Education Policies and Economic Growth

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Abstract

This paper studies the general equilibrium implications of two types of education policy in an overlapping generations growth model with second-best policy. We examine vouchers, which augment inherited private education spending, and public investment on economy-wide human capital, that provides economy-wide externalities to individual human capital accumulation. The government determines jointly the allocation of tax revenues among the two types of education policy and tax policy, subject to the competitive decentralized equilibrium. Using plausible parameter values it is shown that it is socially optimal to spend heavily on economy-wide human capital accumulation and finance government spending by a modest proportional tax on initial human capital and a low tax on inherited private education expenditures.

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JEL classification: H41, H52, I22, C61

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1 Introduction

In Europe, North America and other countries, there is an ongoing debate on potential ways of financing activities that have public good characteristics and involve positive externalities, since competitive equilibria are typically inefficient in such cases.

We focus on education policy, because since the work of Lucas (1988) human capital accumulation has been identified as a fundamental source of long-run growth in modern economies and we have seen significant government intervention in the funding and provision of education worldwide (e.g Gradstein et al., 2000, Thum et al., 2003). In most countries, primary and secondary education are mandatory and provided by the government and higher education is heavily subsidized (e.g by deducting educational spending from taxable income).

Education can be viewed as a mechanism of intergenerational transfers, since it typically takes place at the beginning of the life cycle and it is financed by resources transferred from the old generation. These transfers are altruistically motivated, but affect economic growth, income distribution and welfare through their impact on human capital accumulation (see e.g Lucas, 1988, Azariadis et al, 1990, Barro et al., 1995, Barro 2001, De la Croix et al., 2002). Altruistic decisions yield typically inefficient outcomes and parental decisions regarding children's education, which ignore the positive impact of individual human capital accumulation on the aggregate production, are a classic example of such decisions.

We use an overlapping generations model where human capital accumulation is the engine of long-run growth and relies on private and public investment, initial human capital and time devoted to education. Regarding welfare, members of the old generation have a bequest motive and value education transfers to the next

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1Roughly 5% of GDP is transferred to the young generation through public education in the OECD countries (see Thum et al., 2003).

2Generally, reasons for bequests are altruism on behalf of the parents, provision of incentives
generation \(^3\) as well as consumption and leisure (Zilcha, 2003).

Regarding education policy, a benevolent fiscal authority uses distortionary income taxes to augment human capital accumulation by the provision of educational vouchers (EV) and direct education spending (GH). The allocation of public human capital expenditures among the above two outlays and the associated tax rate are chosen optimally by the fiscal authority, which maximizes the utility of the representative old agent and remain fixed once they are determined.

The basic result of the paper is that it is optimal to undertake high direct education spending and finance it by a modest proportional income tax and a low tax on inherited education transfers, i.e negative education vouchers.

This work is related to three strands of literature. First, it complements the literature on endogenous growth and human capital accumulation (see e.g Lucas 1988, Azariadis et al., 1990). Second, it is related to research, which tries to explain the widespread public provision and financing of education as a way to indoctrinate and instill social norms and values e.g reducing the rent-seeking incentives between competitive groups of heterogeneous agents (Gradstein 2000, Gradstein et al., 2000, 2002, Thum et al, 2003). Third, this work is relevant to the large and diverse literature on alternative ways of financing education (see for example Glomm et al., 1992, Zhang 1996, Epple et al., 1998, Kaganovich et al., 1999, Meier, 2000, Soares 2003).

However, this paper studies an environment where both private and public education spending exist, while other authors (e.g Glomm et al., 1992, Zhang, 1996, Cardak, 1999) analyze the private and public education regimes separately.

\(^3\)Education quality received by the children is assumed to be determined solely by parental funding.

such that their heirs behave according to what parents believe is appropriate and accidental death of retired individuals who are not able to buy actuarially fair annuities. Besides that, in the absence of a bequest motive it would be difficult to explain why even very wealthy individuals maintain large asset balances at death (Azariadis, 1993).
We also use a richer human capital accumulation specification including time devoted to education, two forms of public education spending, private education expenditures and parental human capital than other studies (e.g Kaganovich et al., 1999 and Cardak, 1999). We also include private education transfers directly in the utility function and assume that agents put different weight to the components of utility, in contrast with most of the literature (e.g Glomm et al., 1992, Cardak, 1999, Zhang 1996). Finally, we endogenize the tax rate, that is sometimes taken as exogenous, (see e.g Kaganovich et al., 1999).

The paper is organized as follows. Section 2 solves for the competitive decentralized equilibrium. Section 3 solves for the optimal revenue shares of the two types of public education spending and the associated tax rate and conducts sensitivity analysis with respect to the most important parameters of the model. Section 4 concludes the paper. Technical details are contained in the Appendices.

2 Education Policy and Competitive Equilibrium

2.1 Theoretical framework

Consider an overlapping generations economy populated by $N$ two-period-lived agents. In the second period of life each individual gives birth to another, so population growth is zero. Each generation consists of identical individuals, so it is characterized by a representative agent. Agents derive utility from leisure when young and consumption and education spending (education quality) passed on to the next generation when old. This formulation is standard in the literature (see also Glomm et al., 1992, Zhang, 1996, Cardak 1999, Kaganovich et al., 1999). One unit of time is available to each individual in every period. During the first period, time is allocated to leisure and human capital accumulation, while in the

\footnote{For an examination of the impact of variable population growth on economic growth see e.g. Futagami et al, 2001.}
second period all time is supplied in the labour market.

The two generations alive in every period (i.e young and old) are linked through two channels. First, the stock of human capital of the parents affects children’s learning. This reflects the fact that a young individual inherits partially the human capital of the parents, i.e there is intergenerational transmission of ability and knowledge within the family that does not work through formal schooling. Parental human capital might also affect children’s human capital through the quality of parental tutoring. The second linkage between generations exists through bequests; in our model the bequest is education spending passed on to the next generation, since parents value human capital transfers to their offspring. This reflects education-inclined altruism on behalf of the parents and is referred in the literature as "joy of giving" (or "warm glove"), since parents have a taste for giving (Wigger, 2001, De la Croix et al, 2002).

Education policy operates through two types of government expenditures financed by taxes on private agents’ initial human capital. First, government provides education vouchers which are added to inherited parental spending on schooling (EV), e.g student scholarships, teaching and research assistantships or money permitting parents to send their children to private schools. Second, fiscal authorities incur direct education spending, which provides economy-wide externalities to individual human capital accumulation (GH), e.g public programs for libraries, expenditures on building schools, teachers’ and university professors’ salaries and training. Both GH and EV work as inputs to private human capital formation by complementing private inputs. A possible interpretation of the simultaneous presence of private and public human capital expenditures in the production of human capital is that the majority of public education spending finances primary and secondary education, while private expenditure finances mainly tertiary education and on the job-training (Blankenau et al., 2004).
In an nutshell, individual human capital depends on time devoted to schooling, parental education spending, parental stock of human capital net of taxes, direct public education expenditures and government spending on education vouchers.

Events take place in two stages. First, a centralized fiscal authority chooses the tax rate and the allocation of the associated revenues among the two types of education policy, then private agents choose consumption, education spending on their children and leisure (therefore time devoted to education) given economic policy.

### 2.2 Household behaviour

Solving the problem backwards, in the second stage, the representative agent born in period $t$ chooses $c_{t+1}, n_t$ and $e_{t+1}$ taking educational vouchers, public investment, which provides economy-wide externalities to individual human capital accumulation and the income tax rate as given to maximize the lifetime utility function:

$$a \ln (n_t) + \ln (c_{t+1}) + b \ln (e_{t+1})$$

subject to

$$h_{t+1} = c_{t+1} + e_{t+1}$$

$$h_{t+1} = A \left(1 - n_t\right)^\beta (e_t + v_t)^\gamma G_{t+1}^\zeta \left[(1 - \tau_{t+1}) h_t\right]^\delta$$

where $n_t \in [0, 1]$ is leisure in period $t$, $c_{t+1}, e_{t+1}$ are respectively consumption, education expenditures in period $t+1$. Parameter $a$ represents the preference for

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5 Regarding human capital accumulation, empirical studies show that the quality of education, measured e.g by the student/teacher ratio, term length or relative pay of teachers, influences positively the rate of return of individuals to education, therefore their future income (see Card et al., 1992). Also, the empirical work shows a positive correlation between parental knowledge and
leisure. The last element of the utility function reflects the ad hoc altruism, i.e "joy of giving". The utility from leaving a bequest depends on the size of the bequest. So, $b$ is the altruism factor reflecting the degree of parental altruism towards children expressed via education transfers to the offspring. These parameters are assumed to be constant over time, i.e all generations of every family give the same weight to $n_t$ and $e_{t+1}$.

Also, $h_{t+1}$ is human capital in period $t + 1$, $h_t$ and $e_t$ are predetermined and stand for human capital and inherited private education spending in period $t$. As for $v_{t+1}$ and $G_{t+1}$, they stand for EV, GH respectively and $\tau_{t+1}$ is the uniform proportional tax rate on initial human capital. Finally, $A$ is a technological parameter that stands for total factor productivity in the human capital formation technology and $\beta, \gamma, \delta, \zeta$ exhibit the elasticities of the learning process with regard to time devoted to education, inherited private education spending adjusted for education vouchers (EV), after-tax human capital and government spending on economy-wide human capital (GH) respectively.

Given that agents supply one unit of labour inelastically in period $t + 1$, $h_{t+1}$ stands for income and the wage rate. Equation (2) is the budget constraint of the representative household and states that net human capital (disposable income) is devoted to consumption and education bequests to the decendants. Relation (3) is a Cobb-Douglas production function for human capital, according to which human capital accumulation depends positively on time devoted to schooling $(1 - n_{i,t})$, initial education spending supplemented by education vouchers $(e_t + v_{t+1})$, public spending on economy-wide human capital, $G_{t+1}$, and income in period $t$ adjusted...
for taxes, $[(1 - \tau_{t+1}) h_t]$, where $A > 0$ and $\beta, \gamma, \delta, \zeta \in (0, 1)$, so that all factors exhibit diminishing returns.

Conditions (2) and (3) imply that

$$c_{t+1} + e_{t+1} = A (1 - n_t)^\beta (e_t + v_{t+1})^\gamma G_{t+1}^\zeta [(1 - \tau_{t+1}) h_t]^{\delta}$$  \hspace{1cm} (4)

The first-order conditions give:\footnote{The second-order conditions are also satisfied if $e_t > 2.1$, which is reasonable given that the baseline value for $h_t = 10$ (see below).}

$$n_t = \frac{a}{a + (1 + b) \beta}$$  \hspace{1cm} (5)

$$c_{t+1} = \frac{1}{1 + b} h_{t+1}$$  \hspace{1cm} (6)

$$e_{t+1} = \frac{b}{1 + b} h_{t+1}$$  \hspace{1cm} (7)

$$h_{t+1} = A \left[ \frac{(1 + b) \beta}{a + (1 + b) \beta} \right]^\beta \left( \frac{b}{1 + b} (h_t + v_{t+1}) \right)^\gamma G_{t+1}^\zeta [(1 - \tau_{t+1}) h_t]^{\delta}$$  \hspace{1cm} (8)

Equation (5) implies that time devoted to education is constant in equilibrium and independent of initial human capital and education expenditures, since the income and substitution effects of changes in $e_t$ and $h_t$ balance each other perfectly.\footnote{This holds due to log-linear preferences.}

**Proposition 1** By partial differentiation of (5)-(8) with respect to $a, b, A, \beta, \gamma, \delta, \zeta$, we get:

a) Time devoted to education is a negative function of the preferences over leisure (a) and a positive function of the degree of parental altruism towards children (b) and the elasticity of future human capital with respect to the time spent on education (\beta). The inverse holds for leisure.
b) Consumption and education transfers in period $t + 1$ depend positively on human capital in $t + 1$ ($h_{t+1}$) and education expenditures increase with parental preferences over children’s education quality ($b$).

c) Human capital in period $t + 1$ depends positively on the intensity of parental altruism ($b$), total factor productivity with regard to human capital accumulation, the elasticities of the process of human capital evolution with respect to education expenditures ($\gamma$), net initial human capital ($\delta$) and direct government education spending ($\zeta$). Besides these, human capital in $t + 1$ is a negative function of the elasticity of human capital formation with respect to time devoted to education ($\beta$) and the preference parameter for leisure ($a$).

2.3 Government budget constraint

The government runs a balanced budget. It uses revenues from proportional taxation of initial human capital and allocates them between two types of spending, i.e. educational vouchers ($v_{t+1}$) and expenditures on economy-wide human capital ($G_{t+1}$) keeping their respective shares in total government spending as fixed.

Given that there are $N$ private agents, the government budget constraint is:

$$Nv_{t+1} + G_{t+1} = \tau_{t+1}Nh_t$$

(9)

Without loss of generality, we denote the shares of total tax revenues financing $Nv_{t+1}, G_{t+1}$ as $k_1, k_2$, where

$$k_2 = 1 - k_1$$

(10)

respectively. Thus (9) can be decomposed into:

$$Nv_{t+1} = k_1\tau_{t+1}Nh_t$$

(11)
Equations (9)-(12) imply that economic policy is summarized by \((k_1, \tau_{t+1})\).

2.4 Competitive decentralized equilibrium

Given the policy vector \((k_1, \tau_{t+1})\), the Competitive Decentralized Equilibrium (CDE) is defined as the set of allocations \((n_t, c_{t+1}, e_{t+1}, h_{t+1}, k_1, \tau_{t+1})\) such that: (i) households maximize utility given economic policy; (ii) markets clear; (iii) the government budget constraint is satisfied. We will make use of the specific functional forms and try to obtain closed-form solutions for the elements of the CDE.

Having assumed that private agents are alike\(^8\) and using (10)-(12), we get the following:

\[
G_{t+1} = (1 - k_1)\tau_{t+1} Nh_t
\]

\[
G_{t+1} = (1 - k_1)\tau_{t+1} Nh_t
\]

**Proposition 2** By (5)-(8) and (13)-(14) we have the following result:

In a symmetric competitive decentralized equilibrium (given any economic policy), optimal leisure, consumption, education transfers, individual human capital, education vouchers and public spending on economy-wide human capital are respectively:

\[
n_t = \frac{a}{a + (1 + b) \beta}
\]

\(^8\)This assumption is reasonable, since we focus on the optimal allocation of tax revenues between the two types of public education spending on efficiency grounds and not on inequality or free-riding effects on equilibrium outcomes.
\[ c_{t+1} = \frac{1}{1 + b} A \left[ \frac{(1 + b) \beta}{a + (1 + b) \beta} \right]^\beta \left( \frac{b}{1 + b} h_t + v_{t+1} \right)^\gamma G_{t+1}^\kappa [ (1 - \tau_{t+1}) h_t]^\delta \]  \hspace{1cm} (16) 

\[ e_{t+1} = \frac{b}{1 + b} A \left[ \frac{(1 + b) \beta}{a + (1 + b) \beta} \right]^\beta \left( \frac{b}{1 + b} h_t + v_{t+1} \right)^\gamma G_{t+1}^\kappa [ (1 - \tau_{t+1}) h_t]^\delta \]  \hspace{1cm} (17) 

\[ h_{t+1} = A \left[ \frac{(1 + b) \beta}{a + (1 + b) \beta} \right]^\beta \left( \frac{b}{1 + b} h_t + v_{t+1} \right)^\gamma G_{t+1}^\kappa [ (1 - \tau_{t+1}) h_t]^\delta \]  \hspace{1cm} (18) 

\[ v_{t+1} = k_1 \tau_{t+1} h_t \]  \hspace{1cm} (19) 

\[ G_{t+1} = (1 - k_1) \tau_{t+1} N h_t \]  \hspace{1cm} (20) 

This holds for any fiscal policy, where the latter is represented by the allocation of tax revenues between educational vouchers \( v_{t+1} \), expenditures on economy-wide human capital \( G_{t+1} \) and income tax rate \( \tau_{t+1} \). In the next section, we will endogenize the choice of \( v_{t+1}, G_{t+1} \) and \( \tau_{t+1} \).

### 3 Optimal Economic Policy

To endogenize economic policy, it is sufficient to determine the independent policy instruments \((k_1, \tau_{t+1})\). So, we consider a centralized fiscal authority, that chooses the fraction of total tax revenues devoted to educational vouchers \( k_1 \) and a uniform tax rate \( \tau_{t+1} \). This authority acts as a benevolent Stackelberg leader vis-a-vis the private sector by taking into account the competitive decentralized equilibrium.

As a result, the problem consists in choosing \((k_1, \tau_{t+1})\), which maximize the utility of the representative household given in (1). Substituting (15)-(20) into
(1) and differentiating with respect to \( k_1, \tau_{t+1} \) the first-order conditions are the following:\(^9\)

\[
[\gamma - k_1 (\gamma + \zeta)] \tau_{t+1} - \frac{b\zeta}{1+b} = 0
\]  

\[
-(\gamma + \delta + \zeta) k_1 \tau_{t+1}^2 + \left[(\gamma + \zeta) k_1 - \frac{b(\delta + \zeta)}{1+b}\right] \tau_{t+1} + \frac{b\zeta}{1+b} = 0
\]  

By solving (21)-(22) for the optimal policy vector \((k_1, \tau_{t+1})\), we get the following:

\[
k_1 = \frac{\gamma + b(\gamma - \delta - \zeta)}{\gamma + \zeta + b(\gamma - \delta + \zeta)}
\]  

\[
\tau_{t+1} = \frac{\gamma + \zeta + b(\gamma - \delta + \zeta)}{(1+b)(\gamma + \delta + \zeta)}
\]

Therefore, the portion of tax revenues devoted to education vouchers \((k_1)\) and the optimal (second-best) tax rate \((\tau_{t+1})\) are constant over time and depend on the parental preference intensity over children’s education quality \((b)\) and the elasticities of future human capital with regard to: a) education expenditures bequethed by the parents to their descendants adjusted for vouchers \((\gamma)\); b) after-tax human capital \((\delta)\); c) public spending, which provides economy-wide externalities to individual human capital accumulation \((\zeta)\).

In order to obtain numerical values for the second-best policy \((k_1, \tau_{t+1})\), we need values for \( b, \gamma, \delta, \zeta \), which for \( \gamma, \delta, \zeta \) are chosen from Benhabib et al.,1994, Psararopoulos,1985, Card et al., 1996, Magoula et al.,1997 and Acemoglu et al.,1999. The value for \( b \) is selected so that individuals put more weight on consumption, education transfers to their offspring and leisure sequentially. Thus, the baseline parameter values are \( b = 0.5, \gamma = 0.15, \delta = 0.35, \zeta = 0.15 \). As a

\(^9\)The second-order conditions of the problem are outlined in Appendix A.
result, the elements of the policy vector are calculated as $\tau_{t+1} = 0.28$, $k_1 = -0.09$, therefore $k_2 = 1.09$ by (10). The negative value of $k_1$ means that the government must impose a tax on private education transfers. Therefore, we have:

**Proposition 3** It is socially optimal to spend heavily on activities that display positive economy-wide externalities on human capital accumulation and finance these expenditures by a modest proportional tax on initial human capital and a low tax on private inherited education spending.

This result is expected, because direct education spending implies economy-wide externalities on human capital accumulation, therefore growth, while education vouchers display no external effects. However, since vouchers are growth-enhancing too, government can not levy high taxes on inherited education spending, i.e highly negative vouchers, and must finance expenditure on economy-wide human capital formation mostly by distortionary taxes on private agents’ initial human capital. Finally, the optimal policy mix is contingent on the fact that we examine symmetric equilibria, so only production efficiency and not income distribution matters.

### 4 Dynamics

In this part of our work, we study the dynamics of the economy as reflected in human capital accumulation. To achieve this, we combine (18)-(20), (23)-(24) to get the evolution of income as follows:

$$h_{t+1} = Bh_t^{\gamma+\delta+\zeta}$$  \hspace{1cm} (25)

where

$$B = A(N\zeta)\delta \left[ \frac{(1+b)\beta}{a+(1+b)\beta} \right]^{\beta} \left[ \frac{b}{1+b} + \frac{\gamma + b(\gamma - \delta - \zeta)}{(1+b)(\gamma + \delta + \zeta)} \right]^{\gamma} \left[ \frac{1+2b}{(1+b)(\gamma + \delta + \zeta)} \right]^{\delta+\zeta}$$  \hspace{1cm} (26)
From this law of motion we establish conditions for the existence and uniqueness of the steady-state income.

**Proposition 4** i) If $\gamma + \delta + \zeta \in (0, 1)$, the economy converges monotonically to a steady state human capital $h^* = B^{\frac{1}{\gamma + \delta + \zeta}}$, which is independent of initial human capital, ii) If $\gamma + \delta + \zeta = 1$, there is no steady state human capital and the economy displays endogenous growth or decay equal to $B$. There are three subcases: a) $B = 1$. We have $h_{t+1} = h_t$ and the economy stays at the initial human capital level, b) $B < 1$. Then $h_{t+1} < h_t$ and there is monotonic convergence toward a steady state human capital equal to 0, c) $B > 1$. There is no steady state and the economy exhibits long-run growth, iii) If $\gamma + \delta + \zeta > 1$, the economy converges to one of two equilibria, $h^* = 0$ or $h^* = B^{\frac{1}{\gamma + \delta + \zeta}}$, depending on initial conditions, i.e the equilibria are unstable.

**Proof.** The above findings follow directly from the law of motion (25). ■

As a result, the dynamics of our economy allow for a variety of growth paths depending on parameter values, including the neoclassical and endogenous growth as subcases.

## 5 Sensitivity analysis

### 5.1 Policy instruments

After having computed the shares of spending on economy-wide human capital accumulation and education vouchers in total government expenditure and the associated income tax rate, we conduct a sensitivity analysis of the optimal policy vector $(k_t, \tau_{t+1})$ with regard to the model's parameters, i.e $b$, $\gamma$, $\delta$, $\zeta$. We also calculate the value of the voucher $(v_{t+1})$ and spending on economy-wide human capital $(G_{t+1})$ corresponding to each parameter value. Specifically, we change the value of one parameter at a time and compute the respective value of the policy instrument of interest. The ranges of the parameter values are chosen so that they cover the whole range of empirically plausible values, i.e $b \in [0.29, 0.8]$, 

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Figure 1: Sensitivity of \((k_1, \tau_{t+1}, v_{t+1}, G_{t+1})\) wrt \(b\)

\[
\begin{align*}
\gamma & \in [0.05, 0.25], \quad \delta \in [0.25, 0.45], \quad \zeta \in [0.05, 0.25].
\end{align*}
\]
The results are presented in Figures 1-4 below (the respective tables are shown in Appendix B).

From Figure 1, we observe that as \(b\) increases, \(k_1\) declines, therefore \(k_2\) increases. This is because the stronger the parental preference over education bequests, the larger the portion of government expenditures the agents would like to be devoted to spending on economy-wide human capital (GH), because it has a stronger positive impact on human capital accumulation than education vouchers (EV). As a result, the optimal share of education vouchers in public spending is smaller. Also, as \(b\) rises, the optimal tax rate falls, because the higher share of funding devoted to GH increases human capital for given values of the rest of productive inputs allowing for a lower tax rate to raise the necessary revenues. In addition, \(v_{t+1}\) falls when \(b\) rises, because both \(k_1\) and \(\tau_{t+1}\) decrease \((v_{t+1} = k_1\tau_{t+1}h_t\) by (19)). On the other hand, \(G_{t+1}\) rises because \(k_2\) \((= 1 - k_1)\) rises more than \(\tau_{t+1}\) falls \((G_{t+1} = (1 - k_1)\tau_{t+1}Nh_t\) by (20)). So, economies with high degree of education-inclined altruism towards the young, are expected to
have low vouchers, high direct education spending and low taxes.

In Figure 2, we note that both $k_1$ and $\tau$ are increasing functions of $\gamma$, so $k_2$ is a negative function of $\gamma$. This is because a higher $\gamma$ implies higher productivity of education vouchers relative to direct education spending making it optimal to devote a larger fraction of government spending to EV, therefore a smaller fraction to GH. However, the productivity of vouchers remains lower than that of direct expenditure implying a smaller tax base ceteris paribus, so a rise of the tax rate is necessary to finance the higher voucher spending. So, when $\gamma$ rises, $v_{t+1}$ increases, because both $k_1$, $\tau_{t+1}$ increase, but $G_{t+1}$ falls because $k_2$ declines more than $\tau$ rises. As a result, economies with human capital sensitive to individual-specific education spending, will be characterized by high educational vouchers, low expenditures on economy-wide human capital and high tax rates.

By Figure 3, we see that there is negative relation between both $k_1$, $\tau_{t+1}$ and $\delta$. This happens because, a higher $\delta$ means that both direct educational and voucher expenditures become less productive compared to initial human capital,
so there must be a decline in the tax rate on human capital to promote knowledge accumulation. The component of public spending, which is less productive (EV) takes a lower share of government spending, so the opposite holds for spending on economy-wide human capital. Also, the optimal voucher declines when $\delta$ rises, since both $k_1, \tau_{t+1}$ fall and direct public education investment falls too, since the increase in $k_2$ less than compensates for the fall in $\tau_{t+1}$ (see (19)-(20)). As a consequence, economies with highly sensitive human capital wrt net initial human capital, will be experiencing low education vouchers, low spending on economy-wide human capital and low taxes.

In Figure 4, we point out that $k_1$ is a negative function of $\zeta$ and $\tau_{t+1}$ is a positive function of $\zeta$. This is because a rise in the elasticity of future human capital with regard to public spending on economy-wide human capital makes it optimal to devote a larger share of government spending on this type of expenditure, which implies a lower fraction going to vouchers. But, since the rise in the share of direct education spending is higher than the fall in the fraction of vouchers, a higher tax
rate is necessary to keep the budget balanced. Finally, $v_{t+1}$ declines with the rise in $\zeta$, since the fall of $k_1$ more than offsets the increase in $\tau_{t+1}$, but $G_{t+1}$ rises, because both $k_2$ and $\tau_{t+1}$ increase by (19)-(20). As a consequence, economies with education technology sensitive wrt education spending on economy-wide human capital, will be experiencing low education vouchers, large spending on economy-wide human capital and high tax rates.

Finally, total government spending changes in the same direction with $\tau_{t+1}$, when parameter values vary, since it is equal to $\tau_{t+1} Nh_t$. So, spending on education increases when human capital accumulation is more elastic with respect to individual-specific education spending $(e_t + v_{t+1})$ and public spending on economy-wide human capital $(G_{t+1})$ and falls in case preferences over education quality $(b)$ strengthen and the elasticity of future human capital with regard to initial human capital $(\delta)$ rises.
5.2 Endogenous variables

The sensitivity analysis presented above implies that the policy instruments may take positive as well as negative values depending on the underlying parameter values. This was not taken into account in Proposition 1, where it was e.g. implicitly assumed that $k_1 > 0$, $v_{t+1} > 0$. So, the findings outlined there are not conclusive with respect to the response of human capital, consumption and private education spending in period $t + 1$ ($h_{t+1}, c_{t+1}, e_{t+1}$ respectively) to a change in some model's parameters. As a consequence, a sensitivity analysis is called for with regard to the above variables.10

Concerning $h_{t+1}$, the Figure 5 confirms the conclusions of Proposition 1 for the whole range of parameter values used in our analysis. So, period $t + 1$ human capital is a positive function of the extent of parental human capital-inclined altruism ($b$) and the elasticities of human capital accumulation with respect to individual-specific education expenditures ($\gamma$), net initial human capital ($\delta$) and public spending on economy-wide human capital ($\zeta$). Also, future human capital depends negatively on the elasticity of human capital formation with regard to learning time ($\beta$).

As far as period $t + 1$ consumption and private education spending are concerned, since they are positive functions of $h_{t+1}$ by (6)-(7), they exhibit the same relationship with $b$, $\gamma$, $\delta$, $\zeta$, $\beta$ with period $t + 1$ human capital. The only possible exception is the dependence of $c_{t+1}$ on $b$. This is because, as $b$ rises, $h_{t+1}$ increases, but the fraction in (6) declines. But as Figure 6 indicates, a higher $b$, i.e. stronger preference for education transfers to the young generation, implies lower consumption. This is because as $b$ increases, the weight on consumption falls relative to the weight on education bequests, so optimal consumption falls.

10The tables corresponding to the sensitivity analysis of the endogenous variables and welfare (see next section) are omitted due to space considerations. However, they are available from the author upon request.
Figure 5: Sensitivity of \((h_{t+1}, c_{t+1})\) wrt \((b, \gamma, \delta, \zeta, \beta)\)
As a result, an economy where parental altruism towards children’s education is stronger and/or the elasticities of human capital accumulation w.r.t the various factors of education technology, except time devoted to learning, are higher than in another economies will experience higher growth, but lower consumption.

5.3 Welfare

The analysis of the impact of a variation in the model’s parameters on the policy instruments and private variables would be incomplete if it was not followed by a welfare analysis. The latter is useful even in the context of a representative agent model as ours, since it summarizes the effects of changes in these parameters on utility, which the social planner aims at maximizing through the use of the various policy instruments.

Using the same range of values as before, i.e. $b \in [0.29, 0.8], \gamma \in [0.05, 0.25], \delta \in [0.25, 0.45], \zeta \in [0.05, 0.25], \beta \in [0.05, 0.35]$ and $h_t = 10, N = 10^5$, we present the results in Figure 6. Welfare depends positively on the degree of parental altruism towards the offspring expressed via education transfers ($b$) and the elasticities of human capital accumulation with respect to individual-specific education spending ($\gamma$), net initial human capital ($\delta$) and public education expenditures on economy-wide human capital ($\zeta$).

These findings are expected because the rise in education-inclined altruism is higher proportionately than the fall of the weight given to consumption, inducing a welfare improvement when $b$ increases. Also, as each of the above three elasticities gets higher human capital accumulation in period $t + 1$, therefore income, are higher for given factors of production, so there is room for more consumption and education expenditure in $t + 1$, both of which increase welfare.

Furthermore, a higher elasticity of human capital formation with regard to learning time reduces welfare. This is because a higher $\beta$ reduces human capital...
Figure 6: Sensitivity of welfare wrt \((b, \gamma, \delta, \zeta, \beta)\)
in period $t + 1$, as mentioned above, implying a decline in consumption and/or private education spending in $t + 1$, both of which lower welfare.

Therefore, societies where agents care more about their descendants’ education and the productive factors, except learning time, have a stronger impact on human capital accumulation than others enjoy higher welfare.

6 Conclusions

In this paper we examined the implications in a general equilibrium setting of two types of education policy, i.e. education vouchers (EV) and public investment on economy-wide human capital (GH). We focused on education policy, because human capital accumulation is considered as a fundamental source of long-run growth in modern economies and government intervention in education is widespread. The objective was to determine the optimal allocation of tax revenues between the two types of government spending and the associated tax rate subject to the symmetric competitive decentralized equilibrium in an environment which allows for endogenous time devoted to education and takes into account explicitly the parental altruism for children’s education. This has not been addressed previously in the literature, although there are many papers studying the effects of human capital accumulation on growth.

We found that the optimal policy mix depends on the values of the model’s parameters and for the baseline parameter values it was shown that the government should finance a high level of spending on economy-wide human capital by a modest tax on initial human capital and a low tax on private education transfers.

According to the sensitivity analysis performed, the optimal allocation of tax revenues depends on the relative productivity of the two types of expenditures and the parental preference parameter over education transfers towards children. Although it is difficult to estimate the latter parameter, it would be interesting
to compare various countries in that respect.

Regarding welfare, it was found that it depends positively on the elasticities of human capital accumulation with respect to individual-specific education spending, economy-wide human capital and after-tax initial human capital and parental preferences over children’s education.

We close with possible extensions. First, we could examine an economy where the individuals in each generation are heterogeneous, allowing for more elaborate education policies, e.g. means-tested vouchers, which would allow us to study the impact of education policies on income distribution. We might also assume the more realistic case, where direct government education spending is not a pure public good, but a public good subject to congestion. Furthermore, we might study the case of progressive in addition to proportional taxation of initial human capital and model uncertainty with respect to the characteristics of human capital accumulation. We leave these extensions for the future.
APPENDICES

APPENDIX A

The second-order sufficient conditions for the optimal economic policy are the following:

\[ f_{11} < 0 \]

\[ f_{11} f_{22} - f_{12} f_{21} > 0 \]

where

\[ f_{11} = -2\tau_{t+1}k_1 (\gamma + \delta + \zeta) + (\gamma + \zeta) k_1 - \frac{b(\delta + \zeta)}{1 + b} \]

\[ f_{22} = -(\gamma + \zeta) \tau_{t+1} \]

\[ f_{12} = -(\gamma + \delta + \zeta) \tau_{t+1}^2 + (\gamma + \zeta) \tau_{t+1} \]

\[ f_{12} = f_{21} \]

These conditions hold for the benchmark values used in the analysis of the paper, i.e. \( b = 0.5, \gamma = 0.15, \delta = 0.35, \zeta = 0.15 \) and the respective values of the policy instruments, \( \tau_{t+1} = 0.28, k_1 = -0.09 \).

APPENDIX B

Table B1. Sensitivity analysis of \((k_1, \tau_{t+1}, v_{t+1}, G_{1+1})\) wrt \( b \)
Table B2. Sensitivity analysis of \((k_1, \tau_{t+1}, v_{t+1}, G_{t+1})\) wrt \(\gamma\)

<table>
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<th>(b)</th>
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Table B3. Sensitivity analysis of \((k_1, \tau_{t+1}, v_{t+1}, G_{t+1})\) wrt \(\delta\)

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Table B4. Sensitivity analysis of \((k_1, \tau_{t+1}, v_{t+1}, G_{t+1})\) wrt \(\zeta\)

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References


Cardak, B.A. 1999, Heterogeneous preferences, education expenditures and income distribution. Economic Record, 75, 228, 63-76.


