Restoring monotone power in the CUSUM test

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Abstract

This paper shows that a near-stationarity boundary condition for heteroskedastic and autocorrelation consistent estimators can solve the problem of non-monotone power of the CUSUM test for a single break in the mean of a weakly dependent process.

JEL classifications: C12;C22.

Keywords: Heteroskedastic and autocorrelation consistent estimator; structural break test.

1 Introduction

The CUSUM test for structural breaks is consistent and has good local asymptotic properties for given fixed values in the relevant set of alternative hypotheses (e.g. Ploberger and Kramer, 1990). However in finite samples, its power function can be non-monotone and even reach a zero value as the alternative considered is further away from the null value. This was shown by Perron (1991) and Vogelsang (1999) for a family of tests. We focus on the CUSUM test but the results presented here apply to other tests (e.g. Vogelsang, 1999).

The non-monotone power is due to the variance estimate which scales the CUSUM statistic. The estimated variance is based on the demeaned observed process or the residuals. In the general case of dependence this is the Heteroskedastic and Autocorrelation Consistent (HAC) estimator. The HAC is evaluated under the null hypothesis which implies that under the alternative it is inflated

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resulting in a small scaled CUSUM and thereby loss of power. There are two related sources of overestimation: (i) The variance is usually based on least-squares or recursive residuals that are contaminated by the shift under the alternative. As the shift gets larger the variance increases. (ii) When there is dependence in the process the power problem is exacerbated because the shift induces a bias of the autoregressive coefficient towards one (Perron, 1989) thereby inflating the variance estimator.

The empirical relevance of the nonmonotone power problem is shown using, for instance, the money market rates of two emerging markets. In Figures 10-11 it is evident that the monthly money market rates in Korea and Thailand in the period from the mid 1970s to 2005 experienced a large shift in the mean (as shown by the vertical time lines in the late 1990s) that are associated with financial reforms in these countries. However, the traditional CUSUM test, reported in the first line of Table 2, fails to detect these large structural breaks due to the aforementioned power problem.

This paper shows that a simple near-stationarity boundary condition for the HAC estimator restores the monotone power of the CUSUM test for a mean shift in a weakly dependent process. This is inspired by Andrews (1991) and Sul, Phillips and Choi (2005). We show that this boundary condition solves the inflated variance problem under the change-point alternative and preserves the consistency of the HAC estimator. Extensive simulations support this method. Moreover, the application to the money market rate series illustrates the empirical relevance of this method in detecting structural changes.

2 HAC estimators for the CUSUM test

Consider the following stochastic process for a univariate time series, y_t :

$$y_t = \mu + u_t,\ t = 1, ..., T, \tag{2.1}$$

where u_t is a second-order stationary mean zero error process. The partial sums process $S_t = \sum_{j=1}^{t} u_j$ satisfies the Functional Central Limit Theorem (FCLT), for regularity conditions found, for instance, in Herrndorf (1984), such that $T^{-1/2}S_{[mT]} \to \sigma W(m)$, where $W(m)$ denotes the standard Wiener process defined on $[0,1]$ and $\sigma^2 = \lim_{T\to\infty} E\left[T^{-1}\left(\sum_{t=1}^{T} u_t\right)^2\right]$.

The CUSUM statistic for detecting structural changes in the mean of y_t in (2.1):

$$CUSUM = \left(\sigma\sqrt{T}\right)^{-1} \sup_{1\le j\le T}\left|\sum_{t=1}^{j} y_t - \sum_{t=1}^{T} y_t\right| \to \sup|B(m)| \tag{2.2}$$

converges to the supremum of a Brownian Bridge, $B(m) = W(m) - mW(1)$. Equivalently (2.2) can be expressed in terms of the OLS residuals $\widehat{u}_t^{OLS} = y_t - 1/T\sum_{t=1}^{T} y_t$:

$$CUSUM = \left(\widehat{\sigma}\sqrt{T}\right)^{-1} \sup_{1\leq j\leq T} \left|\sum_{t=1}^{j} \widehat{u}_t^{OLS}\right| \tag{2.3}$$

where $\widehat{\sigma}$ is a consistent estimator under the null of stability. Traditional estimators of σ^2 include the class of non-parametric spectral density estimators such as $\widehat{\sigma}^2 = \sum_{j=-(T-1)}^{T-1} K\left(j/s(T)\right)\widehat{\gamma}_j$, where $K(.)$ is the kernel function, $\widehat{\gamma}_j = T^{-1}\sum_{t=j+1}^{T} \widehat{u}_t\widehat{u}_{t-j}$, $s(T)$ is the bandwidth and $\widehat{\sigma}^2$ is consistent if $s(T)/T \to 0$ and $s(T) \to \infty$ as $T \to \infty$. Andrews and Monahan (1992) propose the prewhitened estimator $\widehat{\sigma}^2_{PW}$ given by:

$$\widehat{\sigma}^2_{PW} = \widehat{\sigma}^2_{\varepsilon}/(1-\widehat{\rho})^2 \text{ and } \widehat{\rho} = \sum_{t=2}^{T} \widehat{u}_t\widehat{u}_{t-1}/\sum_{t=2}^{T} \widehat{u}^2_{t-1}, \text{ where} \tag{2.4}$$

$$\widehat{\sigma}^2_{\varepsilon} = \sum_{j=-(T-1)}^{T-1} K\left(j/\widehat{s}_{PW}(T)\right)\widehat{\gamma}^{\varepsilon}_j, \ \ \widehat{\gamma}^{\varepsilon}_j = T^{-1}\sum_{t=j+1}^{T} \widehat{\varepsilon}_t\widehat{\varepsilon}_{t-j} \ \text{ and } \ \widehat{\varepsilon}_t = \widehat{u}_t - \widehat{\rho}\widehat{u}_{t-1}. \tag{2.5}$$

The data-dependent bandwidth $\widehat{s}_{PW}(T)$ is based on the AR(1) plug-in method and depends on the parameter $\widehat{\alpha}_{PW}$ given by:

$$\widehat{s}_{PW}(T) = 1.3221(\widehat{\alpha}_{PW}(2)T)^{1/5}, \ \widehat{\alpha}_{PW}(2) = 4\widehat{\rho}^2_{\varepsilon}/(1-\widehat{\rho}_{\varepsilon})^4, \ \widehat{\rho}_{\varepsilon} = \sum_{t=2}^{T} \widehat{\varepsilon}_t\widehat{\varepsilon}_{t-1}/\sum_{t=2}^{T} \widehat{\varepsilon}^2_{t-1}, \tag{2.6}$$

which uses the autoregressive estimate $\widehat{\rho}_{\varepsilon}$ obtained from $\widehat{\varepsilon}_t$ instead of $\widehat{u}_t$. In (2.6) the optimal $\widehat{s}_{PW}(T)$ minimizes the asymptotic truncated MSE for the Quadratic Spectral kernel. For the Bartlett kernel the optimal bandwidth is $\widehat{s}_{PW}(T) = 1.1447(\widehat{\alpha}_{PW}(1)T)^{1/3}$ where $\widehat{\alpha}_{PW}(1) = 4\widehat{\rho}^2_{\varepsilon}/(1-\widehat{\rho}^2_{\varepsilon})^2$.

The recoloring procedure in $\widehat{\sigma}^2_{PW}$ in (2.4) involves $\widehat{\rho}$. Andrews and Monahan (1992) suggest replacing any $\widehat{\rho}$ that exceeds 0.97 by 0.97 and is less than -0.97 by -0.97. Andrews (1991) suggests a boundary condition based on the idea of a confidence interval for $\widehat{\rho}$ which can lead to accurate size of a test and reduce its variance. Using such HAC estimators with or without prewhitening the literature shows that the CUSUM test exhibits non-monotone power. Sul, Phillips and Choi (2005) propose another recoloring rule based on T given by the boundary condition $\widehat{\rho}' = \min[1-1/\sqrt{T}, \widehat{\rho}]$. This represents the maximum allowable value for ρ to be unity minus its asymptotic standard error, $1/\sqrt{T}$. We generalize this boundary to represent deviations from unity by some fixed local coefficient c, in the spirit of the 'stationary order of magnitude' distance from unity in Sul, Phillips and Choi (2005), so that any root preserves near-stationarity and recoloring is based on:

$$\widehat{\rho}'_c = \min[1-c/\sqrt{T}, \widehat{\rho}]. \tag{2.7}$$

The effects of this condition on the finite properties of the CUSUM for given c are evaluated via simulations in the next section. One could consider $c = 1, 1.28, 1.65$ in (2.7) where $c = 1$ corresponds to Sul, Phillips and Choi (2005) and $c = 1.28$ and 1.65 are the one-sided confidence interval values for near-stationarity deviations from $\rho = 1$ (given $c > 0$ such that $\rho < 1$) that correspond to the 10% and 5% standard normal probabilities, respectively. Alternatively, in $\widehat{\rho}_c'$ the equivalent of the 0.97 condition applies for large samples e.g. $T \approx 1000$ when $c = 1$.

In order to show that condition (2.7) yields a consistent variance estimator, consider the process (2.1) where for simplicity we assume that u_t follows an AR(1):

$$u_t = \rho u_{t-1} + \varepsilon_t, \varepsilon_t \sim NIID(0, \sigma_\varepsilon^2). \tag{2.8}$$

The limiting distribution of the CUSUM depends on $\widehat{\rho}$ and $\widehat{\sigma}_\varepsilon^2$ that define the long-run variance of u_t in (2.3), σ_u^2. In the parametric model (2.8) the least squares estimate is $\widehat{\sigma}_u^2 = \widehat{\sigma}_\varepsilon^2/(1-\widehat{\rho})^2$. In the non-parametric setting the estimates of $1/(1-\widehat{\rho})^2$ are used in the final stage of recoloring to obtain the HAC estimator (2.4). Perron (1989) shows that large neglected shifts in (2.8) cause $\widehat{\rho} \to 1$. This implies that $\widehat{\sigma}_u^2$ would be inflated and the power of the test would deteriorate. In particular, when u_t is highly persistent e.g. near unit root, then under the alternative of a large break $\widehat{\rho} = 1 + O_p(T^{-1})$, $(1-\widehat{\rho})^2 = O_p(T^{-2})$ and $\widehat{\sigma}_\varepsilon^2/(1-\widehat{\rho})^2 = O_p(T^2)$. Consequently, $\widehat{\sigma}_u$ would diverge at rate T and would not satisfy $\sqrt{T}$ consistency. On the contrary, the near-stationarity boundary (2.7) controls the order of magnitude of the long-run variance under the alternative and yields $\widehat{\sigma}_u^2 = O_p(T/c^2)$.

Turning now to the prewhitened HAC estimators we explain the effects of a spurious unit root due to neglected breaks. Under the alternative of large shifts u_t is biased towards $I(1)$ whereas ε_t towards $I(0)$. Hence $\widehat{\alpha}_{PW}(2) = O_p(T^2)$ which implies $\widehat{s}_{PW}(T) = O_p(T^{3/5})$ in (2.6). This preserves the consistency of the non-parametric variance $\widehat{\sigma}_\varepsilon^2$ in (2.5) since $\widehat{s}_{PW}(T)/T = O_p(T^{-2/5}) \to 0$ as $T \to \infty$. The problem arises at the recoloring stage since the prewhitened HAC estimator (2.4) involves the spurious unit root, $(1-\widehat{\rho}) \to 0$, that reduces power due to an inflated $\widehat{\sigma}_{PW}^2$. In particular, for a near unit root process a large break would cause $\widehat{\rho} = 1 + O_p(T^{-1})$ and thereby $\widehat{\sigma}_{PW}^2 = O_p(T^2)$ which hurts power. In contrast if we adopt the near-stationarity recoloring rule (2.7) then $(\widehat{\rho}_c' - 1)^2 = O_p(c^2 T^{-1})$, where c controls the divergence of $\widehat{\sigma}_{PW}^2$ and affects the power of the test in finite samples.

In the same vein we show that if we adopt a HAC estimator without prewhitening and condition (2.7) we maintain consistency under the alternative. When there is no prewhitening $\widehat{\alpha}(2) = 4\widehat{\rho}^2/(1-\widehat{\rho})^4$ with $\widehat{\rho}$ in (2.4), $\widehat{s}(T) = 1.3221(\widehat{\alpha}(2)T)^{1/5}$ and $\widehat{\sigma}_u^2 = \sum_{j=-(T-1)}^{T-1} K\left(j/\widehat{s}(T)\right)\widehat{\gamma}_j$. The spurious unit root, $\widehat{\rho} \to 1$, would imply an inflated $\widehat{\alpha}(2)$ and $\widehat{s}(T)$ and thereby $\widehat{\sigma}_u^2$. For instance, if u_t is a near unit root process then under the break alternative $\widehat{\alpha}(2) = O_p(T^4)$ and $\widehat{s}(T) = O_p(T)$ which implies that $\widehat{s}(T)/T = O_p(1)$ and $\widehat{\sigma}_u^2$ is inconsistent.[1] We suggest using (2.7) as the plug-in estimate

[1] Vogelsang (1999) and Crainiceanu and Vogelsang (2006) also show that under the alternative the bandwidth increases at rate T as the size of the break increases.

for $\widehat{\alpha}(2)$. This implies that $\widehat{\alpha}(2) = O_p(T^2)$, $\widehat{s}(T) = O_p(T^{3/5})$ and $\widehat{s}(T)/T = O_p(T^{-2/5}) \to 0$ as $T \to \infty$. Hence the near-stationarity boundary (2.7) controls the divergence of the long-run variance and yields consistent variance estimators when used as the plug-in estimate (in $\widehat{\alpha}(2)$ in (2.6)) for HAC estimators with no prewhitening and when used as a recoloring method for prewhitened HAC estimators (for $\widehat{\rho}$ in (2.4)).

3 Simulation and empirical results

A Monte Carlo analysis is performed to evaluate the effects of the near-stationarity boundary (2.7) on the finite sample power of the CUSUM test and on the properties of the HAC estimators. The Monte Carlo design considers the following Data Generating Process:

$$y_t = \mu + \delta D_t + u_t,\ u_t = \rho u_{t-1} + \varepsilon_t,\ t = 1, ..., T, \tag{3.9}$$

where $\varepsilon_t \sim NIID(0,1)$, $D_t = 0$ for $t < \tau$ and 1 otherwise and $\tau = 0.5T$ is the change-point. The break size $\delta = 1, 2, ..., 12$ represents the alternative hypotheses and $\delta = 0$ for the null hypothesis of stability. In (3.9) we consider $\rho = 0.5, 0.7, 0.9$ and $T = 100, 200$. The $H_0 : \delta = 0$ is examined using the statistic (2.3) for alternative $\widehat{\sigma}^2_{PW}$ in (2.4)-(2.6). For $K(.)$ in (2.5) we use here the Quadratic Spectral (QS) that is optimal in asymptotic truncated MSE terms (Andrews, 1991).[2]

The size of the CUSUM for $\widehat{\sigma}^2_{PW}$ (in Table 1 for the 5% significance level) is the same for the 0.97 boundary and the $\widehat{\rho}'_c$ recoloring boundary in (2.7) for all c values, except when $c = 1.65$ and $\rho = 0.9$ where in (2.7) $1 - c/\sqrt{T}$ becomes binding and yields size closer to the nominal as T increases.[3] The power functions of the CUSUM test are presented in Figures 1-6 for $\rho = 0.5, 0.7, 0.9$ and $T = 100, 200$. Each power curve refers to each of the four recoloring rules, 0.97 and $c = 1, 1.28, 1.65$ in (2.7). Three broad results can be drawn when comparing the size adjusted power functions: (i) As the size of the break δ increases the power functions for *all* $c = 1, 1.28, 1.65$ in (2.7) approach one, irrespective of ρ and T. In contrast, as documented previously in the literature, the power functions for 0.97 recoloring yield power as poor as zero for large δ. (ii) There is still some evidence of non-monotone power for $c = 1$ in (2.7) when $\rho = 0.5, 0.7$ and $T = 100, 200$ (in Figures 1,2,4,5), which disappears when $\rho = 0.9$. Similarly $c = 1.28$ yields non-monotone power functions shown in Figures 1 and 5. Therefore for less persistent processes ($\rho = 0.5, 0.7$) and small samples, $\widehat{\rho}'_c$ requires larger deviations from unity corresponding to $c = 1.65$ (that imposes a lower bound on the recoloring condition (2.7) of the HAC) in order to yield monotone power. (iii) In general,

[2] All computations were carried in GAUSS using the random number generator RNDNS from the GAUSS library. The reported simulation results are based on 2000 Monte Carlo replications.

[3] For the CUSUM test the HAC estimators with prewhitening have better size than those without prewhitening, a result which is consistent e.g. with Andrews and Monahan (1992). Hence we focus our discussion on $\widehat{\sigma}^2_{PW}$.

$c = 1.65$ for the near-stationarity boundary condition (2.7) in $\widehat{\sigma}_{PW}$ yields not only monotone power functions but also relatively better size and power compared to $c = 1$ and 1.28. The finite sample empirical densities of the CUSUM statistics for $H_0 : \delta = 0$ and $H_1 : \delta = 5$ (for $\rho = 0.7, T = 200$ for conciseness) are in Figures 7 and 8, respectively. Figure 8 shows that the CUSUM statistic with $c = 1.65$ in (2.7) outperforms in terms of power.

The finite sample efficiency of $\widehat{\sigma}^2_{PW}$ using the Mean Absolute Error (MAE) as a function of the break size, δ (for $\rho = 0.7$, $T = 200$) is found in Figure 9. The inflated MAEs of $\widehat{\sigma}^2_{PW}$ with the 0.97 recoloring under the alternatives are contrasted to MAEs of $\widehat{\sigma}^2_{PW}$ with $\widehat{\rho}'_c$ and especially with $c = 1.65$ that yields the lowest relative MAE. Similar results apply to other T, ρ and $K(.)$.

Robustness checks can be found in Appendices A-G. The above results are robust to early and late change-points given by $\tau = 0.25T, 0.75T$ (Appendix A), to the Bartlett (BT) kernel with data-dependent bandwidth (Appendix E) and to different initial conditions (given by $y_1 = \varepsilon_1$ and $y_1 = (1-\rho^2)^{-1}\varepsilon_1$) found in Appendix F. For the simulated process (3.9) we consider the following cases for the initial condition, y_1: Case (a): $y_1 \sim N(0,1)$ and Case (b): $y_1 \sim N(0,(1-\rho^2)^{-1})$, which refer to y_1 being fixed and a random variable, respectively (see, for instance, Pantula, Gonzalez-Farias, Fuller (1994)). In the simulated processes in Tables F1-F2 (Appendix F) the same ε_t's were used in both cases except for $y_1 = \varepsilon_1$ (Case (a)) and $y_1 = \varepsilon_1(1-\rho^2)^{-1}$ (Case (b)) (see also Pantula et al., 1994). The simulated size of the test is robust to the above initial values as shown in each panel in Table F1. Similarly, the power of the CUSUM in Table F2 shows that it is robust to the initial values comparing the results in Panels A and B for the two initial values when $T = 100$ and similarly Panels C and D for $T = 200$, for alternative ρ. Moreover, similar power functions are found for $\widehat{\sigma}^2_{PW}$ when $\rho = 0.95$ like the ones for $\rho = 0.9$ for different T (Appendix C). Although non-monotone power is mainly a finite sample problem of the CUSUM, for large T (e.g. $T = 300$ and 500 in Appendix B) all c values in (2.7) yield monotone power whereas the 0.97 boundary leads to relatively more serious non-monotone power when $T = 300$. Last but not least, this method also restores monotone power in the Recursive Least Squares (RLS) CUSUM test. The RLS CUSUM results are reported in Appendix D. Table D1 shows the size of the RLS CUSUM test and the relatively improved size of the test when $c = 1.65$ and $\rho = 0.9$ for the near-stationarity boundary condition (2.7) compared to the other boundaries. The power functions of the RLS CUSUM are very similar to the OLS CUSUM and in general for $c = 1.65$ for the near-stationarity boundary condition (2.7) of $\widehat{\sigma}^2_{PW}$ yields monotone and relatively higher power.

Summarizing, the simulation results show that the near-stationarity boundary (2.7) can restore the monotone power functions of the CUSUM test and improve its size as well as yield HAC estimators which are relatively more efficient especially under the change-point alternative.

This section concludes with an empirical illustration of the above method using the money market rate of two Asian economies that went through a financial liberalization period in the 1990s. The data source is the International Financial Statistics and the samples for Korea and Thailand are 8/1976-2/2005 and 1/1977-5/2005, $T = 343$ and 341 monthly data, respectively. The money market rates (in logs) in Figures 10 and 11 show a large permanent shift in the late 1990s.

The empirical results of the CUSUM test for $\widehat{\sigma}^2_{PW}$ using alternative boundaries and QS and BT kernels are summarized in Table 2. Whilst $\widehat{\sigma}^2_{PW}$ with the 0.97 recoloring fails to detect a break, the boundary $\widehat{\rho}'_c$, for all $c = 1, 1.28, 1.65$, estimates July 1998 and September 1998 as the change-points in the money rates of Korea and Thailand, respectively, at the 5% critical level. Moreover the sequential sample segmentation method (e.g. Bai, 1997) does not provide evidence of multiple change-points. The vertical time lines in Figures 10-11 refer to the estimated break dates that relate to reforms in the money market instruments in Korea and to the banking system in Thailand.

4 Final remarks

Other HAC estimators to overcome the non-monotone power problem of the CUSUM test are proposed in Altissimo and Corradi (2003) where the long-run variance is based on local mean estimates. This method directly deals with the inconsistency of the mean and variance estimates under the alternative. The results here are complementary since an alternative approach is pursued that does not involve local mean estimates but annihilates the divergence of the HAC estimator (caused by $\widehat{\rho} \to 1$ due to the mean shift) via the near-stationarity boundary condition (2.7). This condition also retains the consistency of the HAC under the alternative hypothesis of a single break. At the same time this is a simple procedure that restores the monotone power of the CUSUM test as shown by the simulation and empirical results. Last but not least, a comparison of the finite sample performance of our procedure (using (2.7) when $c = 1.65$) with that in Altissimo and Corradi (2003, Table 5, page 222) shows that the two methods have similar power (for $\delta \geq 1$), whilst our method yields a 5% simulated size.

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Table 1: Size of OLS CUSUM using Quadratic Spectral HAC estimators with prewhitening (QS_PW) and different recoloring boundaries

		QS_PW			
		Recoloring boundaries:			
T	ρ	0.97	(1-1/√T)	(1-1.28/√T)	(1-1.65/√T)
100	0.5	0.025	0.025	0.025	0.025
	0.7	0.012	0.012	0.012	0.012
	0.9	0.001	0.004	0.018	0.086
200	0.5	0.032	0.032	0.032	0.032
	0.7	0.023	0.023	0.023	0.023
	0.9	0.001	0.001	0.003	0.037

Note: The 5% significance level finite sample critical values for T=100 and 200 are 1.27 and 1.30, respectively.

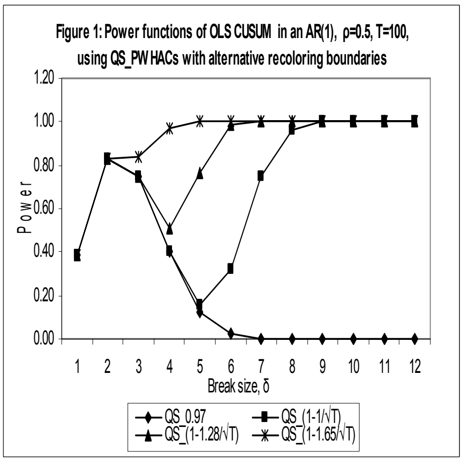


Figure 1: Power functions of OLS CUSUM in an AR(1), ρ=0.5, T=100, using QS_PW HACs with alternative recoloring boundaries

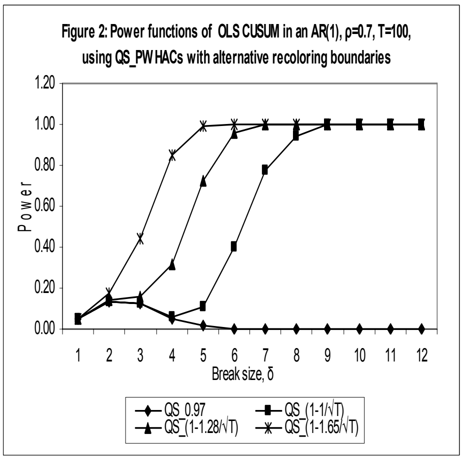


Figure 2: Power functions of OLS CUSUM in an AR(1), ρ=0.7, T=100, using QS_PW HACs with alternative recoloring boundaries

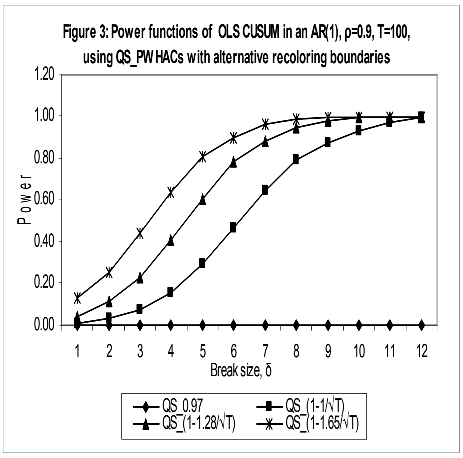


Figure 3: Power functions of OLS CUSUM in an AR(1), ρ=0.9, T=100, using QS_PW HACs with alternative recoloring boundaries

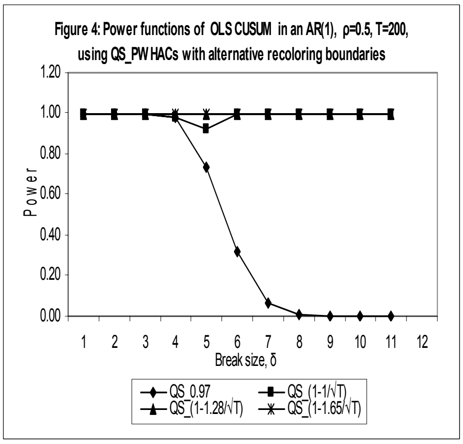


Figure 4: Power functions of OLS CUSUM in an AR(1), ρ=0.5, T=200, using QS_PW HACs with alternative recoloring boundaries

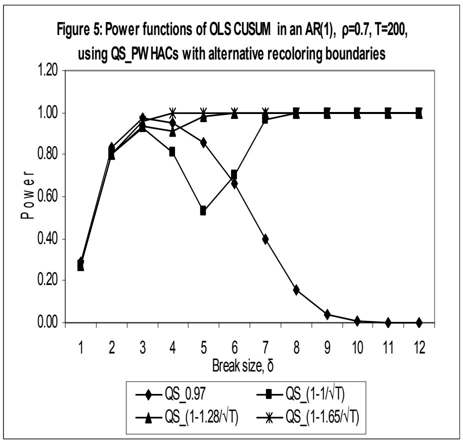


Figure 5: Power functions of OLS CUSUM in an AR(1), ρ=0.7, T=200, using QS_PW HACs with alternative recoloring boundaries

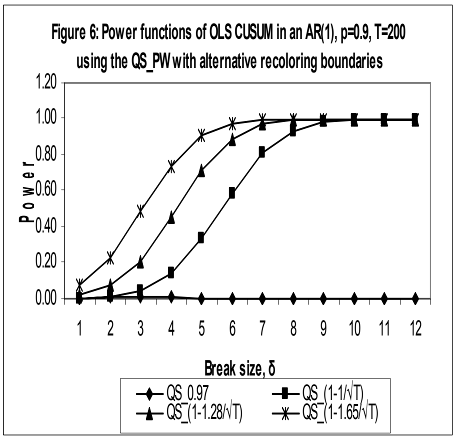


Figure 6: Power functions of OLS CUSUM in an AR(1), ρ=0.9, T=200 using the QS_PW with alternative recoloring boundaries

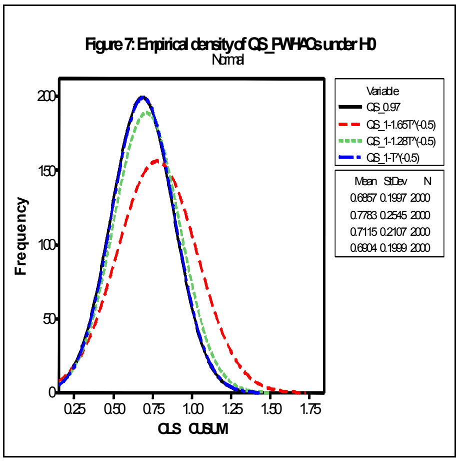


Figure 7: Empirical density of QS_PW HACs under H0

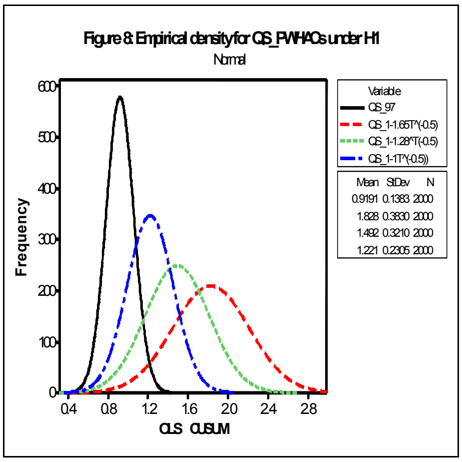


Figure 8: Empirical density for QS_PW HACs under H1

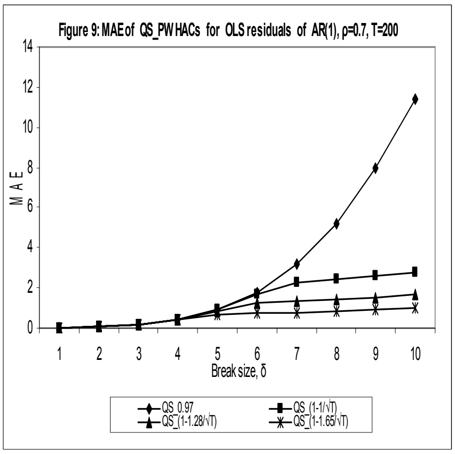
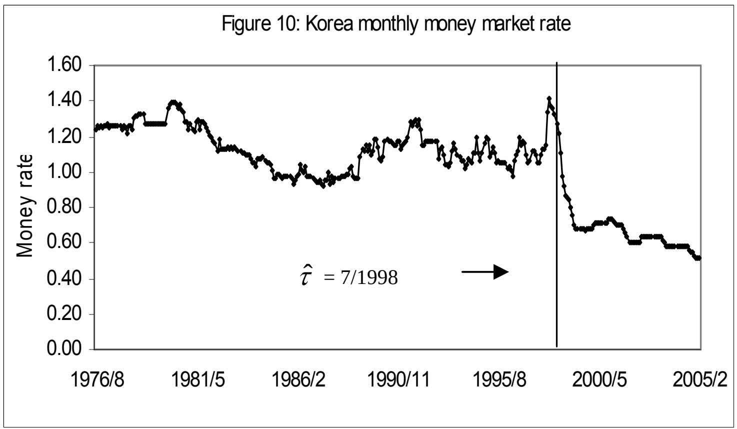


Figure 9: MAE of QS_PW HACs for OLS residuals of AR(1), ρ=0.7, T=200



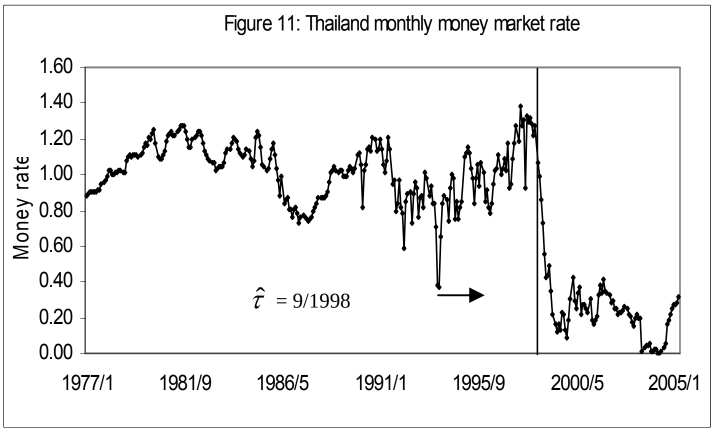


Table 2: OLS CUSUM test for the money market rate and estimated break date

HAC Kernel: Recoloring:	KOREA Bartlett (BT_PW)	KOREA Quadratic Spectral (QS_PW)	THAILAND Bartlett (BT_PW)	THAILAND Quadratic Spectral (QS_PW)
0.97	1.076	1.241	1.051	0.967
1-1/√T	1.483 (7/1998)	1.785 (7/1998)	1.529 (9/1998)	1.737 (9/1998)
1-1.28/√T	1.594 (7/1998)	1.868 (7/1998)	1.634 (9/1998)	2.193 (9/1998)
1-1.65/√T	1.662 (7/1998)	1.797 (7/1998)	1.728 (9/1998)	2.717 (9/1998)

Appendix A

Power functions of OLS CUSUM at early change-points, τ=0.25T, and late change-points, τ=0.75T, for QSHAC estimators with prewhitening and alternative recoloring boundaries

Figure A1: Power functions of OLS CUSUM in of an AR(1), ρ=0.5, T=200, at τ=0.25T using QSHACs with alternative recoloring boundaries

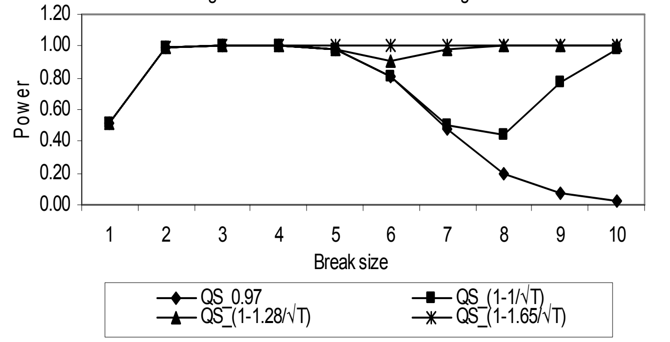


Figure A2 Power functions of OLS CUSUM in an AR(1), ρ=0.7, T=200, at τ=0.25T using QSHACs with alternative recoloring boundaries

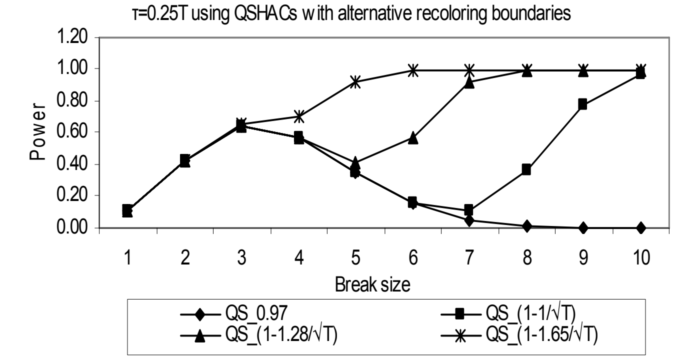


Figure A3: Power functions of OLS CUSUM in an AR(1), ρ=0.9, T=200, at τ=0.25T using QSHACs with alternative recoloring boundaries

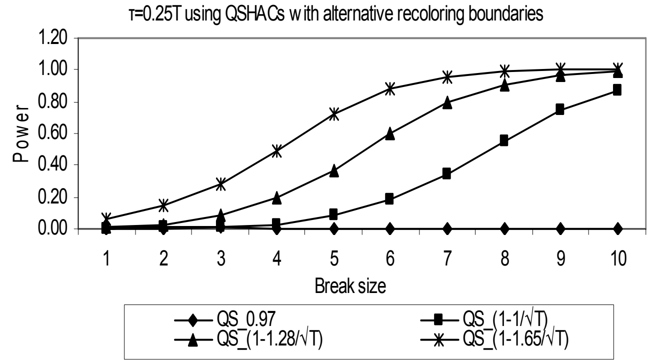


Figure A4: Power functions of OLS CUSUM in an AR(1), ρ=0.5, T=200, at τ=0.75T using QSHACs with alternative recoloring boundaries

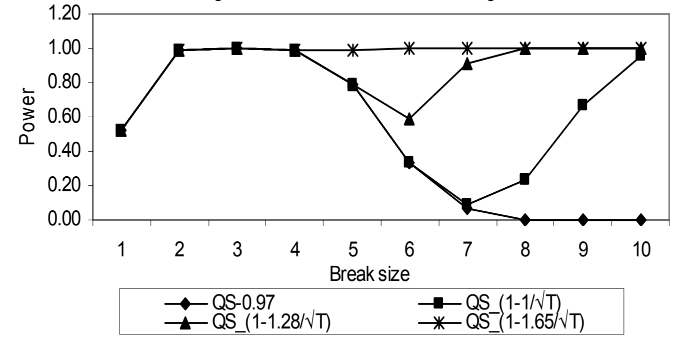


Figure A5: Power functions of OLS CUSUM in an AR(1), ρ=0.7, T=200, at τ=0.75T using QSHACs with alternative recoloring boundaries

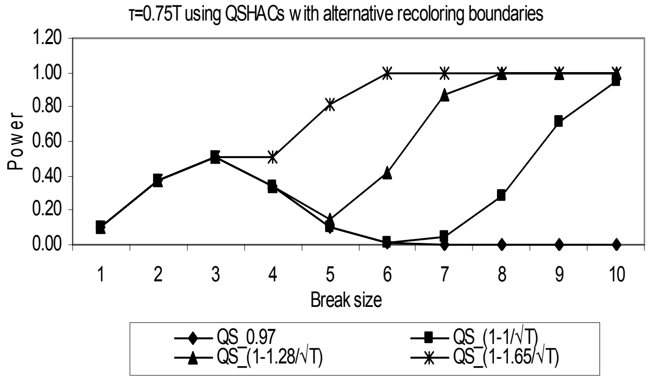
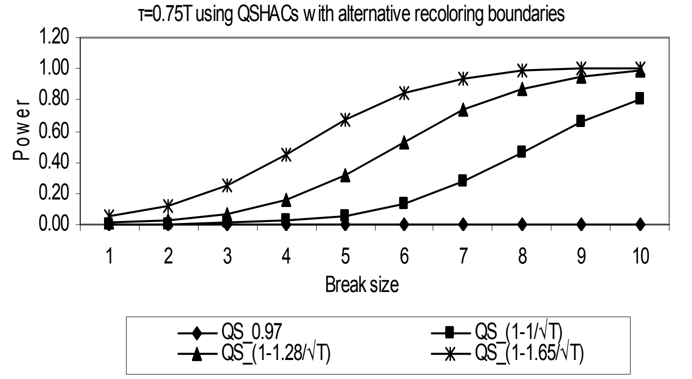


Figure A6: Power functions of OLS CUSUM in an AR(1), ρ=0.9, T=200, at τ=0.75T using QSHACs with alternative recoloring boundaries



Appendix B

Size and power functions of OLS CUSUM at change-point, τ=0.5T, for T=300 and T=500, using the QSHAC with prewhitening and alternative recoloring boundaries

Table B1: Size of OLS CUSUM using Quadratic Spectral HAC estimators with prewhitening (QS_PW) and different recoloring boundaries

		QS_PW			
		Recoloring boundaries			
T	ρ	0.97	(1-1/√T)	(1-1.28/√T)	(1-1.65/√T)
300	0.5	0.044	0.044	0.044	0.044
	0.7	0.039	0.039	0.039	0.039
	0.9	0.011	0.011	0.011	0.025
500	0.5	0.028	0.028	0.028	0.028
	0.7	0.020	0.020	0.020	0.020
	0.9	0.008	0.008	0.008	0.008

Note: The 5% significance level critical values for T=100, 200, 300 and 500 are 1.27,1.30, 1.31 and 1.36 respectively.

Figure B1: Power functions of OLS CUSUM in an AR(1), ρ=0.5, T=300, using QSHACs with alternative recoloring boundaries

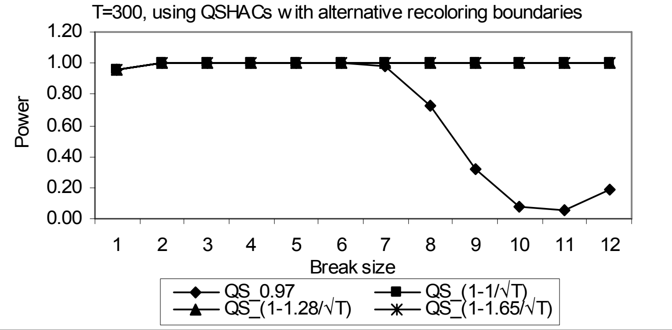


Figure B2: Power functions of OLS CUSUM in an AR(1), ρ=0.7, T=300, using QSHACs with alternative recoloring boundaries

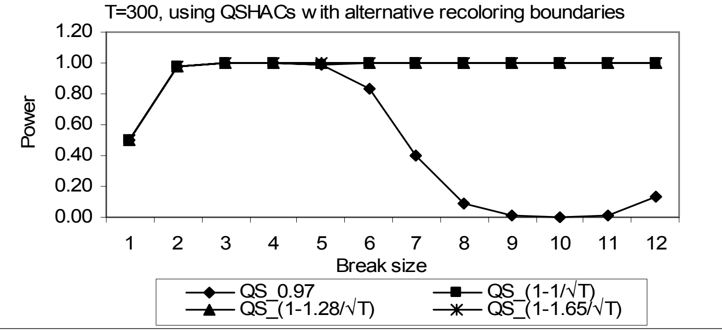


Figure B3: Power functions of OLS CUSUM in an AR(1), ρ=0.9, T=300, using QS HACs with alternative recoloring boundaries

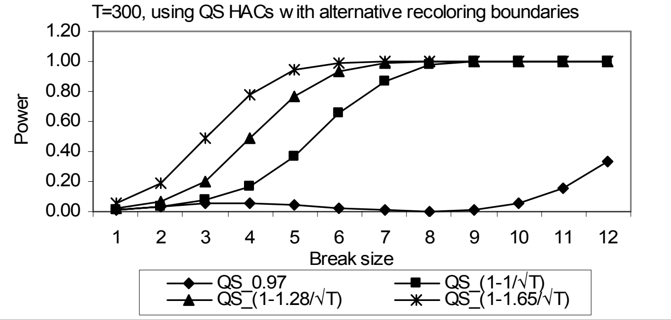


Figure B4: Power functions of OLS CUSUM in an AR(1), ρ=0.5, T=500, using QSHACs with alternative recoloring boundaries

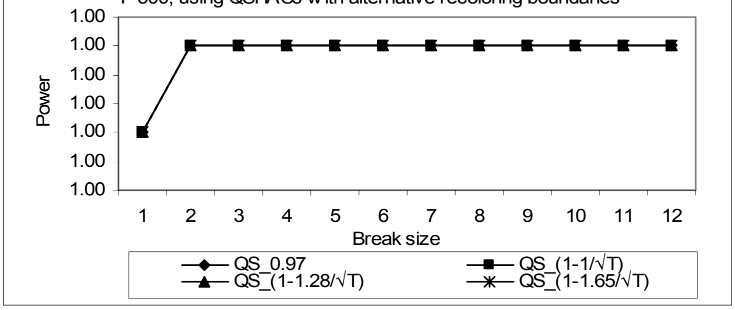


Figure B5: Power functions of OLS CUSUM in an AR(1), ρ=0.7, T=500, using QSHACs with alternative recoloring boundaries

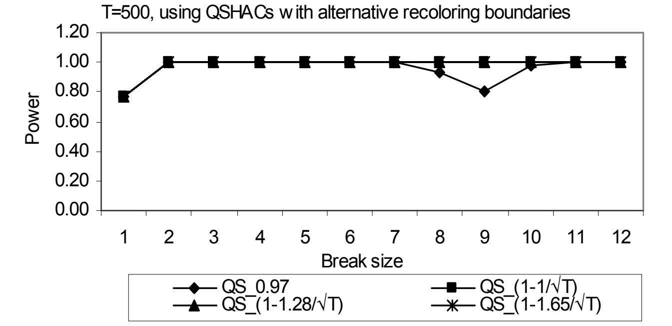
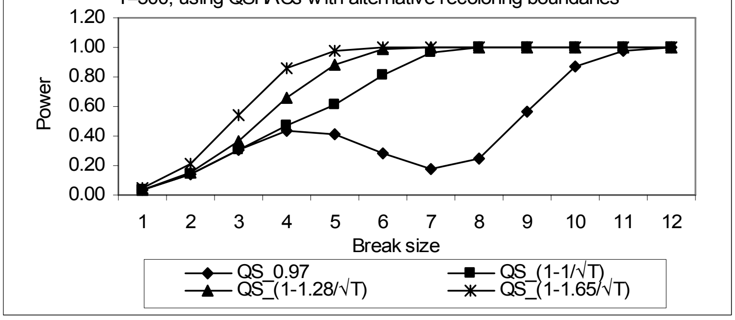


Figure B6: Power functions of OLS CUSUM in an AR(1), ρ=0.9, T=500, using QSHACs with alternative recoloring boundaries



Appendix C

Power functions of OLS CUSUM at change-point, τ=0.5T, and ρ=0.95 (for T=100, 200, 300, 500), using QSHAC with prewhitening and alternative recoloring boundaries

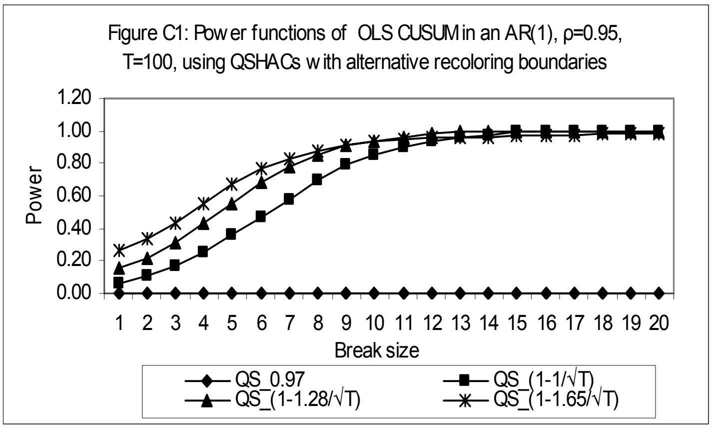


Figure C1: Power functions of OLS CUSUM in an AR(1), ρ=0.95, T=100, using QSHACs with alternative recoloring boundaries

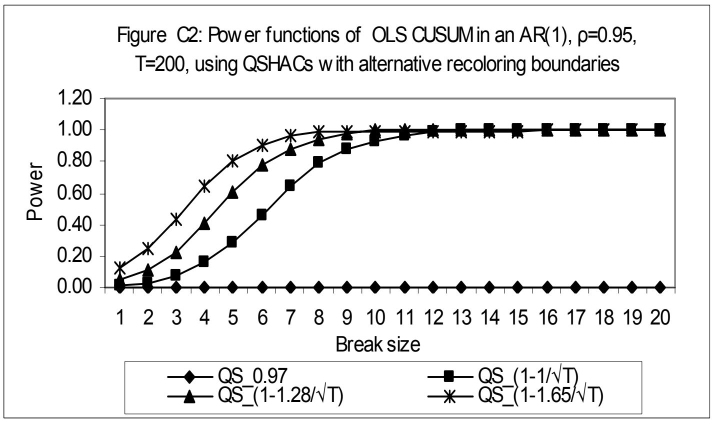


Figure C2: Power functions of OLS CUSUM in an AR(1), ρ=0.95, T=200, using QSHACs with alternative recoloring boundaries

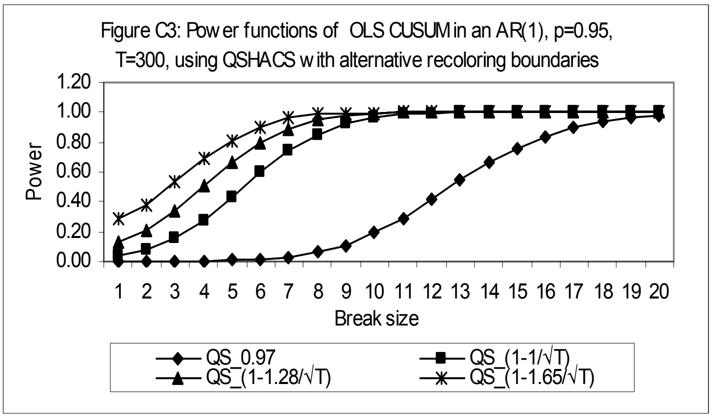


Figure C3: Power functions of OLS CUSUM in an AR(1), ρ=0.95, T=300, using QSHACS with alternative recoloring boundaries

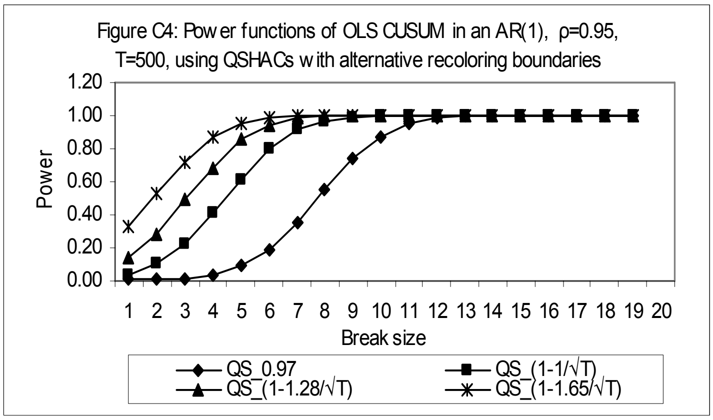


Figure C4: Power functions of OLS CUSUM in an AR(1), ρ=0.95, T=500, using QSHACs with alternative recoloring boundaries

Appendix D

Power functions of Recursive Least Squares (RLS) CUSUM at τ=0.5T for QSHACs with prewhitening and alternative recoloring boundaries using the Brown, Durbin and Evans (1979), BDE, and the Kramer, Ploberger and Alt (1988), KPA, critical values

Table D1: Size of RLS CUSUM and Brown et al. (1975) critical values using QSHAC estimator with prewhitening (QS_PW) and the Bartlett Kernel (BT_PW) and different recoloring boundaries

		QS_PW				BT_PW			
		Recoloring boundaries				Recoloring boundaries			
T	ρ	0.97	(1-1/√T)	(1-1.28/√T)	(1-1.65/√T)	0.97	(1-1/√T)	(1-1.28/√T)	(1-1.65/√T)
100	0.5	0.039	0.039	0.039	0.039	0.043	0.043	0.043	0.043
	0.7	0.026	0.026	0.026	0.026	0.023	0.029	0.029	0.029
	0.9	0.018	0.019	0.021	0.035	0.023	0.021	0.023	0.027
200	0.5	0.047	0.047	0.047	0.047	0.048	0.048	0.043	0.048
	0.7	0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.034
	0.9	0.019	0.019	0.019	0.027	0.020	0.020	0.020	0.023
500	0.5	0.072	0.072	0.072	0.072	0.073	0.073	0.073	0.073
	0.7	0.057	0.057	0.057	0.057	0.060	0.060	0.060	0.060
	0.9	0.027	0.027	0.027	0.027	0.028	0.028	0.028	0.028

Notes: 1. The critical values used refer to the Brown, Durbin and Evans (1975). Similar results were obtained using the Kramer, Ploberger and Alt (1988) boundary.

Figure D1: Power functions of RLS CUSUM in an AR(1), ρ=0.5, T=100, using QSHACs and the BDE critical values

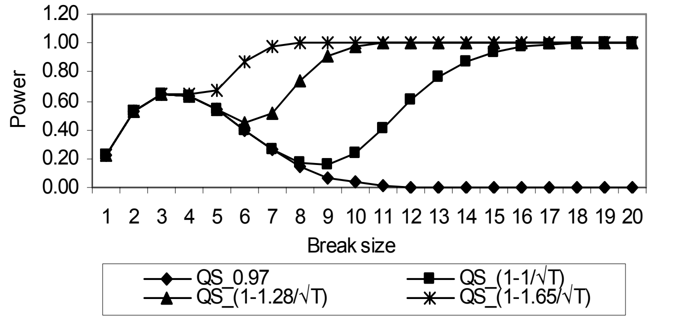


Figure D2: Power functions of RLS CUSUM in an AR(1), ρ=0.7, T=100, using QSHACs and the BDE critical values

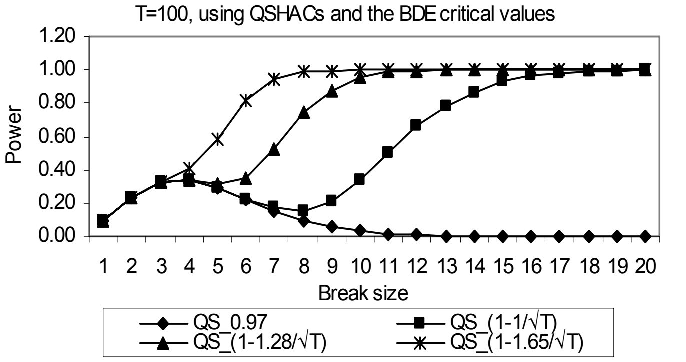


Figure D3: Power functions of RLS CUSUM in an AR(1), ρ=0.9, T=100, using QSHACs and the BDE critical values

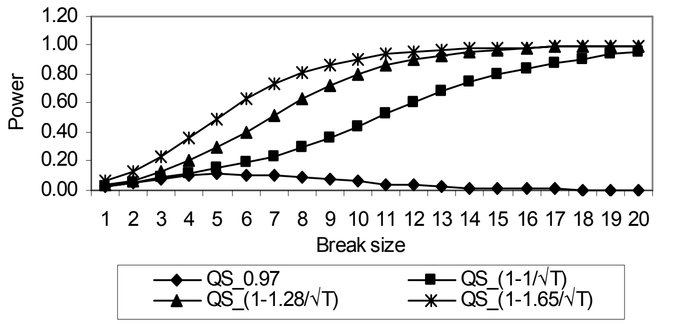


Figure D4: Power functions of RLS CUSUM in of an AR(1), ρ=0.95, T=100, using QSHACs and the BDE critical values

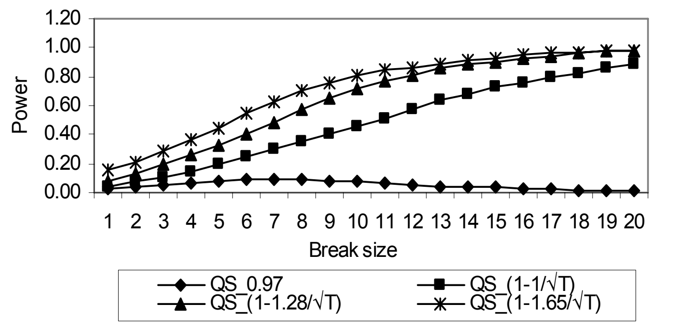


Figure D5: Power functionsof RLS CUSUM in an AR(1), ρ=0.5, T=200, using QSHACs and the BDE critical values

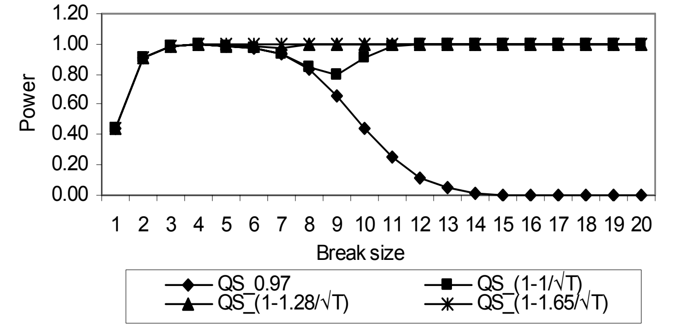


Figure D6: Power functions of RLS CUSUM in an AR(1), ρ=0.7, T=200, using QSHACs and the BDE critical values

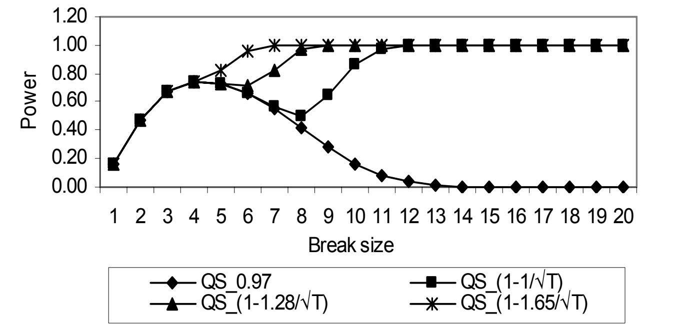


Figure D7: Power functions of RLS CUCUM in an AR(1), ρ=0.9, T=200, using QSHACs and the BDE critical values

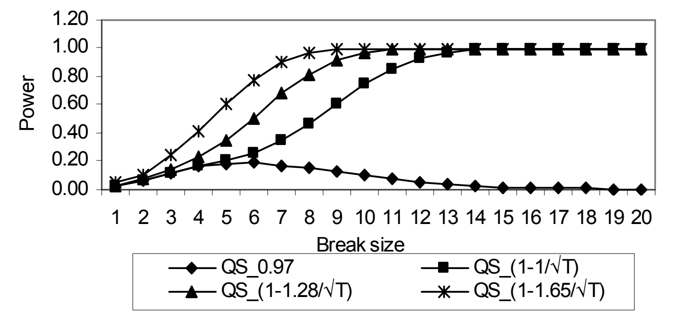


Figure D8: Power functions of RLS CUSUM in an AR(1), ρ=0.95, T=200, using QSHACs and the BDE critical values

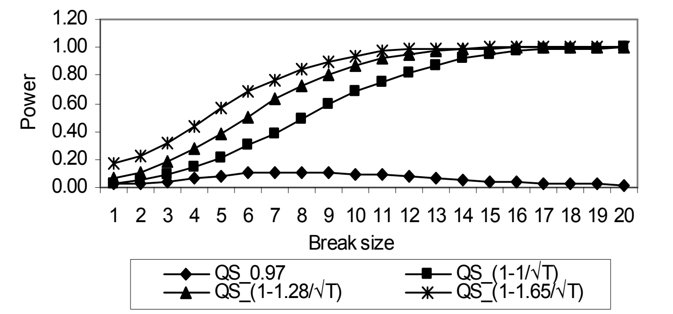


Figure D9: Power functions of RLS CUSUM in an AR(1), ρ=0.5, T=500, using QSHACs and the BDE critical values

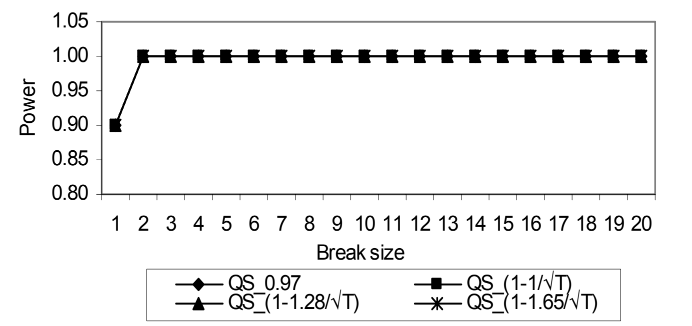


Figure D10: Power functions of RLS CUSUM in an AR(1), ρ=0.7, T=500, using QSHACs and the BDE critical values

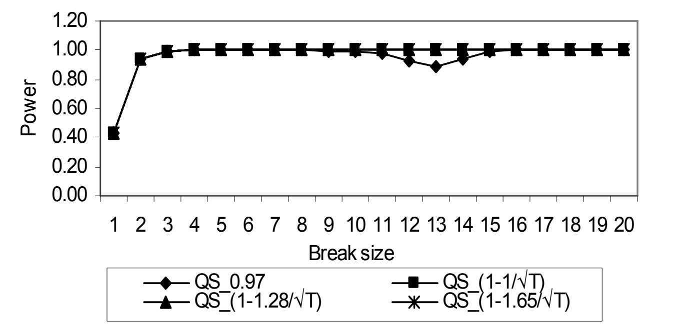


Figure D11: Power functions of RLS CUSUM in an AR(1), ρ=0.9, T=500, using QSHACs and the BDE critical values

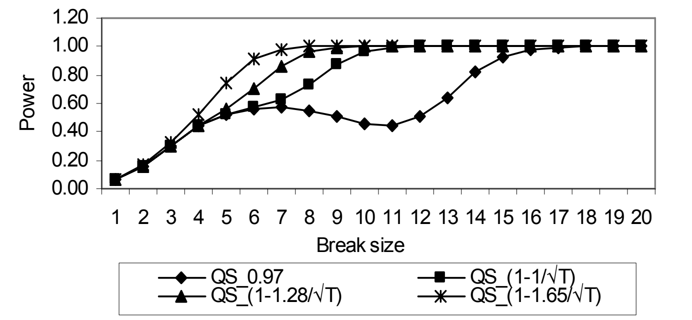


Figure D12: Power functions of RLS in an AR(1), ρ=0.95, T=500, using QSHACs and the BDE critical values

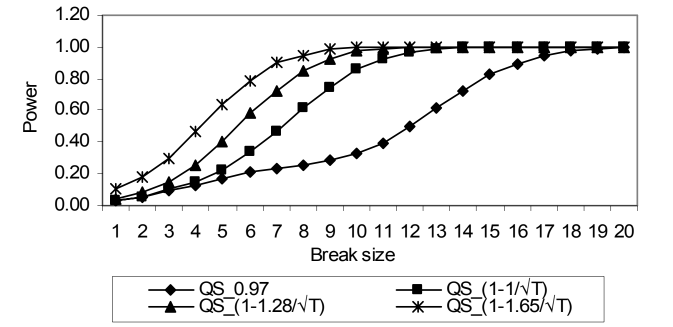


Figure D13:Power functions of RLS CUSUM in an AR(1), ρ=0.5, T=100, using QSHACs and the KPA critical values

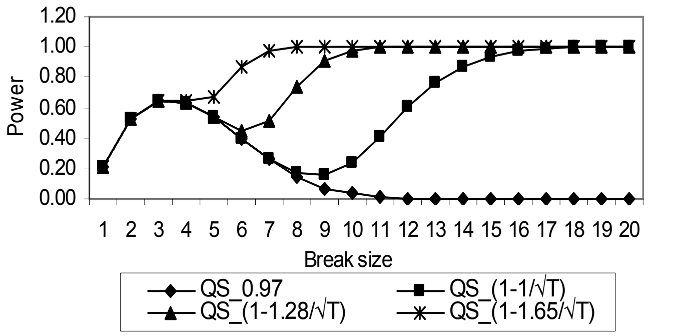


Figure D14: Powerfunctions of RLS CUSUM in an AR(1), ρ=0.7, T=100, using QSHACs and the KPA critical values

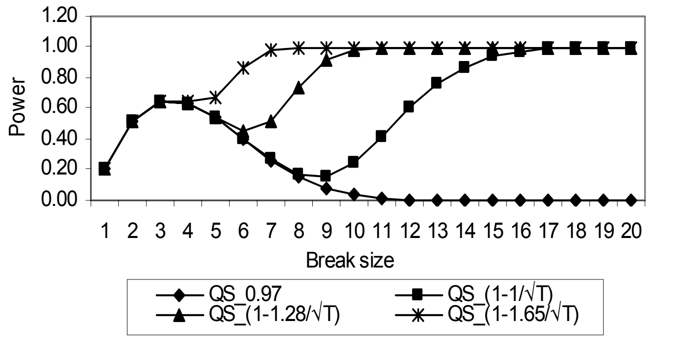


Figure D15: Power functions of RLS CUSUM in an AR(1), ρ=0.9, T=100, using QSHACs and the KPA critical values

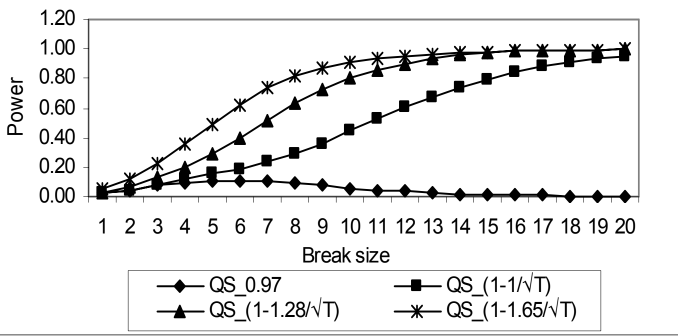


Figure D16: Power function of RLS CUSUM in an AR(1), ρ=0.5, T=200, using QSHACs and the KPA critical values

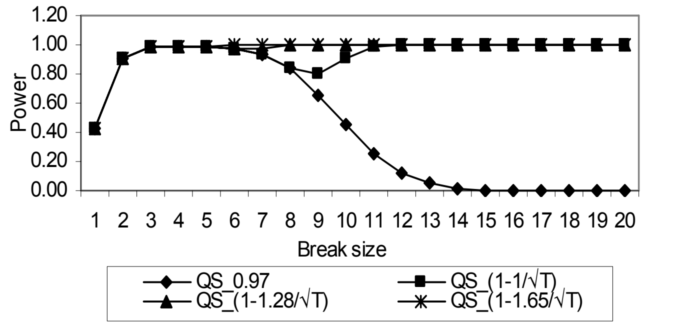


Figure D17: Power functions of RLS CUSUM in an AR(1), ρ=0.7, T=200, using QSHACs and the KPA critical values

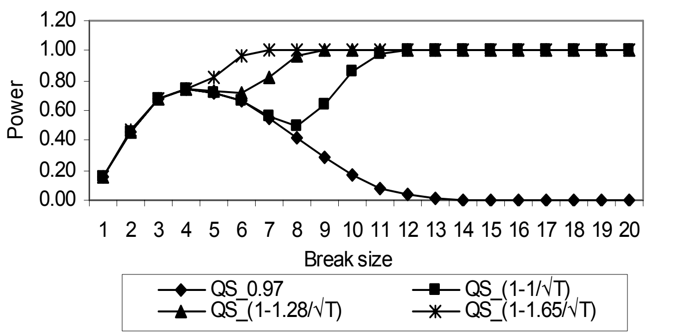
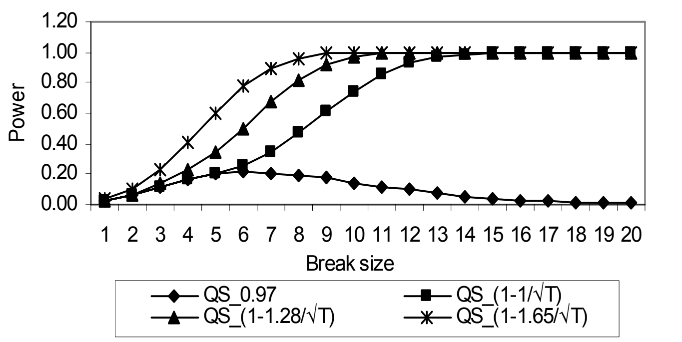


Figure D18: Power functions of RLS CUSUM in an AR(1), ρ=0.9, T=200, using QSHACs and the KPA critical values



APPENDIX E

Size and power functions of OLS CUSUM at change-point, τ=0.5T, using the Bartlett HAC estimator (BTHAC) with prewhitening and alternative recoloring boundaries[1]

Table E1: Size of OLS CUSUM using Bartlett HAC estimators with prewhitening (BT_PW) and different recoloring boundaries

		BT_PW			
		Recoloring boundaries			
T	ρ	0.97	(1-1/√T)	(1-1.28/√T)	(1-1.65/√T)
100	0.5	0.025	0.025	0.025	0.025
	0.7	0.016	0.016	0.016	0.016
	0.9	0.002	0.002	0.009	0.031
200	0.5	0.035	0.035	0.035	0.035
	0.7	0.028	0.028	0.028	0.028
	0.9	0.004	0.004	0.007	0.026
500	0.5	0.029	0.029	0.029	0.029
	0.7	0.022	0.022	0.022	0.022
	0.9	0.011	0.011	0.011	0.011

Note: The 5% significance level critical values for T=100, 200 and 500 are 1.27,1.30, and 1.36 respectively.

Figure E1: Power functions of OLS CUSUM in an AR(1), ρ=0.5, T=100, using BTHACs with alternative recoloring boundaries

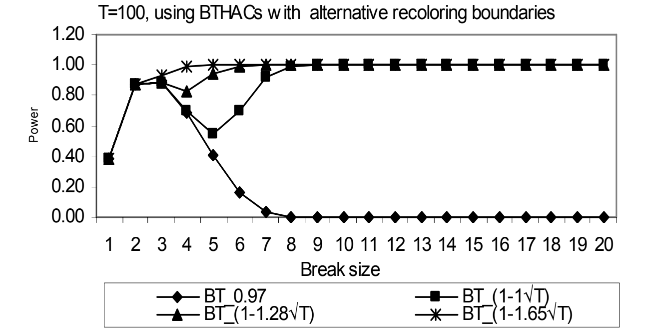


Figure E2: Power functions of OLS CUSUM in an AR(1), ρ=0.7, T=100, using BTHACs with alternative recoloring boundaries

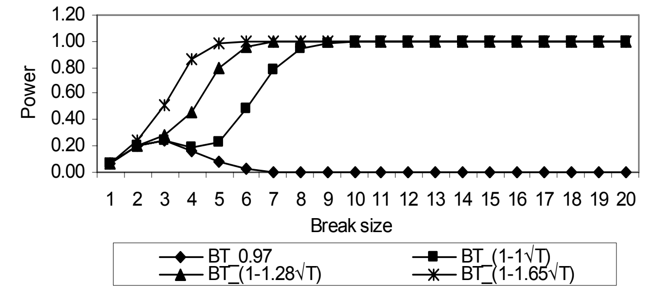


Figure E3: Power functions of OLS CUSUM in an AR(1), ρ=0.9, T=100, using BTHACs with alternative recoloring boundaries

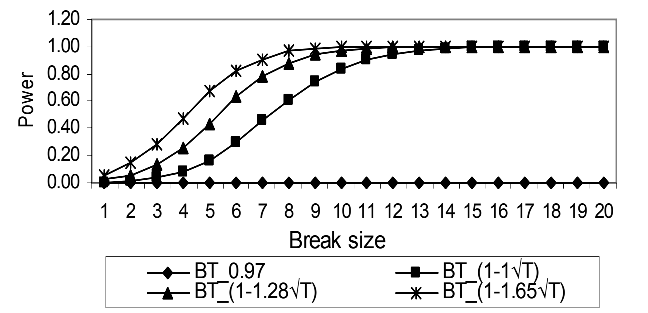
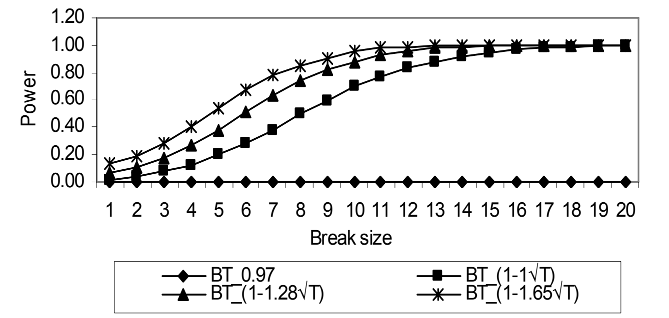


Figure E4: Power functions of OLS CUSUM in an AR(1), ρ=0.95, T=100, using BTHACs with alternative recoloring boundaries



[1] Non-monotone power is found for the recoloring condition of 0.97 for all ρ=0.5, 0.7, 0.9, 0.95 and T=100, 200. Weak evidence of non-monotone power is found for T=500 when ρ=0.9.
Non-monotone power is found for the recoloring condition of c=1 when ρ=0.5, 0.7 and T=200 and when ρ=0.7 and T=100 as well as when c=1.28 when ρ=0.5 and T=100. Only c=1.65 yields monotone power for all ρ and T considered here.

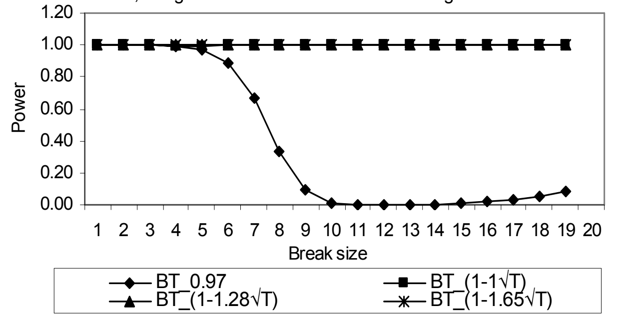


Figure E5: Power functions of OLS CUSUM in an AR(1), ρ=0.5, T=200, using BTHACs with alternative recoloring boundaries

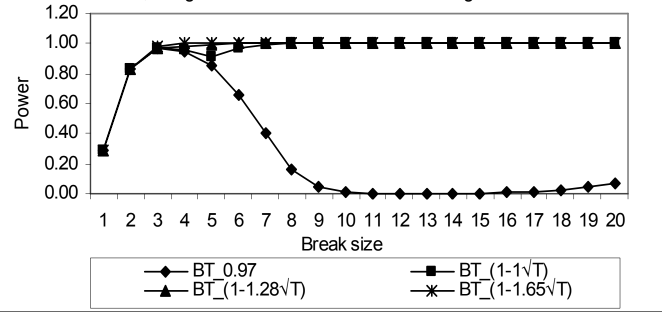


Figure E6: Power functions of OLS CUSUM in an AR(1), ρ=0.7, T=200, using BTHACs with alternative recoloring boundaries

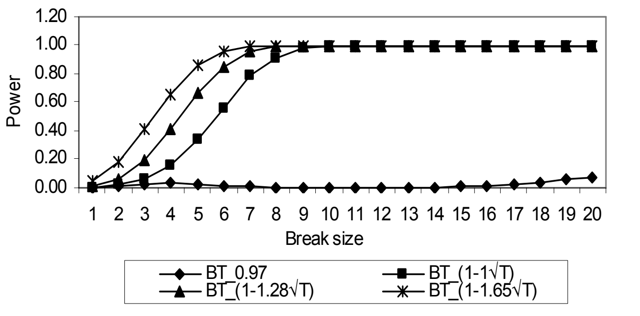


Figure: Power functions of OLS CUSUM in an AR(1), ρ=0.9, T=200, using BTHACs with alternative recoloring boundaries

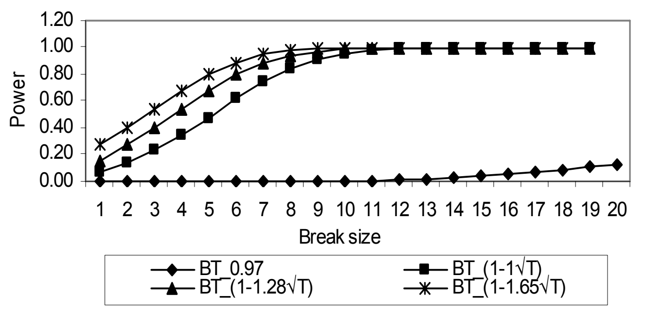


Figure E8: Power functions of OLS CUSUM in an AR(1), ρ=0.95, T=200, using BTHACs with alternative recoloring boundaries

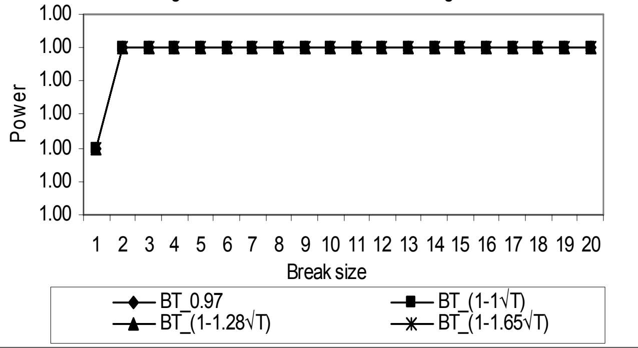


Figure E9: Power functions of OLS CUSUM in an AR(1), ρ=0.5, T=500, using BTHACs with alternative recoloring boundaries

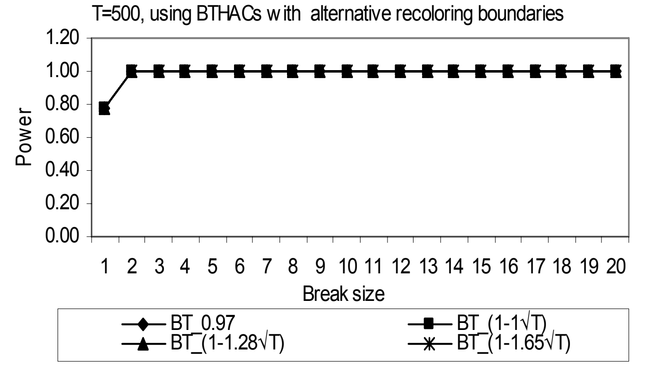


Figure E10: Power functions of OLS CUSUM in an AR(1), ρ=0.7, T=500, using BTHACs with alternative recoloring boundaries

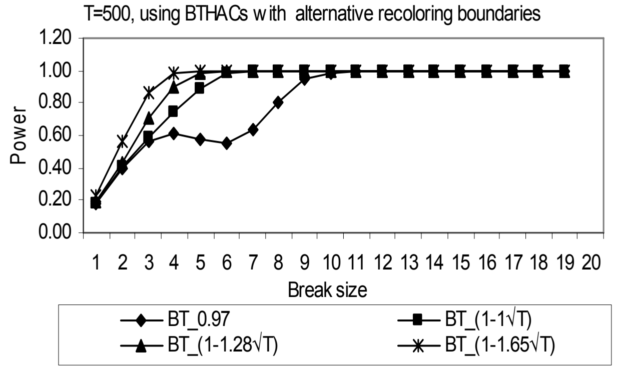


Figure E11: Power functions of OLS CUSUM in an AR(1), ρ=0.9, T=500, using BTHACs with alternative recoloring boundaries

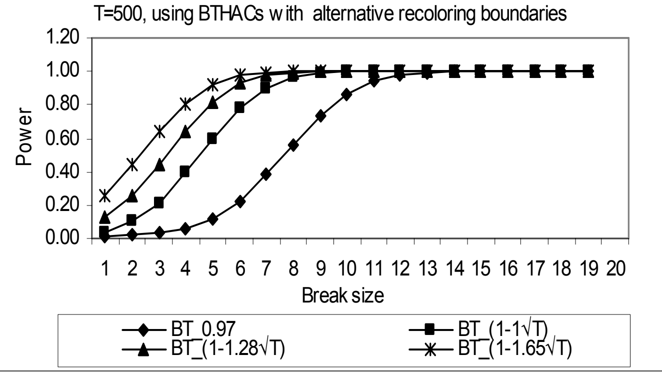


Figure E12: Power functions of OLS CUSUM in an AR(1), ρ=0.95, T=500, using BTHACs with alternative recoloring boundaries

Appendix F

The effects of initial conditions on the size and power of OLS CUSUM

Table F1: The effects of initial conditions on the size of the OLS CUSUM test for a break in the constant of an AR(1) using HAC estimators with prewhitening and alternative recoloring boundaries

Panel A: T=100

Initial Condition	Y_1 ~N (0,1)			Y_1 ~N $(0,(1-\rho^2)^{-1})$		
ρ HAC_PW	0.5	0.7	0.9	0.5	0.7	0.9
BT_0.97	0.025	0.016	0.001	0.026	0.013	0.003
QS_0.97	0.025	0.012	0.001	0.025	0.011	0.000
BT_(1-1√T)	0.025	0.016	0.002	0.026	0.013	0.004
QS_(1-1/√T)	0.025	0.012	0.004	0.025	0.011	0.004
BT_(1-1.28√T)	0.025	0.016	0.009	0.026	0.013	0.019
QS_(1-1.28/√T)	0.025	0.012	0.018	0.025	0.011	0.027
BT_(1-1.65√T)	0.025	0.016	0.031	0.026	0.013	0.046
QS_(1-1.65/√T)	0.025	0.012	0.086	0.025	0.011	0.099

Initial Condition	Y_1 ~N (0,1)			Y_1 ~N $(0,(1-\rho^2)^{-1})$		
ρ HAC_PW	0.5	0.7	0.9	0.5	0.7	0.9
BT_0.97	0.035	0.028	0.004	0.035	0.026	0.004
QS_0.97	0.032	0.023	0.001	0.033	0.024	0.001
BT_(1-1√T)	0.035	0.028	0.004	0.035	0.026	0.004
QS_(1-1/√T)	0.032	0.023	0.001	0.033	0.024	0.001
BT_(1-1.28√T)	0.035	0.028	0.007	0.035	0.026	0.008
QS_(1-1.28/√T)	0.032	0.023	0.003	0.033	0.024	0.007
BT_(1-1.65√T)	0.035	0.028	0.026	0.035	0.026	0.028
QS_(1-1.65/√T)	0.032	0.023	0.037	0.033	0.024	0.041

Notes:
1. The 5% significance level finite sample critical values are 1.27 and 1.30 for T=100 and 200, respectively.
2. Y_1 refers to the initial value.

Table F2: The effects of initial conditions on the power of the OLS CUSUM test for a break in the constant of an AR(1) using HAC estimators with prewhitening and alternative recoloring boundaries

		Panel A: T=100							
Initial Condition				Y_1 ~N (0,1)					
ρ		**0.5**			**0.7**			**0.9**	
HAC_PW	**δ=3**	**δ=5**	**δ=7**	**δ=3**	**δ=5**	**δ=7**	**δ=3**	**δ=5**	**δ=7**
BT_0.97	0.883	0.410	0.037	0.246	0.081	0.006	0.005	0.005	0.003
QS_0.97	0.750	0.122	0.004	0.128	0.015	0.000	0.002	0.001	0.001
BT_(1-1√T)	0.883	0.544	0.918	0.247	0.234	0.789	0.037	0.168	0.454
QS_(1-1/√T)	0.750	0.156	0.745	0.129	0.110	0.773	0.078	0.290	0.641
BT_(1-1.28√T)	0.886	0.946	1.000	0.289	0.794	0.996	0.135	0.435	0.780
QS_(1-1.28/√T)	0.751	0.768	1.000	0.161	0.727	1.000	0.229	0.603	0.880
BT_(1-1.65√T)	0.935	1.000	1.000	0.519	0.987	1.000	0.281	0.669	0.909
QS_(1-1.65/√T)	0.836	0.999	1.000	0.439	0.990	1.000	0.439	0.809	0.962

		Panel B: T=100							
Initial Condition				Y_1 ~N $(0,(1-\rho^2)^{-1})$					
ρ		**0.5**			**0.7**			**0.9**	
HAC_PW	**δ=3**	**δ=5**	**δ=7**	**δ=3**	**δ=5**	**δ=7**	**δ=3**	**δ=5**	**δ=7**
BT_0.97	0.885	0.403	0.031	0.255	0.081	0.006	0.007	0.006	0.003
QS_0.97	0.762	0.130	0.005	0.132	0.016	0.000	0.003	0.002	0.001
BT_(1-1√T)	0.885	0.535	0.911	0.256	0.235	0.779	0.045	0.188	0.457
QS_(1-1/√T)	0.762	0.167	0.748	0.320	0.113	0.768	0.086	0.306	0.637
BT_(1-1.28√T)	0.887	0.940	1.000	0.299	0.786	0.997	0.137	0.455	0.770
QS_(1-1.28/√T)	0.763	0.773	1.000	0.161	0.721	1.000	0.246	0.597	0.874
BT_(1-1.65√T)	0.937	1.000	1.000	0.524	0.984	1.000	0.302	0.666	0.896
QS_(1-1.65/√T)	0.842	1.000	1.000	0.443	0.991	1.000	0.450	0.802	0.951

		Panel C: T=200							
Initial Condition				Y_1~N (0,1)					
ρ		**0.5**			**0.7**			**0.9**	
HAC_PW	**δ=3**	**δ=5**	**δ=7**	**δ=3**	**δ=5**	**δ=7**	**δ=3**	**δ=5**	**δ=7**
BT_0.97	1.000	0.995	0.885	0.972	0.860	0.400	0.027	0.029	0.008
QS_0.97	1.000	0.980	0.322	0.931	0.458	0.016	0.007	0.005	0.002
BT_(1-1√T)	1.000	0.995	0.997	0.972	0.916	0.998	0.057	0.334	0.784
QS_(1-1/√T)	1.000	0.984	0.996	0.931	0.530	0.969	0.043	0.341	0.806
BT_(1-1.28√T)	1.000	0.999	1.000	0.972	0.996	1.000	0.192	0.667	0.958
QS_(1-1.28/√T)	1.000	1.000	1.000	0.933	0.978	1.000	0.205	0.709	0.971
BT_(1-1.65√T)	1.000	1.000	1.000	0.984	1.000	1.000	0.408	0.856	0.995
QS_(1-1.65/√T)	1.000	1.000	1.000	0.961	1.000	1.000	0.482	0.905	0.998

		Panel D: T=200							
Initial Condition				Y_1 ~N $(0,(1-\rho^2)^{-1})$					
ρ					**0.7**			**0.9**	
HAC_PW	**δ=3**	**δ=5**	**δ=7**	**δ=3**	**δ=5**	**δ=7**	**δ=3**	**δ=5**	**δ=7**
BT_0.97	1.000	0.995	0.884	0.970	0.860	0.339	0.030	0.032	0.011
QS_0.97	1.000	0.980	0.324	0.930	0.468	0.020	0.012	0.007	0.002
BT_(1-1√T)	1.000	0.996	0.997	0.970	0.918	0.997	0.067	0.335	0.777
QS_(1-1/√T)	1.000	0.984	0.995	0.930	0.531	0.968	0.051	0.339	0.802
BT_(1-1.28√T)	1.000	0.999	1.000	0.970	0.996	1.000	0.197	0.669	0.951
QS_(1-1.28/√T)	1.000	1.000	1.000	0.933	0.975	1.000	0.209	0.712	0.965
BT_(1-1.65√T)	1.000	1.000	1.000	0.979	1.000	1.000	0.412	0.855	0.990
QS_(1-1.65/√T)	1.000	1.000	1.000	0.958	1.000	1.000	0.487	0.900	0.997

Notes:
1. The 5% significance level finite sample critical values are 1.27 and 1.30 for T=100 and 200, respectively.
2. Y_1 refers to the initial value.

Appendix G

The finite sample distribution of OLS CUSUM

Table G1: Empirical Quantiles for the OLS CUSUM test for AR(1), ρ=0.5, at τ=0.5T using the QSHAC and BTHAC with alternative recoloring boundaries

	T=100, ρ=0.5						T=200, ρ=0.5					
c:	(1-1√T)		(1-1.28√T)		(1-1.65√T)		(1-1√T)		(1-1.28√T)		(1-1.65√T)	
HACs:	BT	QS	BT	QS	BT	QS	BT	QS	BT	QS	BT	QS
NULL HYPOTHESIS												
Q5	0.464	0.442	0.464	0.442	0.464	0.442	0.466	0.455	0.466	0.455	0.466	0.455
Q10	0.517	0.493	0.517	0.493	0.517	0.493	0.513	0.503	0.513	0.503	0.513	0.503
Q25	0.608	0.587	0.608	0.587	0.608	0.587	0.620	0.611	0.620	0.611	0.620	0.611
mean	0.748	0.718	0.748	0.718	0.748	0.718	0.769	0.757	0.769	0.757	0.769	0.757
media	0.778	0.755	0.778	0.755	0.778	0.755	0.797	0.788	0.797	0.788	0.797	0.788
Q75	0.918	0.898	0.918	0.898	0.918	0.898	0.944	0.927	0.944	0.927	0.944	0.927
Q90	1.094	1.073	1.094	1.073	1.094	1.073	1.121	1.120	1.121	1.120	1.121	1.120
Q95	1.188	1.185	1.188	1.185	1.188	1.185	1.235	1.229	1.235	1.229	1.235	1.229
ALTERNATIVE HYPOTHESIS, δ=2												
Q5	1.171	1.153	1.171	1.153	1.171	1.153	1.742	1.710	1.742	1.710	1.742	1.710
Q10	1.244	1.220	1.244	1.220	1.244	1.220	1.809	1.764	1.809	1.764	1.809	1.764
Q25	1.353	1.316	1.353	1.316	1.353	1.316	1.907	1.858	1.907	1.858	1.907	1.858
mean	1.462	1.422	1.462	1.422	1.462	1.422	2.009	1.961	2.009	1.961	2.009	1.961
media	1.455	1.422	1.455	1.422	1.455	1.422	2.005	1.966	2.005	1.966	2.005	1.966
Q75	1.566	1.523	1.566	1.523	1.566	1.523	2.113	2.072	2.113	2.072	2.113	2.072
Q90	1.653	1.627	1.653	1.627	1.653	1.627	2.192	2.169	2.192	2.169	2.192	2.169
Q95	1.701	1.692	1.701	1.692	1.701	1.692	2.245	2.231	2.245	2.231	2.245	2.231
ALTERNATIVE HYPOTHESIS, δ=4												
Q5	1.115	1.046	1.186	1.134	1.339	1.285	1.608	1.511	1.608	1.512	1.635	1.570
Q10	1.160	1.079	1.226	1.161	1.376	1.313	1.671	1.555	1.672	1.555	1.704	1.594
Q25	1.244	1.151	1.296	1.208	1.450	1.374	1.785	1.629	1.785	1.629	1.808	1.655
mean	1.345	1.234	1.381	1.274	1.524	1.456	1.894	1.728	1.894	1.728	1.910	1.741
media	1.350	1.248	1.386	1.290	1.523	1.469	1.888	1.738	1.889	1.738	1.905	1.757
Q75	1.455	1.336	1.470	1.357	1.600	1.551	1.998	1.833	1.998	1.833	2.005	1.840
Q90	1.545	1.430	1.550	1.444	1.662	1.645	2.102	1.939	2.102	1.939	2.103	1.942
Q95	1.610	1.499	1.613	1.508	1.703	1.696	2.146	2.006	2.146	2.006	2.146	2.008
ALTERNATIVE HYPOTHESIS, δ=5												
Q5	1.098	1.039	1.264	1.198	1.486	1.455	1.455	1.343	1.522	1.428	1.732	1.656
Q10	1.136	1.062	1.303	1.222	1.521	1.508	1.524	1.385	1.578	1.454	1.798	1.684
Q25	1.201	1.101	1.369	1.279	1.572	1.590	1.628	1.450	1.673	1.500	1.896	1.755
mean	1.282	1.157	1.441	1.351	1.634	1.685	1.730	1.541	1.761	1.570	1.980	1.837
media	1.284	1.174	1.438	1.362	1.634	1.690	1.725	1.552	1.756	1.588	1.972	1.843
Q75	1.354	1.228	1.507	1.436	1.695	1.788	1.829	1.640	1.844	1.659	2.060	1.919
Q90	1.436	1.309	1.567	1.514	1.751	1.878	1.921	1.733	1.921	1.733	2.132	2.007
Q95	1.487	1.364	1.599	1.564	1.781	1.937	1.966	1.799	1.969	1.804	2.169	2.062
ALTERNATIVE HYPOTHESIS, δ=7												
Q5	1.245	1.175	1.469	1.477	1.577	1.847	1.456	1.354	1.775	1.689	2.080	2.127
Q10	1.283	1.210	1.498	1.515	1.597	1.888	1.514	1.383	1.829	1.731	2.112	2.167
Q25	1.338	1.269	1.544	1.591	1.635	1.974	1.603	1.432	1.914	1.793	2.170	2.246
mean	1.397	1.337	1.598	1.673	1.685	2.072	1.676	1.497	1.994	1.866	2.245	2.326
media	1.397	1.343	1.600	1.676	1.691	2.071	1.667	1.504	1.982	1.871	2.247	2.329
Q75	1.457	1.413	1.655	1.757	1.740	2.167	1.746	1.564	2.058	1.942	2.320	2.407
Q90	1.514	1.480	1.705	1.835	1.797	2.245	1.799	1.635	2.120	2.018	2.388	2.487
Q95	1.544	1.521	1.733	1.884	1.831	2.301	1.833	1.681	2.151	2.074	2.421	2.546

Table G2: Empirical Quantiles for the OLS CUSUM test for AR(1), ρ=0.7, at τ=0.5T using the QSHAC and BTHAC with alternative recoloring boundaries

	T=100, ρ=0.7						T=200, ρ=0.7					
c:	(1-1√T)		(1-1.28√T)		(1-1.65√T)		(1-1√T)		(1-1.28√T)		(1-1.65√T)	
HACs:	**BT**	**QS**	**BT**	**QS**	**BT**	**QS**	**BT**	**QS**	**BT**	**QS**	**BT**	**QS**
NULL HYPOTHESIS												
Q5	0.410	0.410	0.410	0.410	0.410	0.412	0.438	0.434	0.438	0.434	0.438	0.434
Q10	0.464	0.463	0.464	0.463	0.464	0.463	0.479	0.480	0.479	0.480	0.479	0.480
Q25	0.562	0.557	0.562	0.557	0.562	0.557	0.590	0.589	0.590	0.589	0.590	0.589
mean	0.700	0.689	0.700	0.689	0.700	0.689	0.738	0.733	0.738	0.733	0.738	0.733
media	0.726	0.717	0.726	0.717	0.726	0.717	0.763	0.760	0.763	0.760	0.763	0.760
Q75	0.865	0.852	0.865	0.852	0.866	0.853	0.913	0.904	0.913	0.904	0.913	0.904
Q90	1.030	1.016	1.030	1.016	1.030	1.016	1.078	1.075	1.078	1.075	1.078	1.075
Q95	1.124	1.108	1.124	1.108	1.125	1.108	1.185	1.180	1.185	1.180	1.185	1.180
ALTERNATIVE HYPOTHESIS, δ=2												
Q5	0.755	0.750	0.759	0.752	0.769	0.766	1.145	1.136	1.145	1.136	1.146	1.137
Q10	0.842	0.835	0.849	0.839	0.865	0.860	1.219	1.205	1.219	1.205	1.222	1.205
Q25	0.977	0.961	0.981	0.966	1.001	0.991	1.370	1.336	1.370	1.336	1.370	1.337
mean	1.120	1.087	1.125	1.090	1.141	1.119	1.511	1.469	1.511	1.465	1.511	1.465
media	1.107	1.075	1.109	1.078	1.126	1.100	1.496	1.450	1.496	1.450	1.496	1.451
Q75	1.244	1.197	1.245	1.198	1.265	1.225	1.632	1.568	1.632	1.568	1.632	1.568
Q90	1.348	1.298	1.349	1.298	1.365	1.318	1.748	1.673	1.748	1.673	1.748	1.673
Q95	1.411	1.349	1.411	1.349	1.425	1.376	1.816	1.731	1.816	1.731	1.816	1.731
ALTERNATIVE HYPOTHESIS, δ=4												
Q5	0.952	0.926	1.047	1.028	1.183	1.168	1.305	1.218	1.343	1.278	1.466	1.400
Q10	0.991	0.958	1.086	1.059	1.238	1.221	1.354	1.258	1.391	1.306	1.510	1.434
Q25	1.061	1.010	1.172	1.123	1.334	1.335	1.446	1.324	1.473	1.356	1.594	1.494
mean	1.148	1.076	1.255	1.202	1.429	1.462	1.544	1.404	1.559	1.421	1.676	1.571
media	1.156	1.088	1.258	1.218	1.423	1.475	1.545	1.414	1.563	0.144	1.679	1.587
Q75	1.237	1.151	1.345	1.299	1.519	1.606	1.644	1.494	1.653	1.504	1.765	1.664
Q90	1.335	1.236	1.430	1.395	1.595	1.736	1.730	1.576	1.731	1.581	1.849	1.762
Q95	1.404	1.286	1.476	1.459	1.634	1.808	1.784	1.633	1.784	1.635	1.895	1.828
ALTERNATIVE HYPOTHESIS, δ=5												
Q5	1.005	0.976	1.162	1.132	1.344	1.384	1.263	1.189	1.418	1.326	1.645	1.567
Q10	1.047	1.005	1.207	1.173	1.399	1.452	1.313	1.211	1.457	1.354	1.700	1.634
Q25	1.112	1.054	1.292	1.261	1.468	1.578	1.392	1.249	1.534	1.404	1.788	1.733
mean	1.187	1.120	1.372	1.366	1.544	1.705	1.472	1.308	1.602	1.472	1.879	1.846
media	1.190	1.132	1.369	1.377	1.542	1.709	1.477	1.321	1.606	1.485	1.877	1.849
Q75	1.265	1.201	1.455	1.483	1.622	1.842	1.561	1.377	1.682	1.550	1.972	1.959
Q90	1.336	1.279	1.529	1.591	1.679	1.967	1.640	1.452	1.752	1.637	2.048	2.065
Q95	1.384	1.330	1.568	1.660	1.717	2.025	1.688	1.502	1.796	1.691	2.091	2.127
ALTERNATIVE HYPOTHESIS, δ=7												
Q5	1.177	1.150	1.379	1.439	1.501	1.727	1.424	1.317	1.718	1.672	1.948	2.124
Q10	1.218	1.195	1.414	1.495	1.533	1.789	1.469	1.358	1.762	1.719	1.989	2.182
Q25	1.283	1.279	1.471	1.595	1.577	1.904	1.532	1.429	1.832	1.804	2.049	2.277
mean	1.349	1.365	1.537	1.700	1.629	2.029	1.606	1.500	1.912	1.892	2.121	2.387
media	1.351	1.368	1.535	1.700	1.631	2.024	1.605	1.502	1.907	1.895	2.126	2.386
Q75	1.420	1.456	1.598	1.803	1.680	2.144	1.676	1.574	1.984	1.985	2.199	2.493
Q90	1.481	1.541	1.659	1.905	1.732	2.253	1.743	1.642	2.052	2.064	2.278	2.586
Q95	1.525	1.594	1.694	1.956	1.772	2.306	1.781	1.692	2.088	2.118	2.317	2.647

Table G3: Empirical Quantiles for the OLS CUSUM test for AR(1), ρ=0.9, at τ=0.5T using the QSHAC and BTHAC with alternative recoloring boundaries

	T=100, ρ=0.9						T=200, ρ=0.9					
c:	**(1-1√T)**		**(1-1.28√T)**		**(1-1.65√T)**		**(1-1√T)**		**(1-1.28√T)**		**(1-1.65√T)**	
HACs:	**BT**	**QS**	**BT**	**QS**	**BT**	**QS**	**BT**	**QS**	**BT**	**QS**	**BT**	**QS**
NULL HYPOTHESIS												
Q5	0.367	0.383	0.389	0.401	0.419	0.433	0.379	0.387	0.386	0.399	0.404	0.417
Q10	0.414	0.428	0.431	0.449	0.469	0.487	0.432	0.440	0.441	0.450	0.460	0.471
Q25	0.504	0.511	0.531	0.545	0.580	0.600	0.529	0.538	0.541	0.554	0.578	0.589
mean	0.644	0.652	0.679	0.701	0.742	0.778	0.673	0.673	0.686	0.693	0.729	0.750
media	0.661	0.667	0.704	0.725	0.772	0.825	0.689	0.690	0.706	0.711	0.757	0.778
Q75	0.805	0.806	0.855	0.875	0.945	1.002	0.828	0.827	0.854	0.853	0.914	0.932
Q90	0.923	0.920	1.000	1.036	1.121	1.238	0.972	0.965	0.999	0.998	1.090	1.113
Q95	1.003	0.991	1.081	1.134	1.212	1.361	1.059	1.040	1.087	1.080	1.197	1.234
ALTERNATIVE HYPOTHESIS, δ=2												
Q5	0.402	0.415	0.428	0.441	0.471	0.480	0.441	0.449	0.454	0.461	0.475	0.494
Q10	0.464	0.473	0.494	0.507	0.538	0.563	0.516	0.519	0.531	0.539	0.562	0.579
Q25	0.597	0.608	0.633	0.654	0.701	0.728	0.664	0.675	0.691	0.698	0.735	0.758
mean	0.756	0.769	0.828	0.848	0.923	0.986	0.844	0.849	0.890	0.898	0.981	1.018
media	0.766	0.778	0.837	0.878	0.927	1.020	0.838	0.833	0.889	0.897	0.988	1.037
Q75	0.926	0.928	1.023	1.059	1.150	1.270	1.001	0.990	1.074	1.072	1.227	1.270
Q90	1.072	1.081	1.200	1.294	1.329	1.538	1.139	1.101	1.237	1.234	1.420	1.521
Q95	1.148	1.189	1.277	1.445	1.400	1.679	1.228	1.177	1.334	1.347	1.529	1.691
ALTERNATIVE HYPOTHESIS, δ=4												
Q5	0.548	0.570	0.602	0.621	0.675	0.697	0.728	0.740	0.776	0.790	0.874	0.895
Q10	0.658	0.680	0.737	0.752	0.817	0.862	0.814	0.818	0.878	0.892	0.987	1.022
Q25	0.823	0.832	0.934	0.966	1.060	1.145	0.950	0.951	1.062	1.071	1.212	1.281
mean	0.981	0.992	1.114	1.182	1.255	1.410	1.085	1.079	1.243	1.261	1.434	1.576
media	0.969	1.008	1.089	1.193	1.203	1.382	1.084	1.082	1.234	1.280	1.405	1.564
Q75	1.124	1.171	1.283	1.426	1.401	1.643	1.229	1.203	1.432	1.485	1.635	1.849
Q90	1.255	1.365	1.404	1.638	1.493	1.845	1.350	1.348	1.577	1.686	1.777	2.077
Q95	1.333	1.491	1.462	1.762	1.540	1.946	1.428	1.456	1.666	1.822	1.844	2.223
ALTERNATIVE HYPOTHESIS, δ=5												
Q5	0.692	0.711	0.787	0.808	0.884	0.936	0.865	0.884	0.966	0.979	1.119	1.158
Q10	0.793	0.814	0.895	0.925	1.011	1.084	0.941	0.946	1.055	1.073	1.222	1.308
Q25	0.943	0.953	1.080	1.141	1.216	1.342	1.070	1.065	1.234	1.256	1.426	1.576
mean	1.079	1.113	1.236	1.355	1.360	1.571	1.205	1.193	1.415	1.488	1.608	1.851
media	1.069	1.134	1.205	1.352	1.318	1.542	1.210	1.221	1.401	1.492	1.581	1.828
Q75	1.223	1.308	1.371	1.584	1.471	1.776	1.356	1.367	1.589	1.712	1.769	2.092
Q90	1.333	1.496	1.467	1.763	1.552	1.940	1.481	1.527	1.722	1.910	1.889	2.304
Q95	1.394	1.617	1.516	1.877	1.593	2.042	1.556	1.629	1.792	2.033	1.949	2.429
ALTERNATIVE HYPOTHESIS, δ=7												
Q5	0.917	0.931	1.058	1.106	1.187	1.292	1.118	1.123	1.315	1.387	1.516	1.727
Q10	0.995	1.028	1.155	1.229	1.285	1.400	1.185	1.190	1.405	1.492	1.598	1.846
Q25	1.129	1.193	1.290	1.444	1.406	1.575	1.324	1.349	1.548	1.702	1.720	2.062
mean	1.247	1.360	1.398	1.625	1.491	1.752	1.462	1.533	1.686	1.910	1.835	2.271
media	1.233	1.362	1.373	1.607	1.469	1.750	1.457	1.530	1.672	1.895	1.815	2.253
Q75	1.354	1.539	1.486	1.792	1.562	1.931	1.601	1.702	1.811	2.102	1.927	2.458
Q90	1.441	1.685	1.555	1.945	1.616	2.091	1.713	1.851	1.909	2.261	2.009	2.631
Q95	1.484	1.787	1.590	2.031	1.654	2.182	1.770	1.949	1.969	2.375	2.059	2.722