economy consisting of two countries indexed by i and j. The two countries are assumed to be populated by a large and equal number of consumers with identical preferences and their financial markets are perfectly integrated and complete. Consumers in each country maximize the expected value of lifetime utility given by:

$$U = \sum_{t=0}^{\infty} \beta^t \frac{\left[\left(C_t^T\right)^{\alpha} \left(C_t^N\right)^{1-\alpha}\right]^{1-\gamma}}{1-\gamma} \tag{4}$$

where C_t^T (C_t^N) is the consumption of the tradable (nontradable) good, α is the expenditure share of the tradable good, β is the discount factor, and γ is the inverse of the intertemporal elasticity of substitution. In each country, both tradable and nontradable goods are assumed to be produced using labor as an input, according to the following linear technology, $Y_t = A_t L_t$ where A_t, stands for labor productivity. Producers in our economy behave competitively. Their profit maximization problem gives $A_t = W_t$ where W_t denotes the real wage. We assume that the labor market is integrated within countries so that labor costs are the same across the tradable and nontradable sectors within a country, i.e., $W_{i,t}^T = W_{i,t}^N = W_{i,t}$ and $W_{j,t}^T = W_{j,t}^N = W_{j,t}$.

We introduce goods market frictions of the iceberg type by assuming that only a fraction of the of the tradable good shipped actually arrives. We make three assumptions about these frictions. First, bilateral trade costs, denoted by τ, are associated with bringing the tradable good from its point of loading abroad to the point of unloading in the importing country. In this setting, when one unit is shipped, only $\frac{1}{1+\tau}$ units actually arrive. Second, local costs are heterogeneous across countries. Precisely, η_i and η_j represent local costs that are entangled in the movement of tradable good from its point of production or unloading to the point of retailers in the country i and j respectively. Third, as in Burstein et al. (2003), consumption of nontradable goods does not require local costs. The resource constraints faced by each country are then given by:

$$C_{i,t}^{T} = \frac{Y_{i,t}^{T} - X_{i,t}}{1+\eta_i} + \frac{X_{j,t}}{(1+\tau)(1+\eta_i)} \tag{5}$$

$$C_{j,t}^{T} = \frac{Y_{j,t}^{T} - X_{j,t}}{1+\eta_j} + \frac{X_{i,t}}{(1+\tau)(1+\eta_j)} \tag{6}$$

$$0 \leq X_{i,t} \leq Y_{i,t}^{T} \tag{7}$$

$$0 \leq X_{j,t} \leq Y_{j,t}^{T} \tag{8}$$

$$0 \leq Y_{i,t}^{N} \tag{9}$$

$$0 \leq Y_{j,t}^{N} \tag{10}$$

where $X_{i,t}$ is the amount of exports from country i measured before trade and distribution costs, while $\frac{X_{j,t}}{(1+\tau)((1+\eta_i)}$ is the amount of imports from country j measured after trade and distribution costs. The appearance of the cost factors in the denominator is the essence of the iceberg cost assumption: a proportion of the shipped traded good is lost before this arrives at the importing destination.

As in Crucini et al. (2005), retailers combine tradable goods with non-tradable services using a Cobb-Douglas function to place the retail goods in outlets which yields the following expression for the retail price:

$$P_{i,t} = \left(P_{i,t}^{T}\right)^{\delta} \left(P_{i,t}^{N}\right)^{1-\delta} \tag{11}$$

$$P_{j,t} = \left(P_{j,t}^{T}\right)^{\delta} \left(P_{j,t}^{N}\right)^{1-\delta} \tag{12}$$

Assuming that financial markets are frictionless and complete, the model is solved as a central planner problem whose objective is to maximize aggregate utility by choosing the amount of trade:

$$\underset{\{X_{i,t},X_{j,t}\}}{Max} U\left(C_{i,t}^{T}, C_{i,t}^{N}\right) + U\left(C_{j,t}^{T}, C_{j,t}^{N}\right) \tag{13}$$

subject to constraints (5)-(10)

When financial markets are complete, the ratio of marginal utility of consumption between countries is linked to international relative prices. From a standard Lagrangian problem of a central planner, the (logarithm of) international relative price of retail goods defined by a ratio of the retail price of country j to the retail price of country i, is then given by:

$$q_{i,j,t} = \begin{cases} \theta_1(\eta_{i,j} + \tau) - \theta_2 w_{i,j,t} & \text{if } k > \eta_{i,j} + \tau \quad : \text{Country } i \text{ exports} \\ k = \gamma(\eta_{i,j} + w_{i,j,t}) & \textit{otherwise} \quad : \text{No trade} \\ \theta_1(\eta_{i,j} - \tau) - \theta_2 w_{i,i,t} & \text{if } k < \eta_{i,j} - \tau \quad : \text{Country } j \text{ exports} \end{cases} \tag{14}$$

where $\eta_{i,j} = \eta_j - \eta_i$, $w_{i,j} = w_{j,t} - w_{i,t}$, $\theta_1 = \frac{\gamma\alpha - \alpha + \delta}{1 - \alpha + \alpha\gamma}$ and $\theta_2 = \delta(\alpha + \gamma - \alpha\gamma)$. All lowercase letters denote logarithms of the corresponding variables. Equation (14) shows that trade and distribution costs along with wage differentials determine the band of inaction around which trade patterns and resulting international relative prices are characterized. When gains from trade are sufficiently large to cover goods' market frictions, arbitrage takes place and the price in the importing country is higher than in the exporting country by the weighted average of goods' market frictions and wage differences. Even in the absence of distribution costs (i.e., $\eta_j = \eta_i = \tau = 0$), wage differences will still drive a natural wedge between prices across locations. As a result of distribution costs, international relative prices do not move in tandem with wage differences within the band. In the extreme case where all market frictions are eliminated and labor markets perfectly integrated (i.e., $w_{i,t} = w_{j,t}$), the central planner sets the optimal relative consumption equal to unity and corrects any deviations from unity by re-allocating goods, so that international relative prices are equal to unity and the LOP unambiguously holds.

Subtracting $q_{i,j,t-1}$ from both sides of equation (14) and rearranging yields:

$$\Delta q_{i,j,t} = \begin{cases} -(q_{i,j,t-1} - (\eta_{i,j} + \tau)) + (\theta_1 - 1)(\eta_{i,j} + \tau) - \theta_2 w_{i,j,t} & \text{if } q_{i,j,t-1} > \eta_{i,j} + \tau \\ -q_{i,j,t-1} + \gamma\eta_{i,j} + \gamma w_{i,j,t} & \text{otherwise} \\ -(q_{i,j,t-1} - (\eta_{i,j} - \tau)) + (\theta_1 - 1)(\eta_{i,j} - \tau) - \theta_2 w_{i,j,t} & \text{if } q_{i,j,t-1} < \eta_{i,j} - \tau \end{cases}$$

i.e. the model (1) in the text.

Appendix B. Econometric models and estimation

Restrictions on parameters

Since $q_{i,j,t} = -q_{j,i,t}$, the equation (2) can be written as

$$-\Delta q_{j,i,t} = \begin{cases} -\lambda_1^{out,u}(q_{j,i,t-1} + a^u_{i,j}) + \beta^u_a a^u_{i,j} + \beta_w^{out,u} w_{i,j,t} + e^{out}_{i,j,t} & \text{if } -q_{j,i,t-1} > a^u_{i,j} \\ -\lambda_1^{in} q_{j,i,t-1} + \beta_1 \eta_{i,j} + \beta_w^{in} w_{j,i,t} + e^{in}_{i,j,t} & \text{otherwise} \\ -\lambda_1^{out,l}(q_{j,i,t-1} + a^l_{i,j}) + \beta^l_a a^l_{i,j} + \beta_w^{out,l} w_{j,i,t} + e^{out}_{j,i,t} & \text{if } -q_{j,i,t-1} < a^l_{i,j} \end{cases}$$

or, equivalently,

$$\Delta q_{j,i,t} = \begin{cases} \lambda_1^{out,u}(q_{j,i,t-1} - (-a_{i,j}^u)) + \beta_a^u(-a_{i,j}^u) + \beta_w^{out,u}(-w_{i,j,t}) - e_{i,j,t}^{out} & \text{if } q_{j,i,t-1} < -a_{i,j}^u \\ \lambda_1^{in} q_{j,i,t-1} - \beta_1 \eta_{i,j} + \beta_w^{in}(-w_{i,j,t}) - e_{i,j,t}^{in} & \text{otherwise} \\ \lambda_1^{out,l}(q_{j,i,t-1} - (-a_{i,j}^l)) + \beta_a^l(-a_{i,j}^l) + \beta_w^{out,l}(-w_{i,j,t}) - e_{i,j,t}^{out} & \text{if } q_{j,i,t-1} > -a_{i,j}^l. \end{cases}$$

In addition, by observing $\eta_{i,j} = -\eta_{j,i}$, $a_{i,j}^u = \eta_{i,j} + \tau = -(\eta_{j,i} - \tau) = -a_{j,i}^l$ and, similarly, $a_{i,j}^l = -a_{j,i}^u$, and $w_{i,j,t} = w_{j,t} - w_{i,t} = -(w_{i,t} - w_{j,t}) = -w_{j,i,t}$, we can see the equation (2) is equivalent to the following equation (15).

$$\Delta q_{j,i,t} = \begin{cases} \lambda_1^{out,u}(q_{j,i,t-1} - a_{j,i}^l) + \beta_a^u a_{j,i}^l + \beta_w^{out,u} w_{j,i,t} + e_{j,i,t}^{out} & \text{if } q_{j,i,t-1} < a_{j,i}^l \\ \lambda_1^{in} q_{j,i,t-1} + \beta_1 \eta_{j,i} + \beta_w^{in} w_{j,i,t} + e_{j,i,t}^{in} & \text{otherwise} \\ \lambda_1^{out,l}(q_{j,i,t-1} - a_{j,i}^u) + \beta_a^l a_{j,i}^u + \beta_w^{out,l} w_{j,i,t} + e_{j,i,t}^{out} & \text{if } q_{j,i,t-1} > a_{j,i}^u \end{cases} \tag{15}$$

Here, we implicitly assumed $e_{i,j,t}^{out} = -e_{j,i,t}^{out}$ and $e_{i,j,t}^{in} = -e_{j,i,t}^{in}$, which can be justified when we assume $e_{i,j,t}^{out} = e_{j,t}^{out} - e_{i,t}^{out}$ and $e_{i,j,t}^{in} = e_{j,t}^{in} - e_{i,t}^{in}$.

Since indices i and j are nominal, the equation (15) must hold when we call i as j and j as i i.e.

$$\Delta q_{i,i,t} = \begin{cases} \lambda_1^{out,u}(q_{i,j,t-1} - a_{i,j}^l) + \beta_a^u a_{i,j}^l + \beta_w^{out,u} w_{i,j,t} + e_{i,j,t}^{out} & \text{if } q_{i,j,t-1} < a_{i,j}^l \\ \lambda_1^{in} q_{i,j,t-1} + \beta_1 \eta_{i,j} + \beta_w^{in} w_{i,j,t} + e_{i,j,t}^{in} & \text{otherwise} \\ \lambda_1^{out,l}(q_{i,j,t-1} - a_{i,j}^u) + \beta_a^l a_{i,j}^u + \beta_w^{out,l} w_{i,j,t} + e_{i,j,t}^{out} & \text{if } q_{i,j,t-1} > a_{i,j}^u \end{cases} \tag{16}$$

Comparing equations (2) and (16), we can see the following restrictions must hold:

[**Symmetric adjustment speeds**] $\lambda^{out,l} = \lambda^{out,u}$;

[**Reciprocity of threshold functions**] $a_{i,j}^u = -a_{j,i}^l$, $a_{i,j}^l = -a_{j,i}^u$;

[**Ordering of threshold functions**] $a_{i,j,g}^u \geq a_{i,j,g}^l$;

[**symmetric coefficients**] $\beta_a^{out,u} = \beta_a^{out,l}$ and $\beta_w^{out,u} = \beta_w^{out,l}$.

The variables $w_{i,j,t}$ have a property that $w_{i,j,t} = -w_{j,i,t}$. If we are to include a new variable $z_{i,j,t}$ in the model that has a property $z_{i,j,t} = z_{j,i,t}$ such as a constant, then the coefficients of the variable, namely $\beta_z^{out,u}$, $\beta_z^{out,l}$, and/or β_z^{in} similar to $\beta_w^{out,u}$, $\beta_w^{out,l}$, and/or β_w^{in} in (16), must satisfy the following restrictions:

[**Negative symmetric coefficients**] $\beta_z^{out,u} = -\beta_z^{out,l}$ and $\beta_z^{in} = 0$.

Applying these restrictions, have the model (3) in the section 2.1.

Econometric models and estimation

Models we estimated are (M0) $\sim$ (M4) as described in the section 2.1. We estimated all five models per each good category. The good index is omitted for notational simplicity. The definitions and notation of this subsection is based on (M4). The $\mathbf{x}_{i,j,t}^{out,u}(\theta)$, $\mathbf{x}_{i,j,t}^{out,l}(\theta)$, $\mathbf{x}_{i,j,t}^{in}(\theta)$, β^{out}, and β^{in} below may be defined appropriately for other models but change of definitions should be straightforward. Let us define

$$\mathbf{x}_{i,j,t}^{out,u}(\theta) = \begin{bmatrix} q_{i,j,t-1} - \eta_{i,j} - \delta_0 - \delta_1 \ln(dist_{i,j}) \\ q_{i,j,t-2} \\ \eta_{i,j} + \delta_0 + \delta_1 \ln(dist_{i,j}) \\ 1 \\ w_{i,j,t} \\ w_{i,j,t} \ln(dist_{i,j}) \end{bmatrix},$$

$$\mathbf{x}_{i,j,t}^{out,l}(\theta) = \begin{bmatrix} q_{i,j,t-1} - \eta_{i,j} + \delta_0 + \delta_1 \ln(dist_{i,j}) \\ q_{i,j,t-2} \\ \eta_{i,j} - \delta_0 - \delta_1 \ln(dist_{i,j}) \\ -1 \\ w_{i,j,t} \\ w_{i,j,t} \ln(dist_{i,j}) \end{bmatrix},$$

and $\mathbf{x}_{i,j,t}^{in}(\theta) = \begin{bmatrix} q_{i,j,t-1} \\ q_{i,j,t-2} \\ \eta_{i,j} \\ w_{i,j,t} \\ w_{i,j,t} \ln(dist_{i,j}) \end{bmatrix}$. The θ is a coefficient vector $\begin{bmatrix} \eta_U \\ \eta_E \\ \eta_O \\ \delta_0 \\ \delta \end{bmatrix}$ and $\mathbf{x}_{i,j,t}^{out,u}(\theta)$, $\mathbf{x}_{i,j,t}^{out,l}(\theta)$, and $\mathbf{x}_{i,j,t}^{in}(\theta)$ imply that they are functions of θ. Now let us define $\beta^{out} = \begin{bmatrix} \lambda_1^{out} \\ \lambda_2^{out} \\ \beta_a^{out} \\ \beta_0^{out} \\ \beta_w^{out} \\ \beta_{w,z}^{out} \end{bmatrix}$, and $\beta^{in} = \begin{bmatrix} \lambda_1^{in} \\ \lambda_2^{in} \\ \beta_\eta^{in} \\ \beta_w^{in} \\ \beta_{w,d}^{in} \end{bmatrix}$. Then the model (3) is written as follows.

$$\Delta q_{i,j,t} = \begin{cases} (\mathbf{x}_{i,j,t}^{out,u}(\theta))'\beta^{out} + e_{i,j,t}^{out} & \text{if } q_{i,j,t-1} \geq a_{i,j}^u \\ (\mathbf{x}_{i,j,t}^{in}(\theta))'\beta^{in} + e_{i,j,t}^{in} & \text{if } a_{i,j}^l < q_{i,j,t-1} < a_{i,j}^u \\ (\mathbf{x}_{i,j,t}^{out,l}(\theta))'\beta^{out} + e_{i,j,t}^{out} & \text{if } q_{i,j,t-1} \leq a_{i,j}^l \end{cases} \tag{17}$$

To simplify the notation further, let us define

$$\mathbf{x}^{out}_{i,j,t}(\theta) = 1\{q_{i,j,t-1} \geq a^u_{i,j}\} \times \mathbf{x}^{out,u}_{i,j,t}(\theta) + 1\{q_{i,j,t-1} \leq a^l_{i,j}\} \times \mathbf{x}^{out,l}_{i,j,t}(\theta).$$

Then the econometric model becomes

$$\Delta q_{i,j,t} = \begin{cases} (\mathbf{x}^{out}_{i,j,t}(\theta))'\beta^{out} + e^{out}_{i,j,t} & \text{if } 1\{q_{i,j,t-1} \notin (a^l_{i,j}, a^u_{i,j})\} = 1, \\ (\mathbf{x}^{in}_{i,j,t}(\theta))'\beta^{in} + e^{in}_{i,j,t} & \text{if } 1\{q_{i,j,t-1} \in (a^l_{i,j}, a^u_{i,j})\} = 1. \end{cases} \tag{18}$$

Assuming $\begin{bmatrix} e^{out}_{i,j,t} \\ e^{in}_{i,j,t} \end{bmatrix}$ is purely idiosyncratic and follows $N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2_{out} & 0 \\ 0 & \sigma^2_{in} \end{bmatrix}\right)$, and if the true θ, namely θ^*, is known, for given $\mathbf{x}^{out}_{i,j,t}(\theta^*)$ and $\mathbf{x}^{in}_{i,j,t}(\theta^*)$, the likelihood contribution of $(q_{i,j,t}, (\mathbf{x}^{out}_{i,j,t}(\theta^*))', (\mathbf{x}^{in}_{i,j,t}(\theta^*))')$ is

$$\left[\frac{1}{\sigma_{out}}\phi\left(\frac{\Delta q_{i,j,t} - (\mathbf{x}^{out}_{i,j,t}(\theta^*))'\beta^{out}}{\sigma_{out}}\right)\right]^{1\{q_{i,j,t-1} \notin (a^l_{i,j}, a^u_{i,j})\}}$$
$$\left[\frac{1}{\sigma_{in}}\phi\left(\frac{\Delta q_{i,j,t} - (\mathbf{x}^{in}_{i,j,t}(\theta^*))'\beta^{in}}{\sigma_{in}}\right)\right]^{1\{q_{i,j,t-1} \in (a^l_{i,j}, a^u_{i,j})\}},$$

where $\phi(\cdot)$ is the pdf of a standard normal distribution. Therefore we can estimate $\beta(\theta^*) = \begin{bmatrix} \beta^{out}(\theta^*) \\ \beta^{in}(\theta^*) \end{bmatrix}$ by maximizing the following loglikelihood function.

$$\begin{aligned} & L(\beta^{out}, \beta^{in}; \theta^*) \\ = & -\frac{1}{2} \sum_{(i,j,t): q_{i,j,t-1} \notin (a^l_{i,j}, a^u_{i,j})} \left(\ln(\sigma^2_{out}) + \frac{(\Delta q_{i,j,t} - (\mathbf{x}^{out}_{i,j,t}(\theta^*))'\beta^{out})^2}{\sigma^2_{out}}\right) \\ & -\frac{1}{2} \sum_{(i,j,t): q_{i,j,t-1} \in (a^l_{i,j}, a^u_{i,j})} \left(\ln(\sigma^2_{in}) + \frac{(\Delta q_{i,j,t} - (\mathbf{x}^{in}_{i,j,t}(\theta^*))'\beta^{in})^2}{\sigma^2_{in}}\right). \end{aligned} \tag{19}$$

When θ^* is known, the maximization is straightforward. We can simply partition data $\{(\Delta q_{i,j,t}, \mathbf{x}^{out}_{i,j,t}, \mathbf{x}^{in}_{i,j,t}), \text{for } i, j, t\}$ into two: one such that $q_{i,j,t-1} \notin (a^l_{i,j}, a^u_{i,j})$ (namely, partition 'out') and the other that $q_{i,j,t-1} \in (a^l_{i,j}, a^u_{i,j})$ (namely, partition 'in') and we do the Gaussian MLE for each of 'out' and 'in' partition. However, since the θ^* is unknown and is a part of parameters to be estimated, we maximized concentrated loglikelihood function as follows.

(Step 1) Choose a reasonable θ and partition the sample into 'out' and 'in' with the chosen θ.
(Step 2) Do Gaussian MLE with the partitioned samples and obtain $\hat{\beta}^{out}(\theta)$ and $\hat{\beta}^{in}(\theta)$. Let the loglikelihood function value at θ, $L(\hat{\beta}^{out}(\theta), \hat{\beta}^{in}(\theta); \theta)$, be $L(\theta)$.
(Step 3) Try various θ and find $\hat{\theta}$ which gives the greatest $L(\hat{\theta})$.
(Step 4) The $\hat{\theta}$, $\hat{\beta}^{out}(\hat{\theta})$, and $\hat{\beta}^{in}(\hat{\theta})$ are the ML estimators.

Since the [Step 3] involves a search of parameter values in a subspace of the 5-dimensional Euclidean space, a simple grid search takes a lot of time to find a maximizer. We used a rather complicated algorithm to speed up the search. Our codes and detailed information of our algorithm are available upon request.

Bibliography

[1] Alessandria, G., and J. Kaboski, 2011, Pricing-to-Market and the Failure of Absolute PPP, *American Economic Journal:Macroeconomics* 3(1), 91-127.

[2] Alessandria, G., 2009, Consumer Search, Price Dispersion, and International Relative Price Fluctuations, *International Economic Review* 50(3),803-829.

[3] Andrade, P., and M. Zachariadis, 2016, Global Versus Local Shocks in Micro Price Dynamics, *Journal of International Economics* 98, 78-92.

[4] Bergin, R. Paul, Reuven Glick and Jyh-Lin Wu (2013) "The Micro-Macro Disconnect of Purchasing Power Parity" *Review of Economics and Statistics*, 95(3):798-812.

[5] Burstein, A., J. Nerves, and S. Rebelo, 2003, Distribution Costs and Real Exchange Rate during Exchange-Rate-Based Stabilizations, *Journal of Monetary Economics* 50, 1189-1214.

[6] Crucini, M., C. Telmer, and M. Zachariadis, 2005, Understanding European Real Exchange Rates, *American Economic Review* 95, 724-738.

[7] Dixit, A., 1989, Hysteresis, Import Penetration and Exhange Rate Pass-Through, *Quarterly Journal of Economics* 104, 205-28.

[8] Dumas, B., 1992, Dynamic Equilibrium and the Real Exchange Rate in a Spatially Separated World, *The Review of Financial Studies* 5, 153-180.

[9] Glushenkova, M., A. Kourtellos and M. Zachariadis, 2018, “Barriers to Price Convergence”, unpublished manuscript, University of Cyprus.

[10] Krugman, P., 1989, *Exhange Rate Instability* (MIT: Cambridge).

[11] Sercu, P., and R. Uppal, 2003, Exchange Rate Volatility and International Trade: A General Equilibrium Analysis, *European Economic Review* 47, 429-441.

[12] Sercu, P., R. Uppal, and C. Van Hulle, 1995, The Exchange Rate in the Presence of Transaction Costs: Implications for Tests of Purchasing Power Parity, *The Journal of Finance* 50, 1309-1319.

[13] Zachariadis, Marios, 2012, Immigration and International Prices, 2012, *Journal of International Economics* 87:2 298-311.

Table 1. Convergence speed of LOP deviations: Outside the band

		λ_1^{out}					λ_2^{out}	
		M0	M1	M2	M3	M4	M3	M4
Goods	UU	-0.323	-0.438	-0.328	-0.492	-0.395	0.040	0.032
		(1.78)	(1.20)	(1.74)	(1.02)	(1.38)		
	UE	-0.095	-0.239	-0.207	-0.285	-0.268	0.103	0.087
		(6.94)	(2.54)	(2.99)	(2.07)	(2.22)		
	UO	-0.146	-0.208	-0.224	-0.233	-0.237	0.100	0.092
		(4.39)	(2.97)	(2.73)	(2.61)	(2.56)		
Services	UU	-0.209	-0.360	-0.208	-0.228	-0.083	-0.184	-0.146
		(2.96)	(1.55)	(2.97)	(2.68)	(8.00)		
	UE	-0.079	-0.119	-0.109	-0.065	-0.075	-0.013	-0.017
		(8.42)	(5.47)	(6.01)	(10.31)	(8.89)		
	UO	-0.102	-0.151	-0.171	-0.070	-0.063	-0.012	-0.017
		(6.44)	(4.23)	(3.70)	(9.55)	(10.65)		

Notes: We report the mean (averaged across goods) speed at which price differentials between markets revert back to the band once they cross the thresholds, and corresponding half-lives in parentheses below these. M0 stands for the reduced form TAR model. M1 stands for our benchmark structural TAR model. M2 adds interaction effects to M1. M3 is the AR2 version of M1. M4 is the AR2 version of M2. UU signifies comparisons of prices within the US. UE signifies comparisons of prices between the US and European Union countries. UO signifies comparisons of prices between the US and other countries.

Table 2. Convergence speed of LOP deviations: Inside the band

		λ_1^{in}					λ_2^{in}	
		M0	M1	M2	M3	M4	M3	M4
Goods	UU	-0.527	-0.133	-0.148	-0.166	-0.177	0.042	0.052
		(0.93)	(4.86)	(4.33)	(3.82)	(3.56)		
	UE	-0.129	-0.139	-0.170	-0.205	-0.220	0.068	0.072
		(5.02)	(4.63)	(3.72)	(3.02)	(2.79)		
	UO	-0.141	-0.189	-0.183	-0.224	-0.230	0.090	0.094
		(4.56)	(3.31)	(3.43)	(2.73)	(2.65)		
Services	UU	1.906	-0.082	-0.062	-0.008	-0.004	-0.055	-0.058
			(8.10)	(10.83)	(86.30)	(172.94)		
	UE	-0.077	-0.048	-0.089	-0.025	-0.029	-0.037	-0.036
		(8.65)	(14.09)	(7.44)	(27.38)	(23.55)		
	UO	-0.074	-0.112	-0.123	-0.117	-0.138	0.010	0.009
		(9.02)	(5.84)	(5.28)	(5.57)	(4.67)		

Notes: We report the mean (averaged across goods) speed at which price differentials between markets revert back to the band once they cross the thresholds.M1 stands for our benchmark structural TAR model. M2 adds interaction effects to M1. M3 is the AR2 version of M1. M4 is the AR2 version of M2. UU signifies comparisons of prices within the US. UE signifies comparisons of prices between the US and European Union countries. UO signifies comparisons of prices between the US and other countries.

Table 3 Tradability and non-traded input shares

	Tradability	Non-traded input share
Goods		
All	0.718	0.297
- perishable	0.443	0.283
- non-perishable	0.844	0.303
Services	0	0.666

Table 4. Correlations statistics

	UU	UE	UO	Average
Corr(λ_1^{out}, tradability)	0.422	0.415	0.238	0.358
Corr(λ_1^{out}, nontraded input)	-0.807	-0.854	-0.747	-0.803
Corr(λ_1^{in}, tradability)	0.605	0.652	0.405	0.554
Corr(λ_1^{in}, nontraded input)	-0.844	-0.903	-0.814	-0.854

Notes: We report Pearson correlation coefficients computed using weighted means, weighted variances and weighted covariance. UU signifies comparisons of prices within the US. UE signifies comparisons of prices between the US and European Union countries. UO signifies comparisons of prices between the US and other countries.

Table 5. λ_1^{out} by type of tradeable goods

		M1	M2	M3	M4
Perishability of goods					
UU	Perishable	-0.512	-0.337	-0.534	-0.450
	Non-perishable	-0.404	-0.324	-0.473	-0.370
UE	Perishable	-0.291	-0.280	-0.380	-0.356
	Non-perishable	-0.215	-0.174	-0.241	-0.228
UO	Perishable	-0.280	-0.274	-0.372	-0.356
	Non-perishable	-0.175	-0.200	-0.169	-0.183
Type of outlet					
UU	Supermarket/Chain store	-0.464	-0.359	-0.541	-0.445
	Mid-priced/Branded store	-0.386	-0.339	-0.482	-0.344
UE	Supermarket/Chain store	-0.255	-0.228	-0.312	-0.294
	Mid-priced/Branded store	-0.246	-0.225	-0.262	-0.255
UO	Supermarket/Chain store	-0.203	-0.226	-0.255	-0.263
	Mid-priced/Branded store	-0.196	-0.205	-0.214	-0.217

Notes: We report the mean (averaged across goods) speed at which price differentials between markets converge within the band. UU signifies comparisons of prices within the US. UE signifies comparisons of prices between the US and European Union countries. UO signifies comparisons of prices between the US and other countries.

Table 6. Averages of δ_0

		M0	M1	M2	M3	M4
Goods						
All	UU	0.560	0.354	0.120	0.356	0.110
- perishable		0.668	0.413	0.141	0.436	0.104
- non-perishable		0.510	0.327	0.110	0.320	0.113
All	UE	0.614	2.492	2.776	2.620	2.362
- perishable		0.709	2.532	2.551	2.851	2.855
- non-perishable		0.571	2.474	2.878	2.514	2.136
All	UO	0.635	2.712	2.558	2.471	2.957
- perishable		0.669	2.761	2.817	2.114	3.010
- non-perishable		0.620	2.690	2.440	2.633	2.933
Services	UU	0.726	0.443	0.086	0.398	0.098
	UE	0.875	1.826	1.835	1.909	2.086
	UO	1.053	3.039	2.963	2.964	2.499

Notes: We report averages of non-distance-related trade costs coefficients across goods. M0 stands for the reduced form TAR model. M1 stands for our benchmark structural TAR model. M2 adds interaction effects to M1. M3 is the AR2 version of M1. M4 is the AR2 version of M2. UU signifies comparisons of prices within the US. UE signifies comparisons of prices between the US and European Union countries. UO signifies comparisons of prices between the US and other countries.

Table 7. Averages of δ_{dist}

		M0	M1	M2	M3	M4
Goods						
All	UU	-	0.047	0.052	0.045	0.049
- perishable			0.055	0.061	0.053	0.054
- non-perishable			0.043	0.048	0.041	0.046
All	UE	-	0.487	0.492	0.451	0.461
- perishable			0.431	0.442	0.394	0.444
- non-perishable			0.512	0.515	0.477	0.469
All	UO	-	0.373	0.401	0.363	0.408
- perishable			0.397	0.390	0.370	0.411
- non-perishable			0.362	0.406	0.359	0.407
Services	UU	-	0.069	0.062	0.073	0.056
	UE	-	0.478	0.526	0.471	0.497
	UO	-	0.298	0.309	0.329	0.372

Notes: We report averages of distance-related trade costs coefficients across goods. M0 stands for the reduced form TAR model. M1 stands for our benchmark structural TAR model. M2 adds interaction effects to M1. M3 is the AR2 version of M1. M4 is the AR2 version of M2. UU signifies comparisons of prices within the US. UE signifies comparisons of prices between the US and European Union countries. UO signifies comparisons of prices between the US and other countries.

Table 8. Averages of β_{w}^{out} and $\beta_{(w*dist)}^{out}$

		β_{w}^{out}					$\beta_{(w*dist)}^{out}$	
Goods		M0	M1	M2	M3	M4	M2	M4
	UU	-	0.039	0.127	0.021	0.084	-0.012	-0.008
	UE	-	0.016	0.019	0.013	0.099	-0.001	-0.012
	UO	-	0.006	-0.013	0.009	-0.014	0.004	0.005
Services	UU	-	0.049	0.022	0.048	0.067	-0.001	-0.006
	UE	-	0.006	0.042	0.007	0.050	-0.005	-0.006
	UO	-	0.017	0.007	0.008	0.079	0.001	-0.013

Notes: We report averages of wage-related coefficients across goods. M0 stands for the reduced form TAR model. M1 stands for our benchmark structural TAR model. M2 adds interaction effects to M1. M3 is the AR2 version of M1. M4 is the AR2 version of M2. UU signifies comparisons of prices within the US. UE signifies comparisons of prices between the US and European Union countries. UO signifies comparisons of prices between the US and other countries.