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The ‘Invisible Hand’ of Vote Markets

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Abstract

This paper studies electoral competition between two non-ideological parties when voters are free to trade votes for money. We find that allowing for vote trading has significant policy consequences, even if trade does not actually take place in equilibrium. In particular, the parties’ equilibrium platforms are found to converge (hence, there is no reason for vote trading) to the ideal policy of the mid-range voter, instead of converging to the peak of the median voter (as they do when vote trading is forbidden). That is, a market for votes may not change the outcome only by redistributing the political power among voters when the parties’ policy proposals are fixed (e.g., Casella, Llorente-Saguer, and Palfrey, 2012, etc.), but also by acting as an *invisible hand*—modifying parties’ incentives when platform choice is endogenous.

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Keywords: Electoral competition; invisible hand; vote markets; mid-range voter; Downsian model.

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1 Introduction

The analysis in this paper blends two key strands of political economics literature: electoral competition and vote trading. These strands are closely related but have never been combined as they appear to deal with diverse issues. Models of electoral competition have produced a variety of results on how parties choose policy platforms under a variety of electoral rules, but—to our knowledge—there is none that allows voters to exchange votes for money. On the other hand, models of vote trading have extensively studied how voters' cardinal preferences determine their vote-market actions, but they consider policy platforms as fixed and hence overlook parties' reactions.[1] In a more general setting where parties select strategically their policy platforms before voters engage in vote trading, rational parties should anticipate the effects of vote markets on the vote distribution and design their policy platforms accordingly. For instance, they could aim at accommodating the preferences of voters who care the most about the implemented policy—as these individuals are potential vote buyers and may end up with an increased number of votes after vote trading—rather than to attract as many supporters as possible. Hence, without ignoring the various critiques about vote trading, we believe that from a neutral perspective it is interesting to examine the policy implications of vote markets when the parties' policy platforms are endogenous.[2]

We investigate the effects of vote trading on voters' and parties' behavior in a two-party power-sharing system where the policy implemented after the election is a compromise between the competing platforms, with a party's weight on the implemented policy being proportional to its vote share.[3] The study of this electoral rule simplifies our analysis and makes it relevant to many societies where proportional representation is used in a significant extend. For example, many countries are parliamentary democracies, and policies represent a settlement among the competing parties. We stress though—and

[1] See, for instance, Philipson and Snyder (1996), Casella, Llorente-Saguer, and Palfrey (2012), Casella and Turban (2014), Casella, Palfrey, and Turban (2014), Xefteris and Ziros (2017; 2018), Tsakas, Xefteris, and Ziros (2021), Casella and Macé (2021), Casella and Sanchez (2022), among others.

[2] For a complete account of the main criticisms to vote markets, one is referred to the above cited papers that study vote trading with exogenously fixed policy platforms.

[3] Similar approaches can be found in many other works in the political economics literature. See, for instance, Lijphart (1984), Ortuño-Ortín (1997), Grossman and Helpman (1999), Llavador (2006), Merrill and Adams (2007), De Sinopoli and Iannantuoni (2007), Saporiti (2014), Matakos, Troumpounis, and Xefteris (2016), among others.

explain in the end of the paper—that the insights provided by the analysis do not depend crucially on this particular system and extend to alternative ones (e.g., simple majority rule).

Concerning our findings, we demonstrate that for any pair of distinct party platforms and for every generic profile of voters' preferences, there exists an equilibrium at the vote-trading stage where all individuals choose to buy or sell votes. In this equilibrium, bidding for purchases of votes comes only from two voters—the biggest supporter of each party—and all other individuals sell their votes. The party that is supported by the voter who is more concerned about the outcome of the election than any other voter collects the majority of votes. Rational vote-share maximizing parties which expect this behavior, do not have incentives to try to be appealing to as many voters as possible (as when vote trading is not allowed), but only to be preferred by the voter that cares most about the electoral outcome. These incentives lead them to choose platforms that converge to the ideal policy of the mid-range voter; that is, the voter who has equal distance from the two extreme voters.[4] Platform convergence is explained as follows: any move of a party's announced policy away from the ideal policy of the mid-range voter towards an extreme supporter makes the other extreme supporter to be the one most concerned one about the electoral outcome, and hence leads to an increase in the vote-share of the party that still proposes the ideal policy of the mid-range voter.

It follows that voters do not trade votes for money in equilibrium. However, this is not due to voters' ethical or democratic concerns. On the contrary, all voters are willing to trade if there is any profit in doing so, but platform convergence annihilates the incentives. Yet, allowing for vote trading is consequential. Vote markets are shown to be able to affect policy also by acting as an invisible hand: when vote trading is allowed, parties choose the ideal policy of the mid-range voter, and this (generically) differs from the equilibrium outcome when vote trading is not allowed (i.e., the ideal policy of the median voter). While this is reminiscent of electoral competition with costly voting—in equilibrium the two candidates propose the same policy and there is zero turnout

[4]In light of the equilibrium features, our work is associated with Hirata and Kamada (2020), which studies a two-party election where a party's winning probability depends on the amount of contributions it raises.

(Ledyard, 1984)—we note that in that case the outcome is utilitarian, while here it is not. That is, vote markets are neither compatible with majoritarian principles (when the ideal policy of the mid-range voter differs from the one of the median voter), nor they align with utilitarian ones (when the ideal policy of the mid-range voter differs from the utilitarian one).

In what follows, we present the model (Section 2) and the formal results (Section 3).

2 Model

We consider a society of $n > 2$ voters and two parties, $\mathcal{A}$ and $\mathcal{B}$. The two parties compete by simultaneously choosing their platforms in the policy space $Y = [0, 1]$. Let α be the platform of party $\mathcal{A}$ and β that of party $\mathcal{B}$. The vote shares of the two parties after voting are $v_\alpha \in [0, 1]$ and $v_\beta = 1 - v_\alpha$.

Under power sharing, the implemented policy z is a linear combination of party platforms with weights depending on their vote shares, $z = v_\alpha \alpha + (1 - v_\alpha)\beta$. In our setup, parties are non-ideological and aim at maximizing their vote shares.

Each voter has one vote and one unit of money. Voters' ideal policies are distributed along the space $Y = [0, 1]$ and each voter i is characterized by her distinct ideal policy y_i. An individual's utility depends on the implemented policy z and is given by

$$u_i = -(y_i - z)^2 + m_i,$$

where $m_i \geq 0$ is the amount of money she ends up with after vote trading.[5,6] All information is publicly known—that is, there is no uncertainty about voters' preferences.

[5] Considering that a voter's utility declines with the distance between her ideal policy and the implemented one is essential for our main result about platform convergence. This is also the case in standard electoral competition models without vote trading; hence, such a modeling allows for a direct comparison with the results when vote trading is not allowed, which is a main inquiry of this paper.

[6] Most assumptions employed (e.g., two parties, quasilinear preferences) are standard in the vote-trading literature (e.g., Casella, Llorente-Saguer, and Palfrey, 2012, Casella, Palfrey, and Turban, 2014, Xefteris and Ziros, 2017), and provide a convenient way for solving the problem of nonexistence of equilibrium in a market for votes. This literature also abstracts from several empirically relevant factors in real-world settings (e.g., elite influence, lobbies), which future research should consider.

If parties choose distinct platforms, voters are partitioned into two types $t_i \in \{\alpha, \beta\}$, where $t_i = \alpha$ if $|y_i - \alpha| < |y_i - \beta|$ and $t_i = \beta$ if $|y_i - \alpha| > |y_i - \beta|$. If $|y_i - \alpha| = |y_i - \beta|$ individual i votes for party $\mathcal{A}(\mathcal{B})$ with probability 1/2. For distinct party platforms we also define the extreme (or strongest) supporter of each party as the voter with the most intense preferences. Without loss of generality, we assume that if $\alpha < \beta$ the extreme supporter of party $\mathcal{A}$ has $y_i^{\alpha} = 0$ and the extreme supporter of party $\mathcal{B}$ has $y_i^{\beta} = 1$; hence, the mid-range ideology is $y = 1/2$.

The timing of the game is as follows: first, parties choose their platforms; then, vote trading takes place; next, individuals cast the number of votes they have after the vote-trading stage to maximize their utilities; finally, the payoffs of all players are computed.

Vote trading takes place according to the rules of strategic market games (originating in Shapley and Shubik, 1977). There is a trading post where each voter chooses whether to offer her whole vote for sale, $q_i \in \{0, 1\}$, or whether to place a monetary bid for purchase of votes, $b_i \in [0, 1]$, with the restriction that a voter is not allowed to be active on both sides of the market. Hence, the strategy set of voter i at the vote-trading stage is $S_i = \{(b_i, q_i) : b_i \in [0, 1], q_i \in \{0, 1\},\ b_i \cdot q_i = 0\}$.

Given a strategy profile $(b, q) \in \prod_i S_i$, let B and Q denote aggregate bids and offers of all voters, B^T and Q^T denote aggregate bids and offers of all voters of type $T \in \{\alpha, \beta\}$. The number of votes that individual i possesses after vote trading is denoted by x_i.

When $BQ = 0$, no trade takes place and hence the allocation of votes and money is $(x_i, m_i) = (1, 1)$ for all voters. For a strategy profile that results in active trading (i.e., $BQ > 0$), the price of a vote is $p = B/Q$ and the amounts of votes and money of individual i are

$$(x_i, m_i) = \begin{cases} (1 + b_i/p, 1 - b_i) & \text{if } b_i > 0,\ q_i = 0, \\ (0, 1 + p) & \text{if } b_i = 0,\ q_i = 1, \\ (1, 1) & \text{if } b_i = 0,\ q_i = 0. \end{cases}$$

According to this allocation rule, votes offered for sale are distributed among vote buyers in proportion to their bids, whereas vote sellers receive p units of money. Note that votes are perfectly divisible and hence a buyer might end up having a non-integer number of votes. This is perfectly legitimate in our framework as all that matters is the share and not the actual number of votes that a party receives. If $BQ > 0$, the vote shares of the two parties after vote trading are $v_\alpha = \frac{1}{n}(n_\alpha - Q^\alpha + B^\alpha/p)$ and $v_\beta = 1 - v_\alpha = \frac{1}{n}\left(n_\beta - Q^\beta + B^\beta/p\right)$.

Also note that when parties choose identical platforms, the number of votes one has does not affect the implemented policy and hence there is no scope for trade.

Given that the behavior of players at the voting stage is completely unambiguous (i.e., submitting all votes one has to her preferred party is one's dominant strategy), we essentially have a two-stage game, and an equilibrium is defined as a profile of pure strategies that form a subgame perfect Nash equilibrium (SPNE).

3 Results

Proposition 1 considers the second stage of the game and shows that if platforms diverge, apart from the no-trade equilibrium,[7] equilibria involving trade generically exist. In particular, there is always an equilibrium where the two extreme supporters buy votes and all other individuals offer their votes for sale. For the remainder of this paper, we use the term full-trade equilibrium when we refer to this equilibrium.[8]

Proposition 1 *For any subgame with $\alpha < \beta$ and for any distribution of ideal policies there exists an equilibrium where individuals with $y_i \in (0,1)$ sell their votes and the two individuals with $y_i \in \{0,1\}$ buy votes.*

[7] Choosing not to trade is always a best response of an individual when all other individuals choose not to trade.

[8] Let us note that other equilibria where some individuals trade while others prefer not to engage in vote trading cannot be ruled out for particular preference profiles. We focus on the full-trade equilibrium because it exists for every admissible profile of voters' preferences.

Proof. First, let us consider a type α voter who chooses $b_i^\alpha > 0$ in some equilibrium (b, q). This individual faces the problem

$$\max_{b_i^\alpha \in [0,1]} u_i = -(y_i^\alpha - z)^2 + 1 - b_i,$$

where $z = \frac{1}{n}\left(n_\alpha - Q^\alpha + B^\alpha/p\right)\alpha + (1 - \frac{1}{n}\left(n_\alpha - Q^\alpha + B^\alpha/p\right))\beta$,

which is well behaved in $b_i \in [0,1]$.[9] Solving the problem and rearranging we derive that in an interior solution we have

$$y_i^\alpha = \frac{1 + 2z\partial z/\partial b_i^\alpha}{2\partial z/\partial b_i^\alpha}. \tag{1}$$

Similarly, by solving the maximization problem for a type β voter we obtain that in an interior solution it is true that

$$y_i^\beta = \frac{1 + 2z\partial z/\partial b_i^\beta}{2\partial z/\partial b_i^\beta}. \tag{2}$$

Now let us consider the possibility of a full-trade equilibrium where the two individuals with $y_i = \{0, 1\}$ buy votes and the remaining individuals sell their votes (that is, $Q = n - 2$).

In such a case, the equilibrium bids of type α and type β buyers are

$$\bar{b}^\alpha = \frac{2(n-2)}{n}\frac{(\beta-\alpha)(\alpha+\beta(n-1))^2(\beta+\alpha(n-1)-n)}{(2\beta+\alpha(n-2)-(\beta+1)n)^3}, \tag{3}$$

$$\bar{b}^\beta = \frac{2(n-2)}{n}\frac{(\alpha-\beta)(\alpha+\beta(n-1))(\beta+\alpha(n-1)-n)^2}{(2\beta+\alpha(n-2)-(\beta+1)n)^3}, \tag{4}$$

which are both positive and budget feasible for $0 \leq \alpha < \beta \leq 1$ and $n > 2$.[10,11] Straightforwardly, given the concavity of the maximization problem, neither of them wishes to deviate to any other bidding amount.

[9] That is, if at least one other individual sells her vote, the utility function is well defined, differentiable, and strictly concave in $[0, 1]$.

[10] See the Appendix.

[11] We notice that the equilibrium bids of the two buyers are increasing in the size of the electorate, but they are not affected by the ideal policies of all other voters. Moreover, one can easily show that $\bar{b}^\alpha > \bar{b}^\beta$ ($\bar{b}^\alpha < \bar{b}^\beta$), whenever $\frac{\alpha+\beta}{2} > \frac{1}{2}$ ($\frac{\alpha+\beta}{2} < \frac{1}{2}$).

Moreover, the extreme supporters never deviate to any other strategy. The utility of the extreme supporter with $y_i^\alpha = 0$ when she sells her vote is $u_i^\alpha(b_i = 0, q_i = 1) = -\beta^2 + 1 + \frac{1}{n-1}\bar{b}^\beta$ and when she chooses to keep her vote is $u_i^\alpha(b_i, q_i = 0) = -(-\frac{1}{n}\alpha - \frac{n-1}{n}\beta)^2 + 1$. Substituting for the posited bids of type β vote buyer, we derive that $u_i^\alpha(b_i, q_i = 0) > u_i^\alpha(b_i = 0, q_i = 1)$ for $0 \leq \alpha < \beta \leq 1$ and $n > 2$,[12] that is the extreme supporter with $y_i^\alpha = 0$ prefers not to trade than to sell her vote. We can also establish that she prefers to bid the equilibrium amount $\bar{b}^\alpha$ than to keep her vote $(b_i, q_i = 0)$. If $b_i = 0$ was preferable to $\bar{b}^\alpha$ there should have been a local minimum in between, but this is not the case because when the other extreme voter chooses $\bar{b}^\beta$, there is a unique bidding amount $(\bar{b}^\alpha)$ at which the derivative of the expected utility is zero.

With similar arguments we can establish that the extreme supporter with $y_i^\beta = 1$ never deviates to selling her vote or to keeping her vote. Hence, the posited vote-buying strategies of the extreme supporters are their unique best responses.

Next, we show what no individual with ideal policy $y_i \in (0, 1)$ places a monetary bid to acquire more votes. Indeed, given that the supporter with $y_i^\alpha = 0$ is buying votes, no other type α individual is buying votes as expression (1) cannot be satisfied for any $y_i^\alpha > 0$. Similarly, given that the supporter with $y_i^\beta = 1$ is buying votes, no other type β individual is buying votes as expression (2) cannot be satisfied for any $y_i^\beta < 1$.

Finally, we show that if the extremists use the posited vote-buying strategies and all others are expected to sell, then any individual with $y_i \in (0, 1)$ prefers to sell her vote than to simply keep her vote. With such a profile of expected behaviors in place, the utility that a type α supporter with $y_i > 0$ derives from selling her vote is $u_i(b_i = 0, q_i = 1) = -(y_i^\alpha - z)^2 + 1 + \frac{\bar{b}^\alpha + \bar{b}^\beta}{n-2}$, where $z = \frac{1}{n}\left(1 + \frac{\bar{b}^\alpha}{\bar{b}^\alpha + \bar{b}^\beta}(n-2)\right)\alpha + \left(1 - \frac{1}{n}\left(1 + \frac{\bar{b}^\alpha}{\bar{b}^\alpha + \bar{b}^\beta}(n-2)\right)\right)\beta$. On the other hand, the utility that a type α supporter with $y_i > 0$ derives from keeping her vote is $u_i(b_i, q_i = 0) = -(y_i^\alpha - \hat{z})^2 + 1$, where $\hat{z} = \frac{1}{n}\left(2 + \frac{\bar{b}^\alpha}{\bar{b}^\alpha + \bar{b}^\beta}(n-3)\right)\alpha + \left(1 - \frac{1}{n}\left(2 + \frac{\bar{b}^\alpha}{\bar{b}^\alpha + \bar{b}^\beta}(n-3)\right)\right)\beta$. Substituting for the posited bids of the extreme supporters, we derive that for a type α supporter with $y_i^\alpha > 0$ it is true that $u_i(b_i = 0, q_i = 1) > u_i(b_i, q_i = 0)$ for $0 \leq \alpha < \beta \leq 1$ and $n > 2$.[13]

[12] See the Appendix.
[13] See the Appendix.

With similar arguments we can establish that a type β supporter with $y_i^\beta < 1$ always sells her vote in a full-trade equilibrium. Thus, all individuals with $y_i \in (0, 1)$ sell their votes. That is, a full-trade equilibrium always exists when platforms diverge and is characterized by the actions described above. ■

Our results concerning the patterns of vote trading when policy platforms are fixed (i.e., the fact that only the strongest supporter of each party buys votes and the other voters sell their ballots; and that the party supported by the most concerned voter receives a higher share of votes) align with earlier analyses in alternative institutional contexts. Indeed, Casella and Turban (2014) derive sufficient conditions for the existence of a similar equilibrium in a majoritarian system, employing an ex ante competitive equilibrium (a solution notion introduced in Casella, Llorente-Saguer, and Palfrey, 2012). Moreover, Xefteris and Ziros (2018) also describe a similar equilibrium considering that the policy outcome is probabilistic (i.e., the platform of each party is implemented with a probability equal to its vote share).[14]

Next, we examine the behavior of the two parties in the first stage of the game, and we present the main result of the paper.

Proposition 2 *There exists a SPNE where party platforms converge to the mid-range ideology. Moreover, this SPNE is unique among those that involve a full-trade equilibrium in every subgame with distinct platforms.*

Proof. First, we show that the profile $\alpha = \beta = 1/2$ is an equilibrium. If $\alpha = \beta = 1/2$ there is no vote trading and a party's expected vote share is equal to 1/2. Suppose that

[14] In this paper the policy outcome is deterministic (i.e., a weighted average of the parties' platforms, with weights being equal to their vote shares). When voters' utility functions are strictly concave (as they are here), then the probabilistic setup is not equivalent to the deterministic one: in the former case, the relationship between a voter's utility and the vote share of her preferred party is linear, while in the latter the relationship is strictly concave, making the problem distinctly more complicated. For this reason, we cannot simply refer to earlier arguments to establish Proposition 1.

Arguably, the current deterministic setup, where policy is a compromise of the two platforms and not the product of a 'random dictatorship', is a better assumption in many ways and it aligns with how several papers in the literature treat policy formation in the presence of divergent platforms (e.g., Ortuño-Ortín, 1997, Merrill and Adams, 2007, Matakos, Troumpounis, and Xefteris, 2016). Moreover, by employing this assumption, the current paper, beyond its main contribution (i.e., to show how allowing for vote trading affects policy outcomes when platforms are endogenous), makes a secondary point: it establishes that earlier results provided in probabilistic settings, are also valid in more standard settings of deterministic policy formation.

party $\mathcal{A}$ deviates to a platform $\hat{\alpha} \in [0, 1/2)$. Such a deviation induces vote trading and in the full-trade equilibrium, expressions (3) and (4) with $\beta = 1/2$ yield

$$\bar{b}^{\hat{\alpha}} = \frac{2(n-2)}{n} \frac{(\frac{1}{2} - \hat{\alpha})(\hat{\alpha} + \frac{1}{2}(n-1))^2(\frac{1}{2} + \hat{\alpha}(n-1) - n)}{(1 + \hat{\alpha}(n-2) - \frac{3}{2}n)^3},$$

$$\bar{b}^{\beta} = \frac{2(n-2)}{n} \frac{(\hat{\alpha} - \frac{1}{2})(\hat{\alpha} + \frac{1}{2}(n-1))(\frac{1}{2} + \hat{\alpha}(n-1) - n)^2}{(1 + \hat{\alpha}(n-2) - \frac{3}{2}n)^3},$$

for which we can easily derive $\frac{\bar{b}^{\beta}}{\bar{b}^{\hat{\alpha}}} = \frac{-\frac{1}{2}+\hat{\alpha}+(1-\hat{\alpha})n}{-\frac{1}{2}+\hat{\alpha}+\frac{1}{2}n} > 1$ for $\hat{\alpha} \in [0, 1/2)$ and $n > 2$. That is, the type β vote buyer submits a greater bid than the type α vote buyer, and hence after vote trading the vote share of party $\mathcal{A}$ is less than 1/2. In other words, party $\mathcal{A}$ cannot increase her vote share by choosing a different platform. Similarly, there is no profitable deviation for party $\mathcal{B}$.

Next, we show that no profile involving $\alpha = \beta \neq 1/2$ is an equilibrium. If the two parties choose $\alpha = \beta \neq 1/2$ there is no vote trading and the two parties have equal expected vote shares. In such an eventuality, there is always a profitable deviation. Indeed, suppose that party $\mathcal{B}$ deviates to platform $\hat{\beta} = 1/2$. Such a deviation causes vote trading and for the equilibrium bids we have $\bar{b}^{\alpha} < \bar{b}^{\hat{\beta}}$. That is, the type β vote buyer submits a greater bid than the type α vote buyer, and hence after vote trading there is a increase in the vote share of party $\mathcal{B}$.

Finally, we show that no profile involving $\alpha \neq \beta$ is an equilibrium. If $\alpha \neq \beta$ there is vote trading leading either to a tie or to a majority winner. In such an eventuality, using similar arguments as before we can always find a profitable deviation. For example, if for party $\mathcal{A}$ with $\alpha \neq 1/2$ we have that $v_{\alpha} \leq 1/2$, then party $\mathcal{A}$ can always increase its vote share by deviating to $\hat{\alpha} = 1/2$. ■

The reason why the parties converge to the mid-range ideology—and not to the median one—lies in the fact that, when vote trading is allowed, parties compete for the bids of the two extreme voters and not for the votes of centrist citizens as in the standard Downsian model. Indeed, as we saw in the proof of Proposition 1, the electoral outcome does not depend on the preferences of the voters with non-extreme ideal policies. Moreover, the bid of an extreme voter is increasing in the utility difference between the two platforms. Since, the voters' utilities are concave in the implemented policy, the most concerned voter is always the one farthest away from the midpoint between the two platforms, leading parties to converge exactly to the midpoint between the ideal policies of the two extreme voters.

In light of the discussion following the proof Proposition 1, we finally note that our main result remains relevant in many more settings than the one employed by this paper. That is, the finding that parties should be expected to converge to the mid-range ideology—and not to the ideology of the median voter—when vote trading is allowed, does not hinge on the specific voting system and trading mechanism, but holds in alternative settings as well, given the formal results that already exist in the vote-trading literature (e.g., Casella, Llorente-Saguer, and Palfrey, 2012, Casella and Turban, 2014). The reason why we opted for a power-sharing rule and a strategic market game to formally solve this model is twofold: (i) this setting is accepted in the literature as a relevant one when thinking about the policy consequences of vote trading (see, for instance, Casella and Macé, 2021, Tsakas, Xefteris, and Ziros, 2021, Xefteris and Ziros, 2017), and, perhaps more importantly, (ii) this set of assumptions allows us to provide a compact, yet, complete formal argument establishing the main result.

Indeed, the current modeling approach allows us to proceed with a standard SPNE analysis, while if we assumed, instead, a majority rule and an ex ante competitive market for votes—like Casella and Turban (2014)—we could still arrive to a similar conclusion, but we would first need to define a novel solution notion which would combine elements of SPNE (i.e., parties should compete in the first stage taking in account their beliefs regarding their competitor's behavior and the subsequent behavior of the voters), and ex ante competitive equilibrium (for the voters' 'subgames').

Appendix

Calculations showing that expressions (3) and (4) are positive and budget feasible:

In expression (3) the numerator is negative because $\beta + \alpha(n-1) - n < 0$ and the remaining terms are positive for $0 \leq \alpha < \beta \leq 1$ and $n > 2$. The denominator is negative because $2\beta + \alpha(n-2) - (\beta+1)n = \beta(2-n) + n(\alpha-1) - 2\alpha < 0$ for $0 \leq \alpha < \beta \leq 1$ and $n > 2$. Hence, $\bar{b}^{\alpha}$ is positive. Moreover $\bar{b}^{\alpha} < 1$, as each term $\frac{(n-2)}{n}$, $\frac{(\alpha+\beta(n-1))^2}{(2\beta+\alpha(n-2)-(\beta+1)n)^2}$, $\frac{2(\beta-\alpha))(\beta+\alpha(n-1)-n)}{2\beta+\alpha(n-2)-(\beta+1)n}$ is less than one for $0 \leq \alpha < \beta \leq 1$ and $n > 2$.

In expression (4) the numerator is negative because $(\alpha - \beta) < 0$ and the remaining terms are positive for $0 \leq \alpha < \beta \leq 1$ and $n > 2$. The denominator is negative because $2\beta + \alpha(n-2) - (\beta+1)n = \beta(2-n) + n(\alpha-1) - 2\alpha < 0$ for $0 \leq \alpha < \beta \leq 1$ and $n > 2$. Hence, $\bar{b}^{\beta}$ is positive. Moreover $\bar{b}^{\beta} < 1$, as each term $\frac{(n-2)}{n}$, $\frac{2(\alpha-\beta)(\alpha+\beta(n-1))}{2\beta+\alpha(n-2)-(\beta+1)n}$, $\frac{(\beta+\alpha(n-1)-n)^2}{(2\beta+\alpha(n-2)-(\beta+1)n)^2}$ is less than one for $0 \leq \alpha < \beta \leq 1$ and $n > 2$.

Calculations showing that the type α extreme supporter prefers to keep her vote than to sell it:

In the full-trade equilibrium, substituting for the posited bid of the type β vote buyer, the utility of a type α supporter with $y_i^{\alpha} = 0$ from keeping her vote is $u_i^{\alpha}(b_i, q_i = 0) = -(-\frac{1}{n}\alpha - \frac{n-1}{n}\beta)^2 + 1$ and from selling her vote is $u_i^{\alpha}(b_i = 0, q_i = 1) = -\beta^2 + 1 + \frac{2(n-2)}{n(n-1)} \frac{(\alpha-\beta)(\alpha+\beta(n-1))(\beta+\alpha(n-1)-n)^2}{(2\beta+\alpha(n-2)-(\beta+1)n)^3}$. Their difference is $u_i^{\alpha}(b_i, q_i = 0) - u_i^{\alpha}(b_i = 0, q_i = 1) = \frac{1}{n^2}(\beta - \alpha)\left(\alpha + \beta(2n-1) + \frac{2n(n-2)(\alpha+\beta(n-1))(\beta+\alpha(n-1)-n)^2}{(n-1)(2\beta+\alpha(n-2)-(1+\beta)n)^3}\right) =$

$\frac{1}{n^2}(\beta-\alpha)\left(\beta n + (\alpha + \beta(n-1))\left(1 + \frac{2n(n-2)(\beta+\alpha(n-1)-n)^2}{(n-1)(2\beta+\alpha(n-2)-(1+\beta)n)^3}\right)\right) > 0$ for $0 \leq \alpha < \beta \leq 1$ and $n > 2$, because each term $\frac{n-2}{n-1}$, $\frac{(\beta+\alpha(n-1)-n)^2}{(2\beta+\alpha(n-2)-(1+\beta)n)^2}$ is less than one, whereas the absolute value of the term $\frac{n}{2\beta+\alpha(n-2)-(\beta+1)n}$ is also less than one. Thus, the absolute value of the term $1 + \frac{2n(n-2)(\beta+\alpha(n-1)-n)^2}{(n-1)(2\beta+\alpha(n-2)-(1+\beta)n)^3}$ is less than one and $\beta n + (\alpha + \beta(n-1))\left(1 + \frac{2n(n-2)(\beta+\alpha(n-1)-n)^2}{(n-1)(2\beta+\alpha(n-2)-(1+\beta)n)^3}\right) > 0$.

Calculations showing that a type α supporter with $y_i^{\alpha} > 0$ prefers to sell her vote than to keep it:

In the full-trade equilibrium, substituting for the posited bids of the extreme supporters, the utility of a type α supporter with $y_i^{\alpha} > 0$ from selling her vote is $u_i(b_i = 0, q_i = 1) =$

$$\frac{-2(\alpha-\beta)^3+(\alpha-\beta)^2(-3+2\alpha-2\beta-4y_i(y_i-1))n+2(\alpha-\beta)((\alpha-1)y_i(2y_i-1)+\beta(-2+\alpha+(3-2y_i)y_i))n^2-(\beta(y_i-1)+y_i-\alpha y_i)^2n^3}{n((\beta-\alpha)(n-2)+n)^2}$$

and from keeping her vote is $u_i(b_i, q_i = 0) = -\frac{((\alpha-\beta)^2-(\alpha-\beta)(\alpha-2+2y_i)n+(\beta+(\alpha-\beta-1)y_i)n^2)^2}{n^2((\beta-\alpha)(n-2)+n)^2}$.

Their difference is

$u_i(b_i = 0, q_i = 1) - u_i(b_i, q_i = 0) = \frac{(\alpha-\beta)(\beta+\alpha(n-1)-n)\left(-(\alpha-\beta)^2+(\alpha-\beta)(-1+\alpha+4y_i)n+2(1+\beta-\alpha)y_i n^2\right)}{n^2((\beta-\alpha)(n-2)+n)^2} > 0$ for $y_i \in (0,1)$, $0 \leq \alpha < \beta \leq 1$ and $n > 2$, because the product $(\alpha - \beta)(\beta + \alpha(n-1) - n)$ is positive and the term $(-(\alpha-\beta)^2 + (\alpha-\beta)(-1+\alpha+4y_i)n + 2(1+\beta-\alpha)y_i n^2)$, which can be written as $(\beta-\alpha)((\alpha-\beta)+(1-\alpha)n)+(\beta-\alpha)(2n-4)y_i n+2y_i n^2$, is also positive.

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