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A Spatial Model of Perfect Competition

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Abstract

This paper introduces a spatial dimension to an otherwise standard class of strategic market games in order to study the issue of market location in a perfectly competitive setup. In this framework, each player decides strategically where and what quantities she wishes to trade and, hence, the market structure (or simply the distribution of the active trading posts and prices) emerges endogenously. We conduct a comprehensive analysis for a class of simple games with a continuum of traders and we show that (*i*) not all market structures can support a Nash equilibrium, (*ii*) at least some multi-market structures can support a Nash equilibrium and (*iii*) prices in a multi-market Nash equilibrium, generically, diverge.

JEL classification: R32, C72, D41

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1 Introduction

Marketplaces are locations in which economic agents perform their trading activities and their distribution over a geographical area may affect social welfare in a number of ways. A thorough analysis of the determinants of the number and of the distribution of marketplaces should, thus, improve our understanding on various questions addressed in the literature of spatial economics but also on more general issues of economic theory. This paper considers that heterogeneous agents (in terms of consumption preferences and initial endowments), whose residences are distributed over a geographical area, *strategically* decide where and how much to trade and, thus, the number and the distribution of marketplaces over the geographical area emerge endogenously. A formal study of this game with an arbitrarily large number of agents (perfect competition) allows us to characterize the nature and the properties of sustainable market structures.

In order to accommodate trade in a model with instrumental agents, we employ the methodology of strategic market games (*SMG*), which, to the best of our knowledge, has never been used before in the context of location theory. This class of games, originating in Shubik (1973) and in Shapley and Shubik (1977), provide a non-cooperative foundation to perfect competition as they have been particularly successful in showing that in mass markets individuals tend to exhibit price taking behavior[1]. Given that *SMG* are often thought of as an extension of Cournot's analysis of competition to the general equilibrium framework, we consider that they could be associated with (and utilized by) many branches of the existing literature on spatial economics[2].

In the present paper we analyze a simple class of *SMG* with an arbitrarily large number of traders (perfectly competitive setup), enriched with a spatial dimension. That is, unlike common *SMG*, players are not only characterized by preferences on consumption bundles and a set of initial endowments but by a location parameter and an aversion to transportation costs too. The game that we consider is a variant of the "bilateral oligopoly" model of Gabszewicz

[1] For formal studies on the relation between non-cooperative equilibria of SMG and competitive equilibria the reader is referred to Dubey and Shubik (1978), Postlewaite and Schmeidler (1978), Mas-Colell (1982), Sahi and Yao (1989) and Dubey and Shapley (1994).

[2] For example, general equilibrium approaches to location-relevant issues can be found in Wang (1990), Fujita and Smith (1987), Berliant and Wang (1993), Berliant et al. (1990), Karmann (1982), Baesemann (1977), Schweizer et al. (1976).

and Michel (1997), whose key feature is the separation of individuals into two types, where each type has a corner endowment in one of the two goods. Therefore, there are two factors that determine the choice of location; the prices (i.e., the ratio at which the two goods are exchanged) in the different markets (which, in turn, is determined by the populations of individuals of both types and their actions in each market) and the distance of a market from the initial location of an individual[3]. In addition, we assume that the location parameters (residences) of the individuals of the two types follow distinct distributions. In particular, we consider that there is a high concentration of individuals of one type in the left half of the unit interval and a high concentration of individuals of the other type in the right half. This generic assumption is a key feature of the model and it leads to some appealing results about the divergence of prices in multi-market formulations.

Regarding the main results of the paper, we initially prove that there exists a Nash equilibrium where all individuals trade on the same market independently of its location. Then we move on to consider structures with more markets and we prove that some multi-market Nash equilibria do exist[4]. It is also demonstrated that in a multi-market Nash equilibrium no single price clears all markets for a commodity, and thus we show that the "law of one price" fails even in a perfectly competitive setup. A key result of the paper (as shown in Propositions 3,5 is that no equilibrium exists if markets are set on contiguous locations, hence not all market structures can support a Nash equilibrium. As a complement, Proposition 4 proves that there is an upper bound on the number of active markets in equilibrium. The intuition behind these results is clear; as any two active markets exhibit unequal prices, the benefit from moving to the market with the most preferred price offsets the small transportation costs between neighbor markets; hence there is a collapse of trade in some market locations and the given market structures fail to support an equilibrium. Therefore, unlike the standard market games, in our approach the number of marketplaces is determined endogenously. Finally, in addition to the general results, we consider a representative case that provides a numerical analysis of our outcomes and allows for some further results about social welfare.

[3]Our agents' preferences are closely associated with agents' preferences of imperfect competition spatial models. See for instance, Hotelling (1929), D'Aspremont et al. (1979), Economides (1984), Osborne and Pitchik (1987), Salop (1979), Novshek (1980), Economides (1993), Hamilton et al. (1989), Anderson and Neven (1991), Hamilton et al. (1994), Gupta et al. (1997) and Huck et al. (2003).

[4]Equilibria with multiple markets can also be found in several other studies, e.g., in Kung and Wang (2012) for the relevant issue of knowledge transmission or in Fujita and al. (1997) for the formation of employment centers.

Our approach clearly bears no resemblance with most spatial competition models à la Hotelling, where, typically, a finite number of firms compete in terms of prices and locations. Nonetheless, the analysis in Anderson and Engers (1994), which also studies the choice of location under the price-taking hypothesis, is close to our framework. That paper studies the existence of equilibrium in a model where a finite number of firms merely choose their locations, as prices are determined subsequently by a planning authority whose only concern is the maximization of social surplus. The results exhibit that minimum differentiation (firms choosing the same location) occurs in equilibrium only when considering the duopoly case together with completely inelastic demand. On the other hand, minimum differentiation never arises in equilibrium when considering more than two firms or sufficiently elastic demand.

The rest of the paper is organized as follows. In Section 2 we present the model. The general results follow in Section 3. Section 4 presents a representative case of the model. Section 5 offers some concluding remarks. Finally, the proofs of the main results and a brief discussion about the properties of the considered utility functions can be found in the appendix section.

2 The model

We consider that a unit mass of type A (B) individuals is distributed on the $[0,1]$ interval (linear city) according to an absolutely continuous distribution function F_A (F_B). An individual is characterized by her actual location (or location parameter), her initial endowment and her preferences over consumption bundles of two distinct goods. An individual of type A possesses $w > 0$ units of good I and zero units of good II, whereas an individual of type B possesses $w > 0$ units of good II and zero units of good I. If we assume that good I serves as money, then individuals of type A and B can be thought of as the consumers and the producers (with zero production costs) respectively of good II. Consumption preferences for the two types of individuals are described by their utility functions, which we denote with $u_A(x^I, x^{II})$ for individuals of type A and with $u_B(x^I, x^{II})$ for individuals of type B. We suppose that the preferences of the two types are perfectly symmetric, so that $u_A(x^I, x^{II}) = u_B(x^{II}, x^I)$ for all x^I and x^{II} (this assumption can be relaxed at a significant analytical cost). We assume that these functions are increasing, continuous, strictly concave for every $(x^I, x^{II}) \in (0, +\infty)^2$, differentiable in both arguments and that when $x^{II} > 0$ ($x^I > 0$) it is the case that $\lim_{x^I \to +\infty} u_A(x^I, x^{II}) = +\infty$ ($\lim_{x^{II} \to +\infty} u_A(x^I, x^{II}) = +\infty$). For the derivation of our general results we further assume that $u_A(\phi, 0) = u_B(0, \phi) = 0$ for every $\phi \geq 0$ and that $w = 1$, which, obviously, does not make the qualitative part of our results less general compared to an analysis that considers an arbitrary w.

In our model agents will be free to trade *any amount* of their initial endowment at *any possible location* of our linear city. Locations of individuals are indexed by lower case letters and an individual with location parameter h who decides to trade in location l, faces the corresponding transportation cost according to the function $c(h, l)$.

Finally, in order to keep things tractable we further assume that:

Assumption 1 $F_A(z) = 1 - F_B(1 - z)$ (symmetry about the centre of the linear city) and $F_A^{''} > 0$ and $F_A'(0) = 0$ (monotonicity).

Assumption 2 $c(h, l) = |h - l|$,

Assumption 3 $\partial \hat{x}^I / \partial p_{II} = \partial \hat{x}^{II} / \partial p_I = 0$ where $(\hat{x}^I, \hat{x}^{II})$ is the unique solution of the problem $\max_{x^I, x^{II}} u_A(x^I, x^{II})$ s.t. $p_I x^I + p_{II} x^{II} \leq m$ for parameters $p_I, p_{II}, m > 0$ and

Assumption 4 $u_A(\hat{x}^I, \hat{x}^{II}) > 1$ for $p_I = p_{II}$ and $m = p_I w$.

A few comments regarding the above assumptions are in order. Assumption 1 suggests that the greater mass of type A individuals is located in the right part of the interval, whereas the greater mass of type B individuals is symmetrically located in the left part of the interval. Assumption 3, although it seems to be quite restrictive, is satisfied by many popular utility functions. For example, any Cobb-Douglas utility function has this property. It should be stressed at this point that this assumption *is a sufficient* condition for the derivation of our results but *not a necessary one.* Indeed, in the Appendix we show that our results extend to utility functions that fail to satisfy this property. Assumption 4, concerning the utility values at the solution of the maximization problem, just guarantees that the transportation costs that we are introducing in our *SMG* are not so high to make an agent better off by not trading at a "fair" market. This assumption is necessary because our aim is to investigate how trading locations and patterns arise in a spatial *SMG* without making the transportation costs the sole determinant factor of the behavior of any of our agents.[5]

We now turn to describe the rules of trade in our economy[6].

2.1 The strategic market game

The associated market game for this economy is described as follows. Each individual chooses *only* one a location where she places orders for purchases or sales of good $y \in \{I, II\}$ depending on her type.[7] Hence, each type A individual may bid an amount b (with $0 \leq b \leq 1$) of money (good I) in exchange for good II in location $l' \in [0, 1]$, with her bids in all other locations being equal to zero. Similarly, each type B individual may offer an amount q (with $0 \leq q \leq 1$) of good II in exchange for good I in location $l'' \in [0, 1]$, with her offers in all other locations being equal to zero. Hence, the strategy sets are $S_A = \{(l', b) \in [0, 1]^2\}$ for type A individuals and $S_B = \{(l'', q) \in [0, 1]^2\}$ for type B individuals. We denote by $\sigma'_{i,\alpha} = (l', b) \in [0, 1]^2$ the strategy of a type A individual $i \in [0, 1]$ with location parameter $\alpha \in [0, 1]$ and by $\sigma''_{g,\beta} = (l'', b) \in [0, 1]^2$ the strategy of a type B individual $g \in [0, 1]$ with location parameter $\beta \in [0, 1]$. For measuring the size of specific subsets of individuals we employ a Lebesque measure. Specifically, we denote by μ_A^j (μ_B^j) the Lebesque measure of type A (B) individuals who trade in location l^j.

Given a profile of bids and offers with $B^l, Q^l > 0$ (where B^l, Q^l are the total bids and offers in location l) the market clearing price in location l is:

$$p^l = \frac{B^l}{Q^l} \tag{1}$$

If at least one of B^l and Q^l is equal to zero we consider that trade does not take place in location l and agents get back their bids (offers).

[5]One could avoid making this assumption if one assumed instead transportation costs of the form $c(h, l) = t|h - l|$, for $t > 0$ sufficiently small.

[6]An similar investigation of a strategic market game with a continuum of traders can be found in Dubey and Shapley (1994).

[7]After the presentation of our formal arguments it will become evident that all our results hold even when each agent is allowed to trade at any number of locations: in an equilibrium of such a generalized game each agent trades at exactly one location. Hence this assumption is without loss of generality.

According to the allocation mechanism of the game, supplied quantities of the two goods in a specific location are distributed among individuals in proportion to their bids or offers. Hence, the final allocations for the traders participating in market l are as follows: $x_A^I = 1 - b$, $x_A^{II} = b/p^l$ for type A individuals and $x_B^I = p^l q$, $x_B^{II} = 1 - q$ for type B individuals.

Finally, the payoff of a type A individual $i \in [0, 1]$ with location parameter $\alpha \in [0, 1]$ who chooses to trade b units of her initial endowment in location l' given a strategy profile $\hat{\sigma}$ is given by:[8]

$$U_A^\alpha(\sigma'_{i,\alpha}, \hat{\sigma}) = u_A(1 - b, b/p') - c(\alpha, l') \text{ if } B^{l'}, Q^{l'} > 0 \text{ and}$$

$$U_A^\alpha(\sigma'_{i,\alpha}, \hat{\sigma}) = u_A(w, 0) - c(\alpha, l') \text{ otherwise}$$

and the payoff of a type B individual $g \in [0, 1]$ with location parameter $\beta \in [0, 1]$ who chooses to trade q units of her initial endowment in location l'' given a strategy profile $\hat{\sigma}$ is given by:

$$U_B^\beta(\sigma''_{g,\beta}, \hat{\sigma}) = u_B(p''q, 1 - q) - c(\beta, l'') \text{ if } B^{l''}, Q^{l''} > 0 \text{ and}$$

$$U_B^\beta(\sigma''_{g,\beta}, \hat{\sigma}) = u_B(0, w) - c(\beta, l'') \text{ otherwise.}$$

Hence, individuals of type A are viewed as solving the following problem:

$$\max_{\sigma'_{i,\alpha} \in [0,1]^2} U_A^\alpha(\sigma'_{i,\alpha}, \hat{\sigma}) \tag{2}$$

and individuals of type B are viewed as solving the following problem:

$$\max_{\sigma''_{g,\beta} \in [0,1]^2} U_B^\beta(\sigma''_{g,\beta}, \hat{\sigma}) \tag{3}$$

[8] Given that individual deviations from a strategy profile $\hat{\sigma}$ cannot affect prices at any location there is no substantial need to introduce a separate notation for the profile which contains the strategies of all individuals except from the deviating one. This would be necessary in a model with a finite number of players.

An equilibrium is defined as a strategy profile that forms a Nash equilibrium in the ensuing game with the strategic payoff functions that are given above. In particular we will focus in what we call *stable Nash equilibria*. A Nash equilibrium is understood to be stable in this framework if and only if almost all[9] players strictly prefer their equilibrium strategies to any other strategy. That is, stable Nash equilibria are almost strict Nash equilibria. The reason why we prefer to analyze only such Nash equilibria is straightforward: a Nash equilibrium that fails this refinement involves a positive measure of players (infinitely many) who are indifferent between their equilibrium strategy and at least one other strategy and, hence, it is unable to produce robust predictions.

It is evident in the description of our *SMG* that the locations of the active markets emerge endogenously. That is, if for example a mass of consumers and producers all *individually* decide to trade at a certain location l then this location l will be an active market. On the contrary if no individuals or if individuals only of one type decide to trade at location l, l will be an inactive market (a location at which no trade occurs).

Given that equilibrium market structures may be classified according to various characteristics, some further definitions are required at this point.

Definition 1 *An N-market structure is a profile of locations (active markets) $(l^1, \ldots, l^N)$ such that $l^1 < \ldots < l^N$ on the unit interval.*

Definition 2 *An N-market structure satisfies Symmetry if for every market j it is true that $l^j = k + \frac{j-1}{N-1}(1-2k)$. Moreover, an N-market structure, with $N > 1$, satisfies Full-Spreadness and Symmetry (FS) if and only if $l^j = \frac{j-1}{N-1}$.*

The idea behind a Symmetric market structure is that successive markets are equidistant. For the particular case in which $N = 1$ we consider that Symmetry is satisfied if and only if $l^1 = \frac{1}{2}$. In the above definition k stands for the location of market 1, that is the market closer to the left border. Therefore, if we set $k = 0$ we have an *FS* market structure, which indicates that active markets cover the whole linear city, i.e., $l^1 = 0, \ldots, l^N = 1$.

[9] "Almost all players" in this framework means all players except a subset of players whose Lebesgue measure is zero.

Definition 3 *An N-market equilibrium is defined as an N-market structure for which a stable Nash equilibrium exists with active trading in all markets of the N-market structure and nowhere else.*

Before moving to the main inquiry of the paper, let us state a preliminary result about the properties of the equilibria under investigation.

Proposition 1 *If Assumptions 2-4 hold then 1) $\hat{b} = \arg\max_{b\in[0,1]} u_A(1-b, b/p)$ for every $p > 0$ (that is, $\hat{b}$ is independent of p), 2) if in a stable Nash equilibrium positive measures of type A and type B individuals trade in market l^j, we must have that $p^j = \mu_A^j/\mu_B^j$ and 3) any stable Nash equilibrium that involves a positive measure of trading agents is such that all agents trade.*

The first part of Proposition 1 shows that the amount of good I that all individuals of type A trade is always the same no matter the chosen market, or in other words, the submitted level of $\hat{b}$ does not depend on the (potential) price disparities across markets. The second part of the proposition exhibits that no equilibrium features a location where only one type of agents submit positive quantities for exchange. Therefore, at equilibrium, individuals submit their bids or offers only in locations with positive measure subsets of both types of traders. Moreover, similarly to the first part of the result, it is proved that all type A, B individuals who trade in market j submit equal quantities $(\hat{b} = \hat{q})$, and the resulting market clearing price of good 2 in this location is solely determined by the measures of the subsets of traders who choose location l^j. In particular, the price is equal to the ratio of active consumers over active producers in this market. Finally, the last part of Proposition 1 shows that at any stable Nash equilibrium no group of individuals chooses not to trade, as there is always benefit from trading. Hence, all traders are better off by traveling to the trading posts and by taking part in the exchange of the two goods.

3 General results

The first result of this section exhibits that having all individuals trading on a single market (independently of its location) is an equilibrium.

Proposition 2 *For every location* $l^1 \in [0,1]$ *there exists a one-market equilibrium with all agents trading at* l^1*.*

The intuition behind this one-market result is rather clear. Given that all other traders meet in a single market, the best response of an individual, independently of her location parameter or the location of the market, is to participate in the same market rather than no-trade at all. Note also that in such an equilibrium the whole masses of both types of individuals choose the same location, therefore the corresponding price is equal to one, or in other words, one unit of good II is exchanged for one unit of good I.

The above result is now extended to the case of two markets. The first part of Proposition 3 demonstrates that there is always a two-market structure that supports an equilibrium. The second part, however, reveals that not all two-market structures constitute an equilibrium.

Proposition 3 *There always exist two-market equilibria. Not every pair of locations* $(l^1, l^2) \in [0,1]^2$ *though constitute two-market equilibria.*

The first part of Proposition 3 shows that there are market structures (l^1, l^2) for which we can always find the unique location parameters $\dot{\alpha}^1, \dot{\beta}^1$ that define the type A and type B individuals who are indifferent between the two markets. In this case, we can derive that the mass of consumers located to the left of $\dot{\alpha}^1$ and the mass of producers located to the left of $\dot{\beta}^1$ are better off going to l^1 rather than going to any other location, and similarly, the mass of consumers located to the right of $\dot{\alpha}^1$ and the mass of producers located to the right of $\dot{\beta}^1$ are better off going to l^2 rather than going to any other location, thus making the pair (l^1, l^2) a two-market equilibrium. On the other hand, the second part of Proposition 3 reveals that for some pairs of market locations the unique location parameters $\dot{\alpha}^1, \dot{\beta}^1$ of the indifferent type A and type B individuals fail to satisfy the conditions for a two-market equilibrium. In such a case, all type A (or type B) individuals abandon one of the two markets and choose to trade in just one location. As a result the given two-market structure collapses and, thus, fails to form a two-market equilibrium.

The following Lemma complements Proposition 3 by demonstrating that in a two-market equilibrium prices generically diverge. Hence, the "law of one price", namely that in equilibrium there is a single price that clears all markets for a commodity, fails to hold. Moreover, Lemma 1 shows an obvious linkage between equilibrium prices in Symmetric market structures.

Lemma 1 *For any two-market equilibrium we have that* $p^1 < p^2$*. Moreover, for any Symmetric two-market equilibrium we have that* $p^1 = 1/p^2$*.*

We observe in the above lemma that the price is higher in location l^2. This location features increased demand due to the high concentration of type A individuals and reduced supply due the low concentration of type B individuals. Similarly, the cheaper market is located in l^1 due to the high concentration of type B individuals and the low concentration of type A individuals. In our framework, the violation of the "law of one price" is consistent with equilibrium, as individuals cannot profit from the price difference between the two markets due to transportation costs. In order to understand why this is so, let us contemplate an individual of type A with location parameter $a > \dot{\alpha}^1$ (the location of the indifferent type A individual) who shifts her purchasing orders from the more expensive to the cheaper market. This shift has no effect on prices, but increases the transportation costs so as to make such a move unprofitable. Analogous arguments apply for type B individuals. Therefore, there are no opportunities for profitable deviations despite the observed price disparities across markets[10].

We now turn to study the properties of general multi-market equilibria. The next proposition deals with the maximum number of active markets in equilibrium for Symmetric market structures.

Proposition 4 *For any admissible distribution of type A and type B individuals there exists a finite number* $\hat{N}(k)$ *such that no equilibrium exists for a Symmetric market structure with* $N > \hat{N}(k)$ *active markets and* $k \to 0^+$*.*

The above result exhibits that there is an upper bound on the number of active markets in a Symmetric equilibrium, with this number being a function of k. Indeed, as the location

[10]Koutsougeras (2003a,b) proves that of the validitity of the "law of one price" is intimately related to the degree of competitiveness. Indeed, in imperfectly competitive markets, where individuals may have non-negligible effects on market outcomes, it is the case that there exist equilibria where commodities are exchanged simultaneously in two markets at different prices. Our model, provides a *'spatial'* rationale for the observed price differences even in perfect competition.

the first active market approximates the left border of the interval, the market structure covers the whole linear city and more markets can survive in equilibrium. However, there is still a limitation on the number of active markets. An intuitive explanation for this result comes directly from the non-uniform equilibrium prices across markets. In brief, more markets imply greater price divergence, a fact that makes the benefit of a move to the most favorable market greater than the costs of transportation. In such a case, there is at least one market abandoned by all type A (or type B) individuals, leading us to a different market structure with less active markets.

The next result complements Proposition 4 as it uses similar arguments based on price disparities. In particular, it demonstrates that in order for a multi-market structure to support an equilibrium there must be sufficient distance between the locations of markets.

Proposition 5 *For any admissible distribution of type A and type B individuals there exists $\varepsilon > 0$ such that a market structure $(l^1, l^2, ..., l^N)$ with $\frac{1}{2} - \varepsilon < l^1 < l^2 < ... < l^N < \frac{1}{2} + \varepsilon$ cannot constitute a stable Nash equilibrium for any $N > 1$.*

This result shows that multiple markets appear only at sufficient distant from each other so that their existence is purposeful. Survival of two distinct markets which are located very close to each other implies that the prices in these two markets are almost identical. But if two nearby markets are expected to have almost identical prices then one market will attract many type A agents and few type B agents while the other will attract many type B agents and few type A agents - due to the fact that the distribution of individuals of each type is distinct - and this will make the prices of the two markets diverge significantly, hence, contradicting our initial assumption that these two markets will have very similar prices.

4 A representative case

We now focus on a representative case of the general framework presented above. Specifically, we consider that, $F_A(z) = z^2$, $u_A(x^I, x^{II}) = u_B(x^I, x^{II}) = x^I x^{II}$ and that each agent of type A (B) has an initial endowment of two units of good I (II), i.e., $w = 2$. By taking these assumptions we are able to derive extra results using computational techniques. Our aim is to illustrate some of the equilibrium possibilities presented in the previous section in a more elaborate and detailed manner.

Consider that the $[0,1]$ interval stands for a unidimensional geographical region. In this region there is a unit mass of vegetables producers (each producing two units of vegetable stock) and a unit mass of vegetables consumers (each possessing two euros). If consumers are denoted as type A individuals and producers as type B individuals then, since, $F_A(z) = z^2$ the densities of their distribution in the linear region should be the ones depicted in Figure 1.

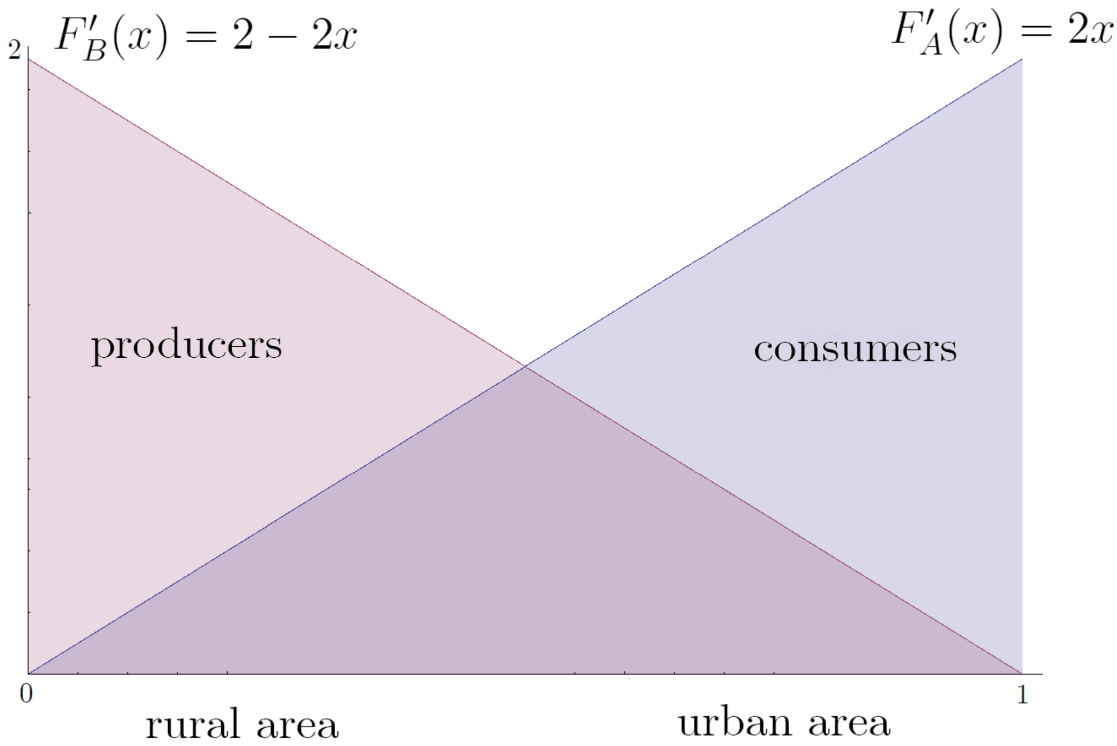

Figure 1. Distribution of vegetable stock producers and consumers.

The high concentration of vegetable producers in the left part of the region and the high concentration of consumers in the right part of the region allows us to refer to the sub-region $[0, \frac{1}{2})$ as the rural part of the region and to the sub-region $(\frac{1}{2}, 1]$ as the urban part of the region. This split is quite intuitive as in any $z \in [0, \frac{1}{2})$ the density of producers is higher than the density of consumers while in any $z \in (\frac{1}{2}, 1]$ the density of producers is lower than the density of consumers.

In our model all consumers and producers are free to strategically decide where and to which extent they wish to trade. Hence, the locations of the markets, the quantities exchanged in each market and the prices are determined endogenously. Having specified the exact distribution that determines how our players are located in the region we can treat most of the general issues analyzed above in a more specific manner.

Our first observation, which corresponds to Proposition 2, is that one-market equilibria can be supported for every single location $l \in [0,1]$. As in the general case the intuition behind this result is trivial. If a consumer (producer), independently of her location parameter, believes that all producers (consumers) will bring their vegetables in $l \in [0,1]$ then she is always better off going to $l \in [0,1]$ too rather than going to any other location. Since the whole mass of consumers and the whole mass of producers meet in a single market and since all agents have the same preferences for consumption of vegetable stock and monetary liquidity (each agent - independently of whether she is a consumer or a producer - she decides to sacrifice exactly one unit of her initial endowments), it directly follows that the price will be one independently of the exact location of the market.

But what about social welfare? An agents' utility does not only depend on the price but on the distance traveled from her initial location to the market too. If we consider that social welfare is the unweighted sum of individual utilities we observe that the single-market equilibrium which maximizes this sum is (as expected) the middle of the region (see Figure 2).

In this case the expression which gives social welfare is

$$SW = \int_0^1 (1 - |l - z|) dF_A(z) + \int_0^1 (1 - |l - z|) dF_B(z) = 1 + 2l - 2l^2.$$

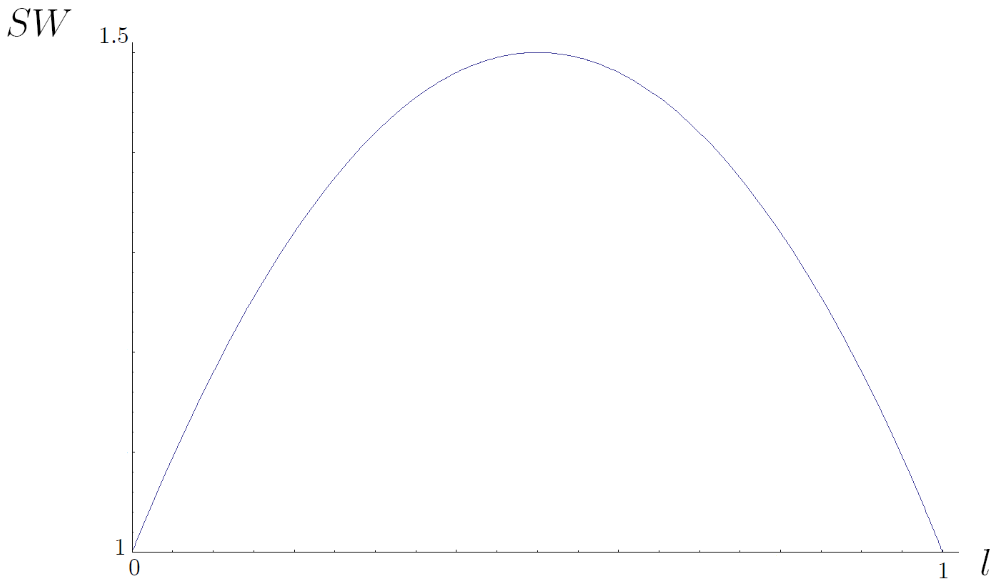

Figure 2. Social welfare as a function of market location.

On the other hand if we consider a measure of social welfare biased in favor of the consumers we see that optimal market location varies accordingly. If we consider that social welfare is a weighted sum in favor of the consumers for example (we multiply the utility of each consumer with φ and the utility level of each producer with $1-\varphi$ for $\omega \in [0,1]$) we can see (Figure 3) that the optimal market location is increasing in φ (it comes deeper in the urban area) and that it does not approach the region's edge even if $\varphi = 1$. That is, for any level of bias the optimal one-market equilibrium is in a sufficiently moderate location.

In this case the expression which gives social welfare is

$$SW = \varphi \int_0^1 (1-|l-z|)dF_A(z) + (1-\varphi)\int_0^1 (1-|l-z|)dF_B(z) =$$

$$= (\tfrac{2}{3} + l - 2l^2 + \tfrac{2l^3}{3})(1-\varphi) + (\tfrac{1}{3} + l - \tfrac{2l^3}{3})\varphi.$$

We observe that the value of this expression is maximized when

$$l^* = \frac{(-2+2\varphi+\sqrt{2}\sqrt{1-2\varphi+2\varphi^2})}{2(-1+2\varphi)}.$$

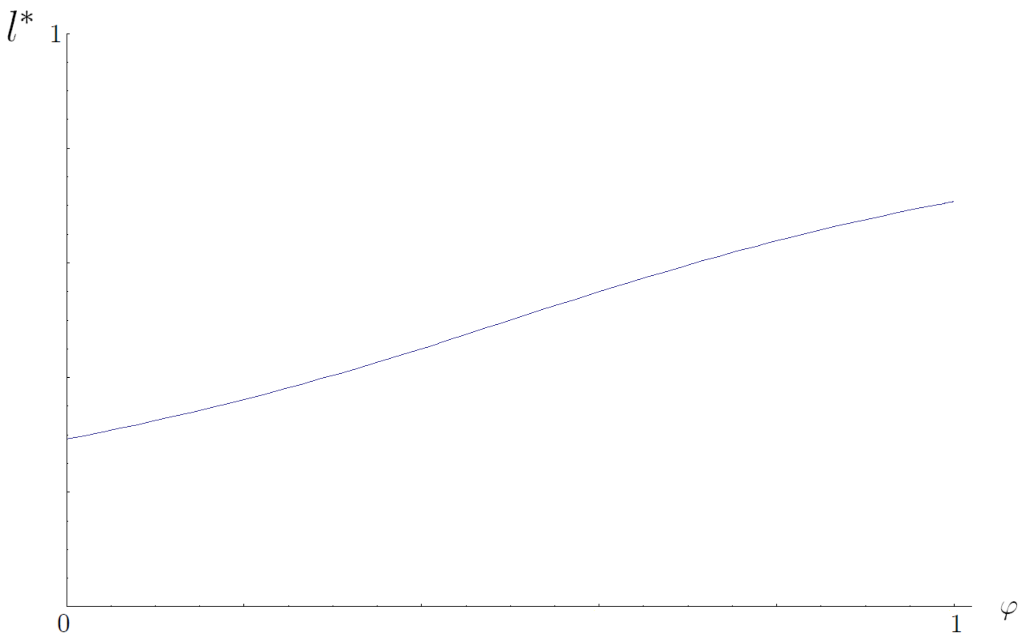

Figure 3. Optimal market location as a function of the social welfare bias.

Notice that $\varphi \in [0, 1]$ can take an alternative interpretation. So far, we have assumed that the measure of consumers is identical to the measure of producers. Consider now that the cumulative mass of the consumers and the producers is of measure $\Phi > 0$ and that $\varphi \in [0, 1]$ represents the fraction of the measure of consumers over Φ. Then, it is straightforward that the unweighted sum of individual utilities (unbiased social welfare) is maximized when the marketplace is located in l^*.

Let us move now to more complicated market structures. As we saw in Proposition 3, a) there always exist equilibria with two active markets and b) not all pairs of locations can support a two-market equilibrium. Let us first study the symmetric location pairs $(l^1, l^2) = (s, 1 - s)$ and check which values of $s \in [0, \frac{1}{2})$ support such two-market equilibria. Our equilibrium notion dictates that there should be a unique location parameter which defines an indifferent consumer between the two markets and, equivalently, a unique location parameter which defines an indifferent producer between the two markets. In general (that is for any number of possible markets) we denote by $\dot{\alpha}^j$ the location parameter of the type A individual who is indifferent between market l^j and market l^{j+1} and, correspondingly, $\dot{\beta}^j$ the location parameter of the type B individual who is indifferent between market l^j and market l^{j+1}. In an equilibrium we know from the above that both $l^j < \dot{\alpha}^j < l^{j+1}$ and $l^j < \dot{\beta}^j < l^{j+1}$ should hold.

Notice that since all agents in this example decide to sacrifice exactly one unit of their initial endowments and because $u_A(x^I, x^{II}) = u_B(x^I, x^{II}) = x^I x^{II}$ it must be the case that $u_A(1-\hat{b}, \frac{\hat{b}}{p^j}) = \frac{1}{p^j}$ and that $u_B(\hat{q}p^j, 1-\hat{q}) = p^j$. For the $(l^1, l^2) = (s, 1-s)$ case $\dot{a}^1$ and $\dot{\beta}^1$ are, hence, the values that solve

$$\frac{F_A(\dot{a}^1)}{F_B(\dot{\beta}^1)} - |\dot{\beta}^1 - s| = \frac{1 - F_A(\dot{a}^1)}{1 - F_B(\dot{\beta}^1)} - |\dot{\beta}^1 - (1-s)|$$

and

$$\frac{F_B(\dot{\beta}^1)}{F_A(\dot{a}^1)} - |\dot{a}^1 - s| = \frac{1 - F_B(\dot{\beta}^1)}{1 - F_A(\dot{a}^1)} - |\dot{a}^1 - (1-s)|$$

conditional on $s < \dot{a}^1 < 1-s$ and $s < \dot{\beta}^1 < 1-s$.

We observe that if $s < \dot{a}^1 < 1-s$ and $s < \dot{\beta}^1 < 1-s$ holds then the above two equations reduce to

$$\frac{F_A(\dot{a}^1)}{F_B(\dot{\beta}^1)} - \dot{\beta}^1 = \frac{1 - F_A(\dot{a}^1)}{1 - F_B(\dot{\beta}^1)} + \dot{\beta}^1 - 1$$

and

$$\frac{F_B(\dot{\beta}^1)}{F_A(\dot{a}^1)} - \dot{a}^1 = \frac{1 - F_B(\dot{\beta}^1)}{1 - F_A(\dot{a}^1)} + \dot{a}^1 - 1$$

which are independent of $s \in [0, \frac{1}{2})$.

Since $F_A(x) = x^2$ and $F_B(x) = 1 - (1-x)^2$ algebraic manipulations give the unique solution $(\dot{a}^1, \dot{\beta}^1) \simeq (0.675, 0.325)$. So for $(l^1, l^2) = (s, 1-s)$ to support a symmetric two-market equilibrium it should be the case that $\dot{a}^1$ and $\dot{\beta}^1$ are constant in s. This further implies that such symmetric two-market structures can be supported only when $s < 0.325$; equilibria with two symmetric active markets exist only if these two markets are sufficiently distant from each other.

When an equilibrium exists, that is, for $s < 0.325$, the equilibrium market prices are $p^1 = \dfrac{F_A(\dot{a}^1)}{F_B(\dot{\beta}^1)} \simeq 0.83$ and $p^2 = \dfrac{1 - F_A(\dot{a}^1)}{1 - F_B(\dot{\beta}^1)} \simeq 1.19$ (Lemma 1); the market located in the rural area offers vegetables in a lower price compared to the market located in the urban area.

Another issue that we would like to address in the framework of the present example relates to the maximum number of active markets in an equilibrium (Propositions 4 and 5). Since the system of equations which characterize an equilibrium is not linear even in this simple example, an inclusion of an additional market increases the complexity of the problem in several orders of magnitude. This is why we will address the question in the most symmetric form possible. That is, considering only market structures which satisfy Full-Spreadness and Symmetry (that is, market structures such that $l^j = \frac{j-1}{N-1}$) we will try to determine which ones can support an equilibrium.

Our results are summarized in the following table.

number of active markets	locations	indifferent consumer	indifferent producer	prices	SW
$N = 2$	$l^1 = 0$ $l^2 = 1$	$\dot{a}^1 \simeq 0.675$	$\dot{b}^1 \simeq 0.325$	$p^1 \simeq 0.83$ $p^2 \simeq 1.19$	1.423
$N = 3$	$l^1 = 0$ $l^2 = \frac{1}{2}$ $l^3 = 1$	$\dot{a}^1 \simeq 0.417$ $\dot{a}^2 \simeq 0.875$	$\dot{b}^1 \simeq 0.125$ $\dot{b}^2 \simeq 0.583$	$p^1 \simeq 0.748$ $p^2 = 1$ $p^3 \simeq 1.335$	1.656
$N = 4$	$l^1 = 0$ $l^2 = \frac{1}{3}$ $l^3 = \frac{2}{3}$ $l^4 = 1$	$\dot{a}^1 \simeq 0.32$ $\dot{a}^2 \simeq 0.67$ $\dot{a}^3 \simeq 0.92$	$\dot{b}^1 \simeq 0.08$ $\dot{b}^2 \simeq 0.33$ $\dot{b}^3 \simeq 0.68$	$p^1 \simeq 0.672$ $p^2 \simeq 0.847$ $p^3 \simeq 1.180$ $p^4 \simeq 1.487$	1.713
$N \geq 5$	no equilibrium				

Table 1: Equilibrium market structures that satisfy Full-Spreadness and Symmetry.

What we observe is that when the number of markets increases an indifferent consumer (producer) comes closer to the market to her right (left). This is expected as more markets imply a higher price dispersion between markets in the extremes and markets in the center of the region. The dispersion in prices cannot exceed though some thresholds as this would make a market be abandoned by all consumers (producers). When $N = 4$ we observe that $\dot{a}^1 \simeq 0.32$ is immensely closer to market $l^2 = \frac{1}{3}$ than to market $l^1 = 0$. Hence, the inclusion of an extra market makes the system collapse (its solution can no longer satisfy the equilibrium constraints).

All these imply that even though we have infinitely many players, at least up to symmetry, the trading pattern that they will strategically decide to follow is quite predictable and simple. Not many markets may emerge in equilibrium (in our case at most four) and the prices, though distinct, cannot diverge immensely. We evaluate social welfare (unweighted sum of individual utilities) of all these multiple-market equilibria (see Table 1) and we observe that the larger the number of markets in equilibrium the larger the value of social welfare becomes. This is mainly driven by the fact that when multiple markets exist consumers and producers need bear low transport costs to trade.

Finally, we discuss some issues regarding transportation costs. According to the utility function that we employed each agent loses a unit of utility for every unit of distance travelled. One could relax this assumption by considering that the travel cost per unit of distance travelled is $c \in [0, +\infty)$.[11] Under this, more general, assumption one can obtain qualitatively similar results with the ones above. For example, for the the $N = 2$ fully-spread and symmetric market structure we can show that the absolute value of the difference between the two equilibrium prices is increasing in c and that it converges to zero when $c \to 0$ and to $\frac{8}{3} \simeq 2.666$ when $c \to +\infty$. Moreover, we observe that the indifferent consumer (who is located at 0.675 for $c = 1$) moves towards the right (left) as c decreases (increases) and reaches the location $\frac{1}{\sqrt{2}} \simeq 0.707$ (0.5) when $c \to 0$ ($c \to +\infty$). That is, the difference between the equilibrium prices and the difference between the measure of players trading in each market vary in c but their variation is bounded by two relatively small numbers. Hence, the qualitative dimension of the equilibrium trading patterns that we presented in Table 1 is robust to general transportation costs.

[11] For values of $c > 1$ one should be very careful to add the assumption that players incur a sufficiently large cost in case they decide not to trade so that full participation in equilibrium trading activities is guaranteed. Otherwise, the equilibrium analysis should be generalized in order to take in account a measure of players strategically deciding not to participate in trading activities in any active market. For any $c \leq 1$ such an additional assumption is redundant.

5 Concluding remarks

This work is an attempt to model spatial competition with a vast number of traders or, in other words, to provide a Walrasian version of location games. We believe that having the great mass of one type being concentrated near one interval border and the great mass of the other type near the other interval extreme resembles a real-world situation where production takes place in the some parts of a region (e.g., rural areas or industrial parks), whereas consumers reside in the urban areas of the same region. The main results of the paper study the agglomeration of economic activities in specific locations, and, more specifically, deal with the number and the distribution of active markets in the linear city. Moreover, by assuming the distinct distributions of the two populations, the paper offers a justification for price disparities even in perfectly competitive markets. The present analysis could also be extended to more general settings, with a general specification of endowments, preferences and transportation costs. Furthermore, different distributions of traders on the unit interval or models with many commodities could be attractive topics for further research. Finally, instead of assuming that individuals act independently without any coordination of actions, it might be attractive to examine settings that can embody the possibility of coordinated actions among groups of traders.

6 Appendix

6.1 Proofs

Proof of Proposition 1. Each agent has expectations about the behavior of all the other agents. Consider a type A agent who is *forced* to trade at a location l^j such that $p^j = B^j/Q^j > 0$; a positive measure of type A and type B individuals are expected to place there certain bids and offers. Then the type A agent that we consider faces the problem:

$$\max_{b\in[0,1]} u_A(1-b, b/p^j)$$

which is equivalent to the programme

$$\max_{x^I,x^{II}} u_A(x^I, x^{II}) \text{ s.t. } x^I + p^j x^{II} = 1.$$

By assumption 3 we know that the first part of the solution $(\hat{x}, \hat{y})$ of the latter programme is invariant to changes in p^j - the agent's optimal bid is such that $\hat{b} = \max_{b\in[0,1]} u_A(1-\hat{b}, \hat{b}/p)$ for every $p > 0$.

To prove the second part of this proposition first observe that a stable Nash equilibrium cannot be such that only a positive measure of type A agents trades in a market. If in a stable Nash equilibrium a positive measure of agents trade in in location l^j, it must be the case that positive measures of both type A and type B agents trade in location l^j. That is, $p^j > 0$.

Then notice that $u_A(x^I, x^{II}) = u_B(x^{II}, x^I)$ along with assumption 3 imply that $\hat{b} = \hat{q}$ for every $p > 0$ where $\hat{b} = \max_{b\in[0,1]} u_A(1-b, b/p)$ and $\hat{q} = \max_{q\in[0,1]} u_B(pq, 1-q)$. Hence, in every market of any stable Nash equilibrium it should be the case that $p^j = B^j/Q^j = \hat{b}\mu_A^j/\hat{q}\mu_B^j = \mu_A^j/\mu_B^j$.

To prove the last part of our proposition we notice that in a stable Nash equilibrium in which a positive measure of agents trade in a market l^j we have $p^j > 0$. If $p^j \leq 1$ then assumptions 2 and 4 along with $u_A(1, 0) = 0$ and the observation that $\max_{b\in[0,1]} u_A(1-b, b/p)$ is equivalent to the programme $\max_{x,\dot{\beta}^1} u_A(x^I, x^{II})$ s.t. $x^I + px^{II} = 1$ indicate that every type A agent is strictly better off by choosing to trade in l^j than not trading at all. That is, a stable Nash equilibrium in which some agents trade in market l^j with $p^j \leq 1$ is such that all type A agents trade at some market. If in a stable Nash equilibrium all type A agents trade at some market and not all type B agents trade then there must exist at least one market l^k such that $\mu_A^k > \mu_B^j$. Then by the second part of this proposition it should be the case that $p^k = \mu_A^k/\mu_B^k > 1$. Hence, by assumptions 2 and 4 along with $u_B(0,1) = 0$ and the observation that $\max\limits_{q\in[0,1]} u_B(pq, 1-q)$ is equivalent to the programme $\max_{x^I, x^{II}} u_B(x^I, x^{II})$ s.t. $\frac{1}{p}x^I + x^{II} = 1$ indicate that every type B agent is strictly better off by choosing to trade in l^k than not trading at all. One can provide symmetric arguments for the case in which $p^j > 1$ and, thus, prove that any stable Nash equilibrium which involves trade is such that all agents trade. ■

Proof of Proposition 2. The proof is straightforward. If all type A and type B individuals ($\mu_A^1 = \mu_B^1 = 1$) place their bids and offers ($\hat{b}$ units each) in location $l^1 \in [0,1]$ then by the second point of proposition 1 we have that $p^1 = 1$ and, hence, by assumptions 2 and 3 we have that $U_A^\alpha(\hat{\sigma}'_{i,\alpha}, \hat{\sigma}) \geq 0$ ($U_B^\beta(\hat{\sigma}''_{g,\beta}, \hat{\sigma}) \geq 0$) for every $i \in [0,1]$ and $\alpha \in [0,1]$ ($g \in [0,1]$ and $\beta \in [0,1]$); every agent prefers to trade in l^1 than to no trade at all. Notice that, for type A agents for example, any strategy (l', b) such that $l' \neq l^1$ is dominated by no-trade and any strategy (l^1, b) such that $b \neq \hat{b}$ is dominated by $(l^1, \hat{b})$. Obviously the same holds for type B individuals too. That is, each agent trading $\hat{b}$ units of her initial endowment in location l^1 is a stable Nash equilibrium of the game. ■

Proof of Proposition 3. Consider a strategy profile $\hat{\sigma}$ such that a) $(l^1, l^2) = (0,1)$, b) $\mu_A^1 = F_A(z)$, $\mu_B^1 = F_B(1-z)$ and c) $\mu_A^2 = 1 - F_A(z)$, $\mu_B^2 = 1 - F_B(1-z)$. For such a profile to be an equilibrium it must be the case that the type A individuals with location parameter $a = z$ should be indifferent between the two markets. That is it should be the case that

$$u_A(1-\hat{b}, \frac{\hat{b}}{F_A(z)/F_B(1-z)}) - |z-0| = u_A(1-\hat{b}, \frac{\hat{b}}{[1-F_A(z)]/[1-F_B(1-z)]}) - |z-1|$$

which can be re-written as

$$u_A(1-\hat{b}, \frac{\hat{b}}{F_A(z)/[1-F_A(z)]}) - z = u_A(1-\hat{b}, \frac{\hat{b}}{[1-F_A(z)]/F_A(z)}) - 1 + z.$$

Notice that there always exists a unique $z^* \in (0,1)$ such that the above equality holds. This is because a) $u_A(1-\hat{b}, \frac{\hat{b}}{F_A(z)/[1-F_A(z)]}) - z$ and $u_A(1-\hat{b}, \frac{\hat{b}}{[1-F_A(z)]/F_A(z)}) - 1 + z$ are both continuous for any $z \in (0,1)$, b) $u_A(1-\hat{b}, \frac{\hat{b}}{F_A(z)/[1-F_A(z)]}) - z$ is strictly decreasing and $u_A(1-\hat{b}, \frac{\hat{b}}{[1-F_A(z)]/F_A(z)}) - 1 + z$ is strictly increasing for any $z \in (0,1)$, c) $\lim_{z\to 0^+}[u_A(1-$

$\hat{b}, \frac{\hat{b}}{F_A(z)/[1-F_A(z)]}) - z] = +\infty$ and $\lim_{z\to 1^-}[u_A(1-\hat{b}, \frac{\hat{b}}{F_A(z)/[1-F_A(z)]}) - z] = -1$ and d) $\lim_{z\to 0^+}[u_A(1-\hat{b}, \frac{\hat{b}}{[1-F_A(z)]/F_A(z)}) - 1 + z] = -1$ and $\lim_{z\to 1^-}[u_A(1-\hat{b}, \frac{\hat{b}}{[1-F_A(z)]/F_A(z)}) - 1 + z] = +\infty$.

Moreover, for this allocation to be an equilibrium we should further have that the type B individuals with location parameter $\beta = 1 - z$ are indifferent between the two markets. That is, it should be the case that

$$u_B(\hat{b}\frac{F_A(z)}{F_B(1-z)}, 1-\hat{b}) - |(1-z) - 0| = u_B(\hat{b}\frac{1-F_A(z)}{1-F_B(1-z)}, 1-\hat{b}) - |(1-z) - 1|$$

which can be re-written as

$$u_A(1-\hat{b}, \frac{\hat{b}}{[1-F_A(z)]/F_A(z)}) - 1 + z = u_A(1-\hat{b}, \frac{\hat{b}}{F_A(z)/[1-F_A(z)]}) - z.$$

That is there exists a unique $z^* \in (0,1)$ such that both relevant equalities hold and hence, $(l^1, l^2) = (0, 1)$ is an equilibrium. We finally note that a) for any type A individual with location parameter $a < z^*$ we have $U_A^\alpha((l^1, \hat{b}), \hat{\sigma}) > U_A^\alpha((l^2, \hat{b}), \hat{\sigma})$, b) for any type A individual with location parameter $a > z^*$ we have $U_A^\alpha((l^1, \hat{b}), \hat{\sigma}) < U_A^\alpha((l^2, \hat{b}), \hat{\sigma})$, c) for any type B individual with location parameter $\beta < 1 - z^*$ we have $U_B^\beta((l^1, \hat{b}), \hat{\sigma}) > U_B^\beta((l^2, \hat{b}), \hat{\sigma})$ and for any type B individual with location parameter $\beta > 1 - z^*$ we have $U_B^\beta((l^1, \hat{b}), \hat{\sigma}) < U_B^\beta((l^2, \hat{b}), \hat{\sigma})$; the equilibrium is stable.

Now consider that a) $(l^1, l^2) = (s, 1-s)$, b) $\mu_A^1 = F_A(\dot{\alpha}^1)$, $\mu_B^1 = F_B(\dot{\beta}^1)$, c) $\mu_A^2 = 1 - F_A(\dot{\alpha}^1)$, $\mu_B^2 = 1 - F_B(\dot{\beta}^1)$ and that d) $s \in (0, \frac{1}{2})$. We first notice that for this allocation to be an equilibrium it must be the case that $(\dot{\alpha}^1, \dot{\beta}^1) \in (s, 1-s)^2$. If for example $0 < \dot{\alpha}^1 \leq s$ then for any type A individual with location parameter $a \leq \dot{\alpha}^1$ we have $U_A^\alpha((l^1, \hat{b}), \hat{\sigma}) = U_A^\alpha((l^2, \hat{b}), \hat{\sigma})$ and, thus, the stability condition we have imposed is violated. Moreover, it should be the case that $\dot{\beta}^1 << x$ (by $<<$ we mean that there exists a non-degenerate positive number λ such that $\dot{\alpha}^1 - \dot{\beta}^1 > \lambda$). This is true because if $\dot{\beta}^1 > \dot{\alpha}^1$ we should have

$$u_B(\hat{b}\frac{F_A(\dot{\alpha}^1)}{F_B(\dot{\beta}^1)}, 1-\hat{b}) - |\dot{\beta}^1 - s| = u_B(\hat{b}\frac{1-F_A(\dot{\alpha}^1)}{1-F_B(\dot{\beta}^1)}, 1-\hat{b}) - |\dot{\beta}^1 - (1-s)| \text{ and}$$

$$u_A(1-\hat{b}, \frac{\hat{b}}{F_A(\dot{\alpha}^1)/F_B(\dot{\beta}^1)}) - |\dot{\beta}^1 - s| < u_A(1-\hat{b}, \frac{\hat{b}}{[1-F_A(\dot{\alpha}^1)]/[1-F_B(\dot{\beta}^1)]}) - |\dot{\beta}^1 - (1-s)|$$

which cannot both hold true at the same time.

This is because

$$u_B(\hat{b}\frac{F_A(\dot{\alpha}^1)}{F_B(\dot{\beta}^1)}, 1-\hat{b}) - |\dot{\beta}^1 - s| = u_B(\hat{b}\frac{1-F_A(\dot{\alpha}^1)}{1-F_B(\dot{\beta}^1)}, 1-\hat{b}) - |\dot{\beta}^1 - (1-s)|$$

can be rewritten as

$$u_A(1-\hat{b}, \hat{b}\frac{F_A(\dot{\alpha}^1)}{F_B(\dot{\beta}^1)}) - |\dot{\beta}^1 - s| = u_A(1-\hat{b}, \hat{b}\frac{1-F_A(\dot{\alpha}^1)}{1-F_B(\dot{\beta}^1)}) - |\dot{\beta}^1 - (1-s)|$$

and because $\dot{\beta}^1 > \dot{\alpha}^1$ implies that $F_A(\dot{\alpha}^1) < F_B(\dot{\beta}^1)$.[12]

The latter suggests that

$$u_A(1-\hat{b}, \hat{b}\frac{F_A(\dot{\alpha}^1)}{F_B(\dot{\beta}^1)}) < u_A(1-\hat{b}, \frac{\hat{b}}{F_A(\dot{\alpha}^1)/F_B(\dot{\beta}^1)})$$

and that

$$u_A(1-\hat{b}, \hat{b}\frac{1-F_A(\dot{\alpha}^1)}{1-F_B(\dot{\beta}^1)}) > u_A(1-\hat{b}, \frac{\hat{b}}{[1-F_A(\dot{\alpha}^1)]/[1-F_B(\dot{\beta}^1)]})$$

and hence

$$u_A(1-\hat{b}, \hat{b}\frac{F_A(\dot{\alpha}^1)}{F_B(\dot{\beta}^1)}) - |\dot{\beta}^1 - s| = u_A(1-\hat{b}, \hat{b}\frac{1-F_A(\dot{\alpha}^1)}{1-F_B(\dot{\beta}^1)}) - |\dot{\beta}^1 - (1-s)|$$

which leads to

$$u_A(1-\hat{b}, \frac{\hat{b}}{F_A(\dot{\alpha}^1)/F_B(\dot{\beta}^1)}) - |\dot{\beta}^1 - s| > u_A(1-\hat{b}, \frac{\hat{b}}{[1-F_A(\dot{\alpha}^1)]/[1-F_B(\dot{\beta}^1)]}) - |\dot{\beta}^1 - (1-s)|$$

and not to the desired

$$u_A(1-\hat{b}, \frac{\hat{b}}{F_A(\dot{\alpha}^1)/F_B(\dot{\beta}^1)}) - |\dot{\beta}^1 - s| < u_A(1-\hat{b}, \frac{\hat{b}}{[1-F_A(\dot{\alpha}^1)]/[1-F_B(\dot{\beta}^1)]}) - |\dot{\beta}^1 - (1-s)|.$$

Moreover we cannot have $\dot{\alpha}^1 \to \dot{\beta}^1$ because then we should have

$u_B(\hat{b}\frac{F_A(\dot{\alpha}^1)}{1-F_A(1-\dot{\alpha}^1)}, 1-\hat{b}) - |\dot{\alpha}^1 - s| \to u_B(\hat{b}\frac{1-F_A(\dot{\alpha}^1)}{F_A(1-\dot{\alpha}^1)}, 1-\hat{b}) - |\dot{\alpha}^1 - (1-s)|$ and

$u_A(1-\hat{b}, \frac{\hat{b}}{F_A(\dot{\alpha}^1)/[1-F_A(1-\dot{\alpha}^1)]}) - |\dot{\alpha}^1 - s| \to u_A(1-\hat{b}, \frac{\hat{b}}{[1-F_A(\dot{\alpha}^1)]/F_A(1-\dot{\alpha}^1)}) - |\dot{\alpha}^1 - (1-s)|$

[12] We have that F_B is first-order stochastically dominant over F_A if and only if $F_B(z) - F_A(z) \geq 0$ for every $z \in [0,1]$. Since $F_B(z) = 1 - F_A(1-z)$ we notice that $F_B(z) - F_A(z) = 1 - F_A(1-z) - F_A(z)$ takes the value 0 when $z \in \{0,1\}$ and that it is strictly convex for every $z \in (0,1)$. That is, first-order stochastic dominance is guaranteed and moreover $F_B(z) - F_A(z) > 0$ for every $z \in (0,1)$. The latter suggests that $F_B(y) > F_A(z)$ for every $y > z$.

which again cannot both hold at the same time with a reasoning similar as above.

Therefore if $(\dot{\alpha}^1, \dot{\beta}^1) \in (s, 1-s)^2$ characterizes an equilibrium it should be that $\dot{\beta}^1 < \dot{\alpha}^1$ and $\max\{|\frac{1}{2} - \dot{\alpha}^1|, |\frac{1}{2} - \dot{\beta}^1|\} >> 0$. Consider without loss of generality that $\dot{\alpha}^1 >> \frac{1}{2}$. We notice that the conditions that $(\dot{\alpha}^1, \dot{\beta}^1) \in (s, 1-s)^2$ should satisfy are independent of s. This is, because

$$u_A(1-\hat{b}, \frac{\hat{b}}{F_A(\dot{\alpha}^1)/F_B(\dot{\beta}^1)}) - |\dot{\alpha}^1 - s| = u_A(1-\hat{b}, \frac{\hat{b}}{[1-F_A(\dot{\alpha}^1)]/[1-F_B(\dot{\beta}^1)]}) - |\dot{\alpha}^1 - (1-s)|$$

and

$$u_B(\hat{b}\frac{F_A(\dot{\alpha}^1)}{F_B(\dot{\beta}^1)}, 1-\hat{b}) - |\dot{\beta}^1 - s| = u_B(\hat{b}\frac{1-F_A(\dot{\alpha}^1)}{1-F_B(\dot{\beta}^1)}, 1-\hat{b}) - |\dot{\beta}^1 - (1-s)|$$

can be re-written as

$$u_A(1-\hat{b}, \frac{\hat{b}}{F_A(\dot{\alpha}^1)/F_B(\dot{\beta}^1)}) - \dot{\alpha}^1 = u_A(1-\hat{b}, \frac{\hat{b}}{[1-F_A(\dot{\alpha}^1)]/[1-F_B(\dot{\beta}^1)]}) - 1 + \dot{\alpha}^1 \text{ and}$$

$$u_B(\hat{b}\frac{F_A(\dot{\alpha}^1)}{F_B(\dot{\beta}^1)}, 1-\hat{b}) - \dot{\beta}^1 = u_B(\hat{b}\frac{1-F_A(\dot{\alpha}^1)}{1-F_B(\dot{\beta}^1)}, 1-\hat{b}) - 1 + \dot{\beta}^1.$$

So for any $(\dot{\alpha}^1, \dot{\beta}^1) \in (s, 1-s)^2$ that characterizes an equilibrium for $(l^1, l^2) = (s, 1-s)$ we can find $(\hat{l}^1, \hat{l}^2) = (\hat{s}, 1-\hat{s})$ such that $\frac{1}{2} < 1-\hat{s} < \dot{\alpha}^1$ and therefore $(\dot{\alpha}^1, \dot{\beta}^1)$ cannot characterize an equilibrium for $(\hat{l}^1, \hat{l}^2) = (\hat{s}, 1-\hat{s})$. We have, thus shown that there are two-market structures which cannot support an equilibrium. ■

Proof of Lemma 1. Consider the two-market equilibrium with $\mu_A^1 = F_A(\dot{\alpha}^1)$, $\mu_B^1 = F_B(\dot{\beta}^1)$, $\mu_A^2 = 1 - F_A(\dot{\alpha}^1)$, $\mu_B^2 = 1 - F_B(\dot{\beta}^1)$ and suppose on the contrary that $p^1 \geq p^2$. Consider the type A individuals with location parameter $\dot{\alpha}^1$ who are indifferent between the two markets. For these individuals it must be true that $u_A(1-\hat{b}, \frac{\hat{b}}{p^1}) - |\dot{\alpha}^1 - l^1| = u_A(1-\hat{b}, \frac{\hat{b}}{p^2}) - |\dot{\alpha}^1 - l^2|$. Given that $\frac{1}{p^1} \leq \frac{1}{p^2}$ we have $\dot{\alpha}^1 - l^1 \leq l^2 - \dot{\alpha}^1$ or $\dot{\alpha}^1 \leq \frac{l^1 + l^2}{2}$.

Consider now the type B individuals with location parameter $\dot{\beta}^1$, who are indifferent between the two markets. For these individuals we have that $u_B(\hat{b}p^1, 1-\hat{b}) - |\dot{\beta}^1 - l^1| = u_B(\hat{b}p^2, 1-\hat{b}) - |\dot{\beta}^1 - l^2|$. Given that $p^1 \geq p^2$ we have $\dot{\beta}^1 - l^1 \geq l^2 - \dot{\beta}^1$ or $\dot{\beta}^1 \geq \frac{l^1 + l^2}{2}$.

Hence, $\dot{\beta}^1 \geq \dot{\alpha}^1$, with the corresponding prices being $p^1 = \frac{F_A(\dot{\alpha}^1)}{F_B(\dot{\beta}^1)} = \frac{F_A(\dot{\alpha}^1)}{1 - F_A(1-\dot{\beta}^1)} < 1$ and $p^2 = \frac{1-F_A(\dot{\alpha}^1)}{1-F_B(\dot{\beta}^1)} = \frac{1-F_A(\dot{\alpha}^1)}{F_A(1-\dot{\beta}^1)} > 1$, which is a contradiction to our initial statement.

Therefore, $p^1 < p^2$.

We now allow for Symmetry by setting $\dot{\alpha}^1 = 1 - \dot{\beta}^1$, which satisfies (as shown Proposition 3) the conditions of a two-market equilibrium. In this case we get $p^1 = \dfrac{F_A(1-\dot{\beta}^1)}{F_B(1-\dot{\alpha}^1)} = \dfrac{1-F_B(\dot{\beta}^1)}{1-F_A(\dot{\alpha}^1)} = 1/p_2$. ■

Proof of Proposition 4. Consider a Symmetric market structure with N markets. We construct upper and lower bounds of the prices in the two extreme markets in an equilibrium.

In market l^1 we should have $\mu_A^j = F_A(\dot{\alpha}^1)$ and $\mu_B^j = F_B(\dot{\beta}^1)$ where $\dot{\alpha}^1$ is the location parameter of a type A individuals who are indifferent between markets l^1 and l^2 and $\dot{\beta}^1$ is the location parameter of a type B individuals who are indifferent between markets l^1 and l^2.

An equilibrium is such that all type A individuals with location parameters smaller than $\dot{\alpha}^1$ strictly prefer market l^1 to any other market and all type B individuals with location parameters smaller than $\dot{\beta}^1$ strictly prefer market l^1 to any other market.

Moreover, by the proof of Proposition 2 we know that $(\dot{\alpha}^1, \dot{\beta}^1) \in (l^1, l^2)^2$. For a Symmetric N-market configuration we have that $l^1 = k$ and $l^2 = k + \frac{1}{N-1}(1-2k)$. Therefore, $\dot{\alpha}^1 \in (k, k + \frac{1}{N-1}(1-2k))$ and $\dot{\beta}^1 \in (k, k + \frac{1}{N-1}(1-2k))$ too.

This implies that $\mu_A^1 \in (F_A(k), F_A(k + \frac{1}{N-1}(1-2k)))$ and that $\mu_B^1 \in (F_B(k), F_B(k + \frac{1}{N-1}(1-2k)))$ and therefore

$p^1 \in (\frac{F_A(k)}{F_B(k+\frac{1}{N-1}(1-2k))}, \frac{F_A(k+\frac{1}{N-1}(1-2k))}{F_B(k)})$ or $p^1 \in (\frac{F_A(k)}{1-F_A(1-k-\frac{1}{N-1}(1-2k))}, \frac{F_A(k+\frac{1}{N-1}(1-2k))}{1-F_A(1-k)})$.

Using equivalent steps we find

$p^N \in (\frac{1-F_A(1-k)}{F_A(1-k-\frac{N-2}{N-1}(1-2k))}, \frac{1-F_A(1-k-\frac{N-2}{N-1}(1-2k))}{F_A(k)})$.

We observe that as N increases these bounds become narrower and in specific when $N \to +\infty$ we have that

$p^1 \to \frac{F_A(k)}{1-F_A(1-k)}$ and that $p^N \to \frac{1-F_A(1-k)}{F_A(k)}$.

We moreover notice that for $k \to 0^+$ we have that $p^1 \to 0$ and that $p^N \to +\infty$.

Continuity of F, and thereafter, continuity of the upper and lower bounds of p^1 and p^N, implies that for high values of N and low values of k an individual of type A would strictly prefer to place her bid in market l^1 rather than in market l^N independently of her location parameter. That is, for high values of N and low values of k an equilibrium for a Symmetric market structure with N active markets is not possible. ■

Proof of Proposition 5. To see why this holds consider an N-market structure $(l^1, l^2, ..., l^N)$ such that $\frac{1}{2} - \varepsilon < l^1 < l^2 < ... < l^N < \frac{1}{2} + \varepsilon$ which is supported by a stable Nash equilibrium. Then, $\mu_A^1 \in (F_A(\frac{1}{2} - \varepsilon), F_A(\frac{1}{2} + \varepsilon))$ and $\mu_A^N \in (1 - F_A(\frac{1}{2} + \varepsilon), 1 - F_A(\frac{1}{2} - \varepsilon))$ and equivalently $\mu_B^1 \in (F_B(\frac{1}{2} - \varepsilon), F_B(\frac{1}{2} + \varepsilon))$ and $\mu_B^N \in (1 - F_B(\frac{1}{2} + \varepsilon), 1 - F_B(\frac{1}{2} - \varepsilon))$. These imply that for $\varepsilon \to 0$ we should have that $p^1 \to \frac{F_A(\frac{1}{2})}{F_B(\frac{1}{2})} = \frac{F_A(\frac{1}{2})}{1-F_A(\frac{1}{2})} << \frac{1}{2}$ and that $p^N \to \frac{1-F_A(\frac{1}{2})}{1-F_B(\frac{1}{2})} = \frac{1-F_A(\frac{1}{2})}{F_A(\frac{1}{2})} >> \frac{1}{2}$ and therefore a type A individual with any location parameter strictly prefers to trade in market l^1 than in market l^N. So given F_A and F_B there always exist $\varepsilon > 0$ such that any market configuration $(l^1, l^2, ..., l^N)$ with $\frac{1}{2} - \varepsilon < l^1 < l^2 < ... < l^N < \frac{1}{2} + \varepsilon$ cannot support an equilibrium. ■

6.2 Utility functions that do not satisfy Assumption 3

Here we address possible concerns regarding the possibility of extending our results to more general classes of utility functions. We have assumed that the part of the utility function that depends on consumption of a bundle of the two goods is such that the optimal consumption choice of the good one initially possesses is independent of how all other players are expected to behave. An attempt to generalize the presented results by relaxing this assumption would be, technically, very challenging - possibly intractable. Regardless of this technical complexity and of the fact that many popular utility functions (for example Cobb-Douglas utility functions) actually satisfy this property, it is essential that we study whether our results are relevant when utility functions do not satisfy Assumption 3. To this end we analyze a certain case that guarantees a desired degree of genericity.

Specifically, we assume that $F_A(z) = z^2$, $u_A(x^I, x^{II}) = u_B(x^I, x^{II}) = 2(\sqrt{x^I} + \sqrt{x^{II}})$ and that each individual of type A (B) has an initial endowment of one unit of good I (II). In this case we have that $\arg\max_{b\in[0,1]} u_A(1-b, b/p) = \frac{p}{1+p}$ and hence Assumption 3 does not hold. Existence of a one-market equilibrium is straightforward. If we assume that a two-market equilibrium exists in locations $(s, 1-s)$ for some $s < \frac{1}{2}$ then one can show (by the means of computational methods) that the indifferent type A (B) individual must be situated approximately near location 0.64 (0.36). These locations of the indifferent type A and type B individuals are independent of the exact value of s. This implies that a two-market equilibrium in locations $(s, 1-s)$ exists if and only if s is sufficiently small (smaller than 0.36). Hence, two-market equilibria exist, but not all two-market structures support an equilibrium, precisely as in the main part of our analysis. The price in the market at s will be about 0.82 and the price in the market at $1-s$ will be about 1.21. The bids (offers) of type A (B) individuals in the first market (at s) will be about 0.54 (0.45) of their initial endowment of good I (II) while the bids (offers) of type A (B) individuals in the second market (at $1-s$) will be about 0.45 (0.54) of their initial endowment of good I (II).

Notice that, as in our main analysis, participation (positive trade) of everybody is guaranteed in a two-market equilibrium of this case. This is due to the fact that the smallest level of total utility (consumption utility minus transportation costs) that an individual may enjoy here is about 2.3, which is larger than the utility level of the no-trade option. Therefore, we feel confident to conjecture that our results apply to general classes of utility functions which fail to satisfy Assumption 3.

References

[1] Anderson S, Engers M. (1994) Spatial competition with price-taking firms. Economica 61; 125-136.

[2] Anderson S, Neven D. (1991) Cournot competition yields spatial agglomeration. International Economic Review 32; 793–808.

[3] Baesemann RC. (1977) The formation of small market places in a competitive economic process - The dynamics of agglomeration. Econometrica 45; 361-374.

[4] Berliant M, Papageorgiou YY, Wang P. (1990) On welfare theory and urban economics. Regional Science and Urban Economics 20; 245-261.

[5] Berliant M, Wang P. (1993) Endogenous formation of a city without agglomeration externalities or market imperfections: Marketplaces in a regional economy. Regional Science and Urban Economics 23; 121-144.

[6] D' Aspremont C, Gabszewicz JJ, Thisse JF. (1979) On Hotelling's stability in competition. Econometrica 47; 1145-1150.

[7] Dubey P, Shapley L. (1994) Non-cooperative general exchange with a continuum of traders: Two models. Journal of Mathematical Economics 23; 253-293.

[8] Dubey P, Shubik M. (1978) A theory of money and financial institutions: The non-cooperative equilibria of a closed economy with market supply and bidding strategies. Journal of Economic Theory 17; 1-20.

[9] Economides N. (1984) The principle of minimum differentiation revisited. European Economic Review 24; 345-368.

[10] Economides N. (1993) Hotelling's 'main street' with more than two competitors. Journal of Regional Science 33; 303-319.

[11] Fujita M, Smith TE. (1987) Existence of continuous residential land-use equilibria. Regional Science and Urban Economics 17; 549-594.

[12] Fujita M, Thisse JF, Zenou Y. (1997) On the endogeneous formation of secondary employment centers in a city. Journal of Urban Economics 41; 337-357.

[13] Gabszewicz JJ, Michel P. (1997) Oligopoly equilibrium in exchange economies. In Eaton BC, Harris RG (Eds.), Trade, Technology and Economics: Essays in Honor of Richard G. Lipsey. Edwar Elgar; Cheltenham; 217-240.

[14] Gupta B, Pal D, Sarkar J. (1997) Spatial Cournot competition and agglomeration in a model of location choice. Regional Science and Urban Economics 27; 261–282.

[15] Hamilton J, Klein JF, Sheshinski E, Slutsky SM. (1994) Quantity competition in a spatial model. Canadian Journal of Economics 27; 903–917.

[16] Hamilton J, Thisse JF, Weskamp A. (1989) Spatial discrimination: Bertrand vs. Cournot in a model of location choice. Regional Science and Urban Economics 19; 87–102.

[17] Hotelling H. (1929) Stability in competition. The Economic Journal 39; 41-57.

[18] Huck S, Knoblauch V, Müller W. (2003) On the profitability of collusion in location games. Journal of Urban Economics 54; 499-510

[19] Karmann A. (1982) Spatial barter economies under locational choice. Journal of Mathematical Economics 9; 259-274.

[20] Koutsougeras LC. (2003a) Non-Walrasian equilibria and the law of one price. Journal of Economic Theory 108; 169-175.

[21] Koutsougeras LC. (2003b) Convergence to no arbitrage equilibria in market games. Journal of Mathematical Economics 39; 401-420.

[22] Kung F, Wang P. (2012) A spatial network approach to urban configurations. Canadian Journal of Economics 45:1; 314-344.

[23] Mas-Colell A. (1982), The Cournotian foundations of Walrasian equilibrium theory: An exposition of recent theory. In Hildenbrand W (Ed.), Advances in Economic Theory. Cambridge University Press: Cambridge; 183-224.

[24] Novshek W. (1980) Equilibrium in simple spatial (or differentiated product) models. Journal of Economic Theory 22; 313-326.

[25] Osborne MJ, Pitchik C. (1986) Price competition in a capacity-constrained duopoly. Journal of Economic Theory 38; 238-260.

[26] Postlewaite A, Schmeidler D. (1978) Approximate efficiency of non-Walrasian Nash equilibria. Econometrica 46; 127-135.

[27] Sahi S, Yao S. (1989) The non-cooperative equilibria of a trading economy with complete markets and consistent prices. Journal of Mathematical Economics 18; 325-346.

[28] Salop S. (1979) Monopolistic competition with outside goods. Bell Journal of Economics 10; 141-156.

[29] Schweizer U, Varaiya P, Hartwick J. (1976) General equilibrium and location theory. Journal of Urban Economics 3; 285-303.

[30] Shapley L, Shubik M. (1977) Trade using one commodity as a means of payment. Journal of Political Economy 85; 937-968.

[31] Shubik M. (1973) Commodity money, oligopoly, credit and bankruptcy in a general equilibrium model. Western Economic Journal 11; 24-38.

[32] Wang P. (1990) Competitive equilibrium formation of marketplaces with heterogeneous consumers. Regional Science and Urban Economics 20; 295-304.