LABOR-MARKET VOLATILITY IN A MATCHING MODEL WITH WORKER HETEROGENEITY AND ENDOGENOUS SEPARATIONS

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Abstract

This paper shows that introducing worker heterogeneity into a standard search and matching model can help increase the volatility of unemployment without violating the tight negative correlation between vacancies and unemployment, i.e., the Beveridge curve. In the model, periods of high job destruction and unemployment correspond with periods of more severe mismatch between the demands of firms and the qualifications of job seekers. A more severe mismatch translates into fewer successful employment matches conditional on the number of contacts per firm and, as a result, into a higher expected recruitment cost per worker hired, with adverse effects on incentives to open vacancies.

JEL classification: E24; E32; J63; J64

Keywords: search and matching, endogenous separations, worker heterogeneity, unemployment and vacancies volatility

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INTRODUCTION

Consider a negative shock to labor productivity in the standard search and matching model. On impact, firm profitability worsens and fewer vacancies are opened. Unemployment increases through fewer matches. But the decline in the number of new vacancies and the rise in unemployment make the labor market less tight, with a lower number of vacancies per unemployed worker. This creates a positive externality on firms and ultimately puts a break on the decline in vacancies and consequent increase in unemployment. This externality is the main reason why, as shown in Shimer (2005), a reasonably calibrated version of the textbook matching model grossly fails to account for the observed volatility of unemployment. The model can achieve more unemployment volatility only if a mechanism is introduced to offset the positive externality on vacancy creation.

A natural candidate for such a mechanism is an endogenous job destruction caused by the productivity shock. There is ample evidence that both transitions in and out of unemployment contribute to the cyclical volatility of unemployment, with the inflow rate contributing about one third to one half of the volatility of unemployment.\(^1\) With a negative productivity shock, job destruction rises and the entry into unemployment is increased. The rise in unemployment is reinforced, adding to the volatility due to the lower matching rate. But the rise in job destruction also reinforces the positive externality on vacancy creation, because the further increase in unemployment lowers the vacancy to unemployment ratio even more. As Mortensen and Nagypál (2007b) demonstrate, in the most widely used model of endogenous job destruction, due to Mortensen and Pissarides (1994), the externality is strong enough to make firms increase vacancies, leading to a negatively sloped Beveridge curve.

This paper shows that the positive externality on vacancy creation from the fall in tightness can be mitigated when there is heterogeneity in the labor force. In a recession, firms become more selective in terms of the profitability of the employment relationships they choose to commence and their threshold for hiring a worker becomes tighter than usual. Firms also become more selective with respect to what workers they retain, thus job destruction rises and unemployment entry is increased. But the employment relationships that are endogenously terminated in a recession are those with workers whose

\(^{1}\)See, e.g., Shimer (2007) and Fujita and Ramey (2009). Evidence reported in Rogerson and Shimer (2011) suggest that spikes in job destruction drive part of the initial decline in unemployment during most downturns. Moreover, evidence from Barsky et al. (1994), Bowles et al. (2002) and Liu (2003) that the average labor quality increases in economic downturns, suggest that at least some part of job separations is driven by endogenous decisions in response to aggregate productivity shocks.
qualifications are not strong enough to secure positive rents to firms, i.e., workers that are less likely to be employable during bad times. Consequently, at times of low aggregate productivity and high job destruction there are a lot more unemployed workers looking for jobs, but at the same time, there is larger dispersion between the demands of firms and the qualifications offered by unemployed workers. Because a firm searches for a good match among a heterogeneous group of workers, larger dispersion translates into fewer successful matches, conditional on the vacancy to unemployment ratio, i.e., into a deterioration in matching efficiency, with adverse effects on incentives to post vacancies. This idea is formalized in a relatively standard search and matching model extended to allow for worker heterogeneity in terms of ability. I show quantitatively that the model with heterogeneity is capable of increasing the volatility of unemployment without violating the tight negative correlation between vacancies and unemployment.

Recent empirical literature suggests that matching efficiency can, indeed, decline substantially in a recession. For instance, Elsby et al. (2010) report that the outflow rate from unemployment, conditional on the vacancy-unemployment ratio, has been very low during the 2008-2009 recession. Likewise, Davis et al. (2010) show a dramatic decline in the vacancy yield during the same period. Evidence of a strong cyclical component in matching efficiency can also be found in Barnichon and Figura (2010, 2011), Daly et al. (2011) and Sahin et al. (2011). Further, a large empirical literature studying the matching function links changes in matching efficiency to aggregation issues often disguised under the term mismatch: the disparity between the characteristics of job seekers and the requirements of firms. In line with the mismatch hypothesis, the model developed in this paper captures an endogenous mechanism that generates mismatch-driven cyclical changes in matching efficiency.

The model also accounts for the coexistence of a large number of short unemployment spells with a small number of workers who stay unemployed for much longer. Shimer

2The model I propose relies on unobservable heterogeneity and emphasizes a type of mismatch that is more likely to occur within small segments of the labor market, due to firms becoming more selective with respect to what workers to hire and retain. But the degree of disaggregation that can be achieved by available data is limited. For this reason, empirical studies that examine the mismatch hypothesis rely on broad disaggregations such as differences in the distribution of locations or industrial sectors between unemployed workers and vacancies (see, e.g., Lilien 1982, Elsby et al. 2010, Barnichon and Figura 2010, Daly et al. 2011 and Sahin et al. 2011). The hypothesis that the losses in matching efficiency observed during the recent recession can be attributed to imbalances in labor supply and demand across industries has received only mixed support in the empirical literature. If such imbalances were present, we would expect growing industries facing much larger falls in vacancy yields than struggling industries. In contrast, vacancy yields have been below expectations across all industries, suggesting that the shortfalls in vacancy yields are due to firms with vacancies becoming more selective about filling them, in line with my assumptions.
documents a strong negative duration dependence in re-employment probabilities and interprets his finding as evidence for the view that workers who fail to quickly find jobs need to wait for new vacancies to come into the market. Likewise, Coles and Petrongolo (2003) find that the re-employment rate of some of the newly unemployed workers depends statistically on the inflow of new vacancies and not on the vacancy stock. They interpret this finding as evidence that some newly unemployed workers are on the long-side of their market. Consistent with these findings, in the model presented here, some of the newly unemployed workers - particularly those with lower ability - will manage to exit unemployment only when economic conditions improve and the hiring margin becomes looser again, in the sense that workers with lower ability are hired.

A few other papers also explore the role of heterogeneity in generating more cyclical volatility in matching models. The papers most related to this one are Pries (2008) and Bils et al. (2010). Pries (2008), incorporates worker heterogeneity in terms of productivity into a relatively standard matching model to demonstrate that the changing composition of unemployment can increase labor market volatility. In particular, Pries argues that if during downturns the unemployment pool consists of a larger than usual share of low-productivity workers, who generate lower surplus to employers, then firms have less incentive to open vacancies. My model is complementary to Pries’s model in that I allow for endogenous separations, while Pries allows for only exogenous separations. But my analysis emphasizes a different channel through which worker heterogeneity can generate more volatility in job creation. In the model developed here, the driving force behind the enhanced responsiveness of vacancies to productivity shocks are not the compositional changes in the unemployment pool, but the adjustments in the match ac-

Another related paper is Guerriero (2007). She also pursues the idea that cyclical adjustments in the hiring margin can be a potential source of volatility in job creation, but her model differs from mine in several aspects. It is a competitive search model with homogeneous workers and only exogenous separations. Her main finding is that in such a model the adjustments in the hiring margin do not contribute to more volatility in job creation. Some other studies, such as Krause and Lubik (2007), Nagypál (2007) and Tasci (2007), also explore the interaction between worker heterogeneity and labor market volatility, but focus on the heterogeneity introduced by on-the-job search.

In Pries’s model, the shift in the unemployment pool towards low-productivity workers is imposed exogenously by considering a larger increase in the exogenous separation rate for low-productivity workers. It is not clear cut, however, that a model with endogenous separations can predict such a shift. In such a model, just as in the present model, the workers laid off in a recession are more productive than those laid off in a boom, meaning that in a recession the share of low-productivity workers in the unemployment pool is smaller, not larger, than usual. Evidence suggest that the skill composition of the unemployed does not change much over the business cycle (see, e.g., Elsby et al. 2010 and Barnichon and Figura 2010), while evidence based on unobservable heterogeneity point to a reversed impact than the one assumed by Pries. Using CPS data Mueller (2011) finds that the average residual wage of the unemployed is strongly countercyclical.
ceptance/continuation threshold that cause procyclical changes in matching efficiency. Pries’s model precludes such effects due to the assumption that all matches are acceptable at all times.

Bils et al. (2010) consider a variant of the Mortensen and Pissarides (1994) where workers are risk averse and therefore heterogeneous in terms of their willingness to trade search for work (i.e., in terms of their reservation wage). The question they ask is whether their model can produce both realistic fluctuations in unemployment and a realistic dispersion in wage growth within matches. For this reason, they abstract from other sources of heterogeneity and consider only match-quality shocks, which are necessary to generate wage changes within matches. In their model, differences in worker’s reservation wages reflect differences in their wealth, that in turn, reflect differences in their histories of match qualities and unemployment spells. Having both worker heterogeneity and a match acceptance decision, their model captures a channel for volatility in vacancies that is similar to the one emphasized in this paper. Just as in the present model, endogenous separations may depress vacancy creation, if they are concentrated on workers with high reservation wages, who are less likely to be employable during bad times. However, in their calibrated model the dispersion in reservation wages across workers is relatively small, meaning that differences in rents across matches reflect mainly match-quality shocks. The Beveridge curve correlation in their calibrated model is therefore weak, because the matches that endogenously break up in their model are mainly low-quality matches, as opposed to matches with workers whose reservation wage is high. Freeing up workers by destructing such low-quality matches facilitates the creation of new more productive jobs.

The paper is organized as follows. Section I lays out the set up of the model under study and characterizes the steady-state equilibrium. Section II presents steady-state comparisons that characterize the response of key labor-market variables to aggregate productivity shocks. Section III presents some quantitative results. Section IV briefly discusses some of the model’s assumptions and Section V concludes.

I The Model

The model is in discrete time. The economy is populated by ex-ante heterogeneous risk-neutral workers of measure one and firms of a large measure. Workers differ in terms of

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5In two related papers (Bils et al. 2007 and 2009) the same authors consider versions of the model where workers are also heterogeneous in terms of labor market ability, but assume that markets are segmented by ability.
their ability, which is measured by $x$. Ability is distributed according to the cumulative distribution function $F(\cdot)$ with support $X \equiv [\underline{x}, \bar{x}]$ and associated density function $f(\cdot)$. In any period, a worker may be either employed or unemployed, while a firm may be either matched with a worker and producing or posting a vacancy. A type-$x$ worker produces $y_t p(x)$ units of output, where $y_t$ is a stochastic aggregate productivity component and $p(x)$ is a constant worker-specific productivity component that increases with the worker’s ability: $p'(x) > 0$. Unemployed workers receive a constant flow benefit $b$ per period. Firms that post a vacancy pay a constant cost $c$ per period. The number of vacancies is determined by free entry. Hence, firms open vacancies until the expected value of doing so becomes zero.

Ability is assumed to be observable to the firm, but only when the firm actually meets with the worker. Firms cannot learn about the workers’ abilities prior to meeting with them. For this reason, they cannot direct their search to workers of a particular ability level. There is therefore a single matching market with a meeting function determining the number of contacts/matches. More precisely, let $u_t(x)$ and $v_t$ denote the number of unemployed workers of type $x$ and posted vacancies, respectively, in period $t$. The total number of matches between searching workers and firms in period $t$ is determined by a matching function $M(v_t, u_t) = v_t^{1-\alpha} u_t^\alpha$, where $u_t = \int_{\underline{x}}^{\bar{x}} u_t(x) dx$ gives the total number of unemployed workers in period $t$.

The probability that a worker matches with a firm can be written as $m(\theta_t)$, where $\theta_t = \frac{v_t}{u_t}$ measures the tightness of the labor market. Likewise, a vacancy matches with a worker (of any type) with probability $q(\theta_t)$ and with a worker of type $x$ with probability $q(\theta_t) u_t(x)$.

Each period, before production takes place, matched workers and firms (including those in ongoing employment relationships) negotiate on a contract that divides the surplus of the match according to the Nash Bargaining solution. The worker’s bargaining

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6 The idea is that observationally equivalent workers (i.e., workers with similar education and experience) may actually differ in terms of their ability to perform similar jobs and that employers can learn about these differences only during the interview process. After all, one of the most important reasons employers interview their applicants is to learn about their inherent abilities, because other characteristics, such as education and experience are usually known to employers prior to the interview. Consequently, even if firms can effectively direct their vacancies towards workers with a particular education/experience level, they still have to search among a pool of applicants that is heterogeneous in terms of ability. These assumptions are consistent with a large empirical literature that documents significant wage differences among observationally equivalent workers (see, e.g., Mortensen 2003).

7 With the term “match” I refer to a meeting between a searching worker and a firm with a vacancy. As I explain below, a meeting may or may not lead to the beginning of a new employment relationship, because an agreement may not be reached. I use the terms “employment relationship” and “employment match” to refer to the cases where an agreement has been reached and the pair has decided to start producing.
weight is $\beta$ and the disagreement point is separation. Let $S_t(x)$ denote the surplus of a match between a firm and a worker of type $x$ in period $t$. A worker and a firm will choose to continue or begin an employment relationship only if $S_t(x) > 0$, and will agree to separate if $S_t(x) = 0$, in which case separation is jointly optimal. Since the surplus of an employment relationship is increasing in the productivity and therefore the ability of the worker, there will be a reservation productivity $p(R_t)$ and a reservation ability $R_t$, such that $S(R_t) = 0$. Hence, the worker and the firm will choose to continue or commence any employment relationship with $x > R_t$. Aside from the jointly optimal separations, known as endogenous separations, employment matches also face a risk of separating for exogenous reasons with a probability $s$.

The timing of events and decisions within a period is as follows. At the beginning of each period matches between unemployed workers and vacancies are realized. At the same time, a randomly selected fraction $s$ of ongoing employment relationships is destroyed for exogenous reasons. Subsequently, aggregate productivity, $y_t$, is realized. Upon observing $y_t$, workers and firms in surviving relationships bargain a new wage if there is still a surplus to share, i.e., if $x > R_t$. In the opposite case, they optimally separate. Likewise, the newly matched workers decide whether or not to begin an employment relationship with the wage reflecting worker-firm bargaining. If, given the realization of aggregate productivity, the worker’s ability is sufficiently large, i.e., if $x > R_t$, so that the surplus of the employment relationship is positive, then a new employment relationship begins. Otherwise, the firm and the worker continue searching. Finally, production takes place and unemployed workers and vacancies engage in search.

**Value Functions**

The unemployment value, $U_t(x)$, and the value of a match, $W_t(x)$, to a worker of ability $x$ satisfy:

$$U_t(x) = b + \gamma E_t [m(\theta_t)W_{t+1}(x) + (1 - m(\theta_t))U_{t+1}(x)]$$

(1)

$$W_t(x) = \max \{w_t(x) + \gamma E_t [sU_{t+1}(x) + (1-s)W_{t+1}(x)], U_t(x) \}$$

(2)

where $E_t$ is the expectation operator, $w_t(x)$ the wage rate and $\gamma = \frac{1}{1+r}$ the discount factor.

The value of a vacancy is given by

$$V_t = -c + \gamma E_t \left[ q(\theta_t) \int_x^z \frac{u_t(x)}{u_t} J_{t+1}(x) dx + (1 - q(\theta_t))V_{t+1} \right]$$

(3)
and the value of a match with a type-x worker to a firm is given by

$$J_t(x) = \max \left\{ y_t p(x) - w_t(x) + \gamma E_t \left[ s V_{t+1}(x) + (1 - s) J_{t+1}(x) \right], V_t \right\}$$  \hspace{1cm} (4)$$

In (1) the payoff in the current period for an unemployed worker is $b_t$; with probability $m(\theta_t)$ the worker matches with a vacancy yielding a value $W_{t+1}(x)$ and in the opposite case the worker remains unmatched yielding a value $U_{t+1}$. The first term in the bracket of equation (2) is the value of an employment relationship to a type-x worker. An employed type-x worker earns the wage $w_t(x)$ and faces the risk of an exogenous separation that occurs with probability $s$. If the relationship exogenously breaks up, the worker becomes unemployed yielding a value $U_{t+1}$, but if the relationship survives, the continuation value is $W_{t+1}(x)$. A worker will choose to stay in (or commence) an employment relationship only if the value of being in the employment relationship is greater than the value of being unemployed. Accordingly, the value of a match to the worker is the maximum between the two. Likewise, in (4), the value of a match with a type-x worker to the firm is the maximum between the value of being in an employment relationship with that worker and the value of being vacant. If the firm chooses to commence (or stay in) an employment relationship with the worker, it produces output $y_t p(x)$, pays the wage $w_t(x)$ and faces the risk of an exogenous separation that occurs with probability $s$. In (3), a firm with a vacancy incurs a cost $c$ and matches with a type-x worker with probability $q(\theta_t) \frac{u_t(x)}{u_t}$, yielding a value $J_{t+1}(x)$. With probability $1 - q(\theta_t)$ the firm fails to match with a worker yielding a value $V_{t+1}$.

The wage rate, $w_t(x)$, satisfies the Nash conditions, $W_t(x) - U_t(x) = \beta S_t(x)$ and $J_t(x) - V_t = (1 - \beta) S_t(x)$. Moreover, in a free-entry equilibrium $V_t = 0$ holds for all $t$. Using these conditions we can write the surplus of a match, when the worker’s ability is $x$ as

$$S_t(x) = \max \left\{ y_t p(x) - b + \gamma E_t S_{t+1}(x) \left[ 1 - s - \beta m(\theta) \right], 0 \right\}$$ \hspace{1cm} (5)$$

and the value in (3) as

$$\frac{c}{q(\theta_t)} = \gamma (1 - \beta) E_t \int_x^\bar{x} \frac{u_t(x)}{u_t} S_{t+1}(x) dx \hspace{1cm} (6)$$

The law of motion for the unemployment of a type-x worker is given by

$$u_{t+1}(x) = u_t(x) + s (f(x) - u_t(x)) - u_t(x) m(\theta_t) I + \delta(x) \hspace{1cm} (7)$$
where $\delta(x)$ captures discrete jumps from employment to unemployment due to endogenous separations, and I is an indicator function, which takes the value of 1 if the worker’s ability is equal or above the reservation ability and 0 otherwise. Specifically, $\delta(x) = 0$, I = 1, if $x > R_t$, and $\delta(x) = (1 - s)(f(x) - u_t(x))$, I = 0, otherwise.

Equations (5) to (7) determine the free-entry equilibrium path of $\theta_t$ for given realizations of the aggregate productivity process.

The steady-state equilibrium

Here I characterize the properties of the non-stochastic steady state, where the aggregate state, $y$ and the distribution of unemployment across different types of workers are constant.

The steady-state surplus of a match when the worker’s type is $x$ is given by

$$S(x) = \max\left\{ \frac{y p(x) - b}{\gamma (r + s + \beta m(\theta))}, 0 \right\}$$

(8)

It is evident from (8) that the steady-state reservation productivity satisfies:

$$p(R) = \frac{b}{y}$$

(9)

The steady-state unemployment is given by

$$u(x) = \begin{cases} \frac{s f(x)}{s + m(\theta)}, & \text{if } x > R \\ f(x), & \text{otherwise} \end{cases}$$

(10)

The overall unemployment rate, $u = \int_{2}^{R} f(x) dx + \int_{R}^{\bar{x}} s f(x) dx$, can be written as

$$u = \frac{s + F(R)m(\theta)}{s + m(\theta)}$$

(11)

Employed workers with $x > R$ face only the risk of an exogenous separation that occurs with probability $s$. However, workers with $x \leq R$ separate at rate 1, because even if they manage to survive the exogenous separation shock, they will separate endogenously. The overall separation rate, denoted by $\tilde{s}$, is therefore given by $\tilde{s} = \int_{2}^{R} f(x) dx + \int_{R}^{\bar{x}} s f(x) dx$ and can be written as

$$\tilde{s} = F(R)(1 - s) + s$$

(12)
It is clear from (9) that $R$ is decreasing in $y$, meaning $F(R)$ is also decreasing in $y$. The model therefore features countercyclical fluctuations in the separation rate: a reduction in $y$ raises the reservation productivity leading to more endogenous separations.

The average job finding and job filling rates differ from the matching rates $m(\theta)$ and $q(\theta)$, respectively, because only matches with workers whose productivity is above the reservation productivity will continue as employment matches. In particular, the average job finding and filling rates can be calculated as $\tilde{m} = \int_{R}^{\infty} \frac{u(x)}{\theta} m(\theta)$ and $\tilde{q} = \int_{R}^{\infty} \frac{u(x)}{\theta} q(\theta)$, which give:

$$\begin{align*}
\tilde{m} &= \phi(R, \theta)(1 - F(R))m(\theta) \\
\tilde{q} &= \phi(R, \theta)(1 - F(R))q(\theta)
\end{align*}$$

where

$$\phi(R, \theta) = \frac{s}{s + F(R)m(\theta)}$$

The term $\phi(R, \theta)$ measures the probability that an unemployed worker has ability $x > R$.

The free-entry condition that determines the steady-state value of $\theta$ is given by,

$$\frac{c}{q(\theta)} = \gamma(1 - \beta) \int_{\bar{y}}^{x} \frac{u(x)}{\theta} S(x)dx$$

With (8) and (10) substituted in, the free-entry condition can be written as

$$\frac{c}{q(\theta)\phi(R, \theta)} = \frac{(1 - \beta) \int_{R}^{\infty} (yp(x) - b)f(x)dx}{\gamma(r + s + \beta m(\theta))}$$

The free-entry condition is such that the expected surplus from filling a vacancy equals the expected recruitment cost. If the expected surplus is higher than the expected recruitment cost (i.e., if the right-hand side of (16) is higher than its left-hand-side), firms open more vacancies per job seeker until all rents are exhausted.

The main difference between this model and other models that allow for endogenous separations is the presence of the term $\phi(R, \theta)$ in the free-entry condition. A larger $\phi(R, \theta)$ means that the number of employment relationships that are expected to be formed, conditional on the number of contacts per firm, is larger. In other words, a larger $\phi(R, \theta)$ implies an improvement in matching efficiency, and therefore, a decline in the recruitment cost a firm expects to pay on average in order to fill a vacancy. Notice from (14) that $\phi(R, \theta)$ is decreasing in $R$, meaning that a rise in the reservation productivity
(and thus ability) deteriorates matching efficiency and causes the expected recruitment cost to rise, with adverse effects on incentives to open vacancies. Intuitively, when firm profitability is lower and thus firms are more selective with the workers they are willing to hire, they are also more reluctant to open vacancies, because they anticipate that they will have more difficulty finding suitable workers to fill them. Hence, the model captures a new source of cyclical fluctuations in vacancies, that comes from the impact of changes in the reservation productivity on the expected recruitment cost. In this model, the rise in unemployment that occurs in a recession due to the rise job destruction, i.e., due to the rise in $R$, lowers the vacancy to unemployment ratio without reinforcing the positive externality on vacancy creation. In contrast, the further increase in unemployment acts to further depress job creation, because the workers that enter unemployment due to endogenous separations are those whose productivity falls below the firms’ acceptance threshold. Such workers congest the market during downturns, making it more difficult for firms to locate workers whose productivity is above the acceptance threshold. These workers will make it easier for firms to fill their vacancies only when economic conditions improve and the acceptance threshold falls again, as captured by the negative relation between $R$ and $\phi(R, \theta)$.

For the results below, it is also useful to characterize the replacement ratio. The replacement ratio in the model is $\tilde{b} = \frac{b}{\tilde{p}}$, where $\tilde{p} = \int_0^1 p(x) \frac{dF(x)}{1-F(R)}$ is the average worker-specific productivity among the employed.

II Steady-State Comparisons

Next, I derive results that describe how the key labor-market variables in the model respond to changes in aggregate productivity. By taking logs of (11) and differentiating the result with respect to $\ln y$ we obtain the

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8 It is perhaps useful to clarify that this feature of the model is not due to endogenous separations being concentrated on low-ability workers. Even if one assumed that profits per worker are non-monotonic in ability, or even falling monotonically with ability, so that the workers laid off are not necessarily the least able, this feature would still be present. For instance, one could assume that higher ability workers generate lower profits to employers, say because they have a much better outside option and so need to paid much more. If this was the case, the workers laid off in a recession would be the most able. Nevertheless, the congestion effects mentioned here would still be present, because the most able workers in this case would be those that generate smaller profits, making employers reluctant to hire them during downturns.

9 I follow a common practice and derive elasticities of the key labor-market variables with respect to aggregate productivity to gauge the cyclical response of the model. Examples of studies that follow the same approach are Mortensen and Nagypál (2007a,b) and Pissarides (2009). For a discussion of how good such an approximation is, see Mortensen and Nagypál (2007b).
following expression for the elasticity of the unemployment rate with respect to aggregate productivity:

\[
\frac{\partial \ln u}{\partial \ln y} = - \frac{(1 - \alpha)m(\theta)}{s + m(\theta)} \left[ \frac{s(1 - F(R))}{s + F(R)m(\theta)} \right] \frac{\partial \ln \theta}{\partial \ln y} + \frac{m(\theta)f(R)R}{s + F(R)m(\theta)} \frac{\partial \ln R}{\partial \ln y}
\]  

(17)

where it may be recalled that \(\alpha\) denotes the elasticity of the matching function with respect to the unemployment rate. The first term captures the effect of changes in aggregate productivity on the unemployment rate through the impact of such changes on market tightness. The second term captures the impact of changes in the reservation ability. Clearly, the negative response of the reservation ability amplifies the negative response of unemployment to aggregate productivity shocks. Thus, this model can generate a larger volatility of unemployment than the model with a constant separation rate (canonical model, henceforth), analyzed in Shimer (2005). However, if the model fails to also generate sufficiently larger volatility in tightness than the canonical model, then it will fail to generate a realistic Beverage curve, because

\[
\frac{\partial \ln v}{\partial \ln y} = \frac{\partial \ln \theta}{\partial \ln y} + \frac{\partial \ln u}{\partial \ln y}
\]  

(18)

If the positive elasticity of \(\theta\) with respect to aggregate productivity is not much larger, while the negative elasticity of unemployment is much larger than in the canonical model, then the resulting elasticity of vacancies will be small or even negative; equivalently, if the elasticity of \(\theta\) with respect to aggregate productivity is not sufficiently larger than that of the canonical model, the model will generate a very small or even positive covariance between unemployment and vacancies.

Substituting (17) into (18) yields

\[
\frac{\partial \ln v}{\partial \ln y} = \left[ 1 - \frac{(1 - \alpha)m(\theta)}{s + m(\theta)} \frac{s(1 - F(R))}{s + F(R)m(\theta)} \right] \frac{\partial \ln \theta}{\partial \ln y} + \frac{m(\theta)f(R)R}{s + F(R)m(\theta)} \frac{\partial \ln R}{\partial \ln y}
\]  

(19)

Because the term in the bracket is positive, a larger response in market tightness implies a larger (positive) response in the vacancy rate. But the negative response of the reservation ability dampens the response of the vacancy rate, as captured by the second term in the above expression. This means that the model can explain jointly the cyclical behavior of unemployment and vacancies, only if in addition to the larger volatility in unemployment, it generates a sufficiently larger volatility in market tightness than the canonical model.

The most widely used model of endogenous separations, due to Mortensen and Pis-
sarides (1994), predicts a counter-cyclical vacancy rate, because it delivers significantly
larger volatility in unemployment, but no more volatility in market tightness than the
canonical model. The elasticity of tightness in the canonical model with only a constant
separation rate \( s \) and homogeneous workers that produce \( p \) is given by
\[
\frac{\partial \ln \theta}{\partial \ln y} = \left[ \frac{r + s + \beta m(\theta)}{\alpha(r + s) + \beta m(\theta)} \right] \left[ \frac{1}{1 - \tilde{b} \phi} \right]
\]  
(20)
where \( \tilde{b} \phi = \frac{k}{yp} \) gives the replacement ratio. As shown in Mortensen and Nagypál (2007b),
the elasticity of market tightness in the Mortensen and Pissarides model (the MP model,
henceforth) is observationally equivalent to that in the canonical model, given in (20).
Thus, when both models are calibrated in the same way, i.e., given equal replacement
ratios, average job finding and separation rates, and parameter values for \( \alpha, r \) and \( \beta \),
they yield identical elasticities of market tightness. As mentioned in the introduction, the
reason behind this result is the endogenous response of vacancies to the fall in tightness,
caused by the rise in unemployment.

The current model has the potential to deliver more volatility in market tightness,
because, as explained above, it captures a new mechanism that can help mitigate the
positive externality on vacancy creation from the fall in tightness. By taking logs of the
free-entry condition in (16) and differentiating the result with respect to \( \ln y \) we obtain
the following expression for the elasticity of market tightness in the current model:
\[
\frac{\partial \ln \theta}{\partial \ln y} = \left[ \frac{r + s + \beta m(\theta)}{\alpha(r + s) + \beta m(\theta)} \right] \left[ \frac{\partial \ln \phi(R, \theta)}{\partial \ln y} + \frac{1}{1 - \tilde{b} \phi} \right]
\]  
(21)
Notice that the term \( \frac{\partial \ln \phi(R, \theta)}{\partial \ln y} \) that reflects the impact of changes in aggregate productivity on matching efficiency enters with a positive sign in (21) but is absent from (20).
Consequently, if this term is positive, the elasticity of tightness with respect to aggregate productivity in the present model can be higher than that in the MP model.

By taking logs of (14) and differentiating with respect to \( \ln y \) we obtain:
\[
\frac{\partial \ln \phi(R, \theta)}{\partial \ln y} = - \frac{(1 - \alpha)F(R)m(\theta) \partial \ln \theta}{s + F(R)m(\theta) \partial \ln y} - \frac{m(\theta)f(R)R \partial \ln R}{s + F(R)m(\theta) \partial \ln y}
\]  
(22)
The first term reflects the impact of changes in market tightness: an increase in \( \theta \), and
as a consequence, an increase in the workers’ matching rate, \( m(\theta) \), deteriorates matching efficiency, because it lowers the share of workers with \( x > R \) in the unemployment pool.
The second term captures the impact of changes in the reservation ability: an increase in $R$ implies a lower share of workers with $x > R$ in the unemployment pool and therefore a deterioration of matching efficiency. While the effect of changes in market tightness is negative on the elasticity of $\phi(R, \theta)$, the effect of changes in the reservation ability is positive, because the response of the reservation ability is countercyclical. As shown in Section III, for realistic parameter values the effect of changes in reservation ability dominates that of changes in market tightness so that $\frac{\partial \ln \phi(R, \theta)}{\partial \ln y} > 0$.

The elasticity of the job finding rate, $\hat{m}$, with respect to aggregate productivity can be expressed as:

$$\frac{\partial \ln \hat{m}}{\partial \ln y} = \left[ \frac{s(1 - \alpha)}{s + F(R)m(\theta)} \right] \frac{\partial \ln \theta}{\partial \ln y} - \left[ \frac{f(R)R}{1 - F(R)} \right] \frac{\partial \ln R}{\partial \ln y}$$

(23)

The job finding rate responds to changes in aggregate productivity due to the impact of such changes on both the market tightness (first term) and the reservation ability (second term).

The separation rate responds to aggregate productivity shocks due to the impact of such changes on the reservation ability. Specifically,

$$\frac{\partial \ln \hat{s}}{\partial \ln y} = \frac{f(R)R(1 - s)}{\hat{s}} \frac{\partial \ln R}{\partial \ln y}$$

(24)

Finally, by taking logs of (9) and differentiating with respect to $\ln y$, we can write the elasticity of the reservation ability with respect to aggregate productivity as:

$$\frac{\partial \ln R}{\partial \ln y} = \frac{1}{\epsilon_p(R)}$$

(25)

where $\epsilon_p(R)$ denotes the elasticity of the productivity function $p(x)$ with respect to $x$, evaluated at $x = R$.

It is also worth mentioning here that an additional channel through which the model can generate a larger change in the vacancy rate relative to that in labor productivity is the divergence between aggregate and labor productivity, which is a common feature of models that allow for endogenous separations. Because the reservation productivity moves countercyclically, the average worker-specific productivity among the employed workers, $\bar{p}$ is lower at higher $y$. For this reason, a percentage increase in the aggregate component of productivity, $y$, translates into a smaller percentage increase in average labor productivity,
\[
\frac{\partial \ln y \tilde{\rho}}{\partial \ln y} = 1 + \frac{f(R)R}{1 - F(R)} \frac{\partial \ln R}{\partial \ln y} \tag{26}
\]

Apparently, \(\frac{\partial \ln y \tilde{\rho}}{\partial \ln y}\) is less than one, because \(\frac{\partial \ln R}{\partial \ln y}\) is negative. When confronting the model with the data, the appropriate measure of the changes in a variable, say \(z\), relative to the changes in labor productivity is given by

\[
\frac{\Delta_y \ln z}{\Delta_y \ln y \tilde{\rho}} \equiv \frac{\partial \ln z/\partial \ln y}{\partial \ln y \tilde{\rho}/\partial \ln y} \tag{27}
\]

This means that the change in variable \(z\) relative to that in labor productivity is larger in this model than that in the canonical model. Moreover, since both models are calibrated to match the empirical volatility of the average productivity of labor, \(y\)-shocks are larger in this model than in the canonical model. In turn, larger \(y\)-shocks generate larger fluctuations in the key labor market variables.

### III Some Quantitative Results

Next, I present some quantitative results of the model. In my baseline calculations I use the same parameter values and targets used by Shimer (2005), who reports the results of the canonical model. Hence, aggregate productivity is normalized to \(y = 1\) and the quarterly discount rate is \(r = 0.012\). I set the elasticity parameter to \(\alpha = 0.72\), let worker’s bargaining power take the same value, \(\beta = 0.72\) and set the replacement ratio to 0.40. Finally, I target an average separation rate of 0.10 and an average job finding rate of 1.355.

With the above calibration approach we can obtain the model-implied elasticities of the job finding rate, tightness, vacancies and unemployment, for a given endogenous fraction \(F(R)\) and elasticity of the separation rate with respect to aggregate productivity. In order to derive the fraction \(F(R)\) and the separations elasticity we need information about the distribution of productivity across employment matches. Since the exact shape of this distribution matters only for the volatility of separations, I choose not to impose a particular shape for this distribution.\(^{10}\) Instead, I set the separations elasticity equal to its

\(^{10}\)Because there is no obvious empirical counterpart to which this distribution should be matched, the shape of this distribution is usually chosen in a rather ad hoc fashion. For instance, Mortensen and Nagypáll (2007b) derive the results of the MP model assuming that the productivity distribution is log-normal over \([0, 1]\). Pissarides (2007), on the other hand, calibrates a version of the MP model where, as in Mortensen and Pissarides (1994), the productivity distribution is uniform in the range \([\gamma, 1]\) and
empirical counterpart, which based on Table 1 in Shimer (2005) equals $-1.97$ and derive results for different values of $F(R)$. This enables me to examine whether the model can generate realistic fluctuations in both unemployment and vacancies for reasonable amount of variation in job separations, which is the central issue here.

I use the elasticities derived in Mortensen and Nagypál (2007b) to also compute results for the MP model, using the same calibration approach. I set equal replacement ratios, separation rates, job finding rates and separations elasticities and let the parameters $y, r, \alpha$ and $\beta$ take the same values in both models. This implies that any differences in the predicted volatilities of tightness, vacancies and unemployment found between the two models must come from the cyclical changes in matching efficiency that are present in the current model, but absent from the MP model. Comparing the results of the two models, derived with this calibration, helps quantify the role of worker heterogeneity in amplifying the volatility of job creation.

Table 1 reports the model-implied elasticities of the key labor-market variables both with respect to aggregate and labor productivity. I use the notation $\epsilon_{i,j}$ to denote the elasticity of the variable $i$ with respect to variable $j$. The table also reports the results of the MP model (in parentheses), the results of the canonical model and the relevant empirical responses (labeled as data) based on Table 1 in Shimer (2005). The model has no trouble generating a large enough (negative) change in the unemployment rate relative to changes in labor productivity and generates significantly larger volatility in tightness than both the canonical and the MP model. The response of tightness to aggregate productivity shocks in the MP model is the same as in the canonical model, while in the current model it is much larger. Hence, endogenous job destruction does not contribute to more volatility in tightness in the MP model, but has a significant impact on the volatility of tightness in the current model. The current model generates realistic fluctuations in unemployment, and at the same time, predicts a procyclical vacancy rate.

The model with worker heterogeneity clearly outperforms the MP model, but still, for the selected parameter values, it cannot explain the magnitude of variation in tightness,

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11 In the current model, $F(R)$ measures the probability that a worker is unemployable, while in the MP model the probability that a match-specific productivity draw is below the reservation productivity. In deriving the results below I let $F(R)$ take the same value in both models. Moreover, it is reasonable to assume that $F(R)$ is small. I therefore choose small values for this fraction.

12 As Mortensen and Nagypál (2007b) point out, the empirical equivalent to the elasticity of $x$ with respect to change in $y$ in the model with endogenous separations is the OLS coefficient $\rho_{xy} \frac{\sigma_x}{\sigma_y}$, where $\rho_{xy}$ is the correlation between $\ln x$ and $\ln y$ and $\sigma_x$ is the standard deviation of $\ln x$. Table 1 reports the same OLS coefficients.
and as a consequence, in the vacancy rate we observe in the data. This may be due
to Shimer’s replacement ratio of 0.4, being too low. From equations (20) and (21) it is
evident that a higher replacement ratio in our calibrations implies a larger elasticity of
market tightness. This is because a higher replacement ratio reduces the firm’s profits
so that cyclical shocks have a bigger proportional impact on profits, and thus vacancy
creation. Hagedorn and Manovskii (2008) agree that Shimer’s replacement ratio is too
low, because it does not include the value of leisure or home production, but they suggest
a replacement ratio of 0.955, which seems implausibly large. Hall and Milgrom (2008)
improve on this by estimating the value of additional leisure using evidence on the Frisch
elasticity of labor supply. Their suggested replacement ratio, which includes both unem-
ployment insurance and the value of leisure, is 0.71; a value that is commonly used in
recent studies.\textsuperscript{13} As shown in Table 2, setting the replacement ratio equal to the value
suggested by Hall and Milgrom improves the results considerably. The elasticities of mar-
ket tightness and vacancies obtained from the present model are now very close to their
empirical equivalents, while those obtained from the MP model are still much smaller

\footnotesize{\textsuperscript{13}See, for instance, Pissarides (2009) and Brugemann and Moscarini (2010).}
Table 2: Model results at $\tilde{b} = 0.71$

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<th>$F(R)$</th>
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<th>$\epsilon_{u,y}$</th>
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<th>$\epsilon_{v,y}$</th>
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<td>0.95</td>
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</table>

| data   | 7.56                   | 3.68             | -3.88            | 2.34             |
| canonical | 3.55                  | 2.63             | -0.93            | 0.99             |

than in the data.

IV Discussion

The cyclical changes in matching efficiency emphasized above, are partly driven by the assumption that workers who are not employable, i.e., workers whose productivity falls below the threshold for hiring, stay in the unemployment pool, thereby making it more difficult for searching firms to locate more productive workers. In the current setting, these workers have an incentive to stay in the unemployment pool, because matches are realized prior to the aggregate state. There is therefore always a chance that aggregate conditions will improve and the reservation productivity will fall by the time these workers find a match. It may be worthwhile, however, to comment on some additional reasons that such workers stay attached.

First, workers may have limited information for how well their attributes match with the demands of available jobs. One of the most important reasons employers interview
their applicants is to learn about their inherent abilities and other characteristics that cannot be identified prior to meeting with them. Likewise, workers may want to meet with potential employers and obtain more information about their demands, before they can assess whether they would be employable in available jobs or not. Hence, it may be the case that workers cannot determine whether they are employable in available jobs, unless they search for them. Second, even if jobs are hard to find, workers have an incentive to stay attached, because they want to be entitled to unemployment benefits.

Finally, suppose that a match-specific productivity component of productivity is randomly drawn each time a new match is formed. In such a setting, the worker-specific reservation productivity (or ability) would be lower on matches whose match-specific productivity component is higher. Thus, the possibility of a match-specific productivity draw that is high enough to bring the overall productivity of their match above the acceptance threshold, would give an additional incentive for marginal workers to stay attached. The approach taken in this paper has been to keep the model simple in order to make the role of worker heterogeneity more transparent. Match productivity has therefore been assumed to depend only on the worker’s ability. But the current setting can be viewed as the limiting case of this generalized set up, where both worker- and match-specific heterogeneity are present. To understand why, suppose that during periods of low aggregate productivity some marginal workers cannot be hired in new jobs unless their draw of match-specific productivity turns out to be exceptionally high. As the probability of such an exceptionally high productivity draw approaches zero, the labor market volatility in this generalized setting approaches the one in the simpler setting developed here.

V CONCLUSION

In this paper I have shown that introducing worker heterogeneity in a relatively standard search and matching model can help in amplifying the responsiveness of unemployment to productivity without violating the Beveridge curve correlation. An interesting property of the model is that it reconciles endogenous separations with the Beveridge curve without introducing complex features relative to the most widely used model of endogenous separations, due to Mortensen and Pissarides (1994). The only difference is that this model allows for match productivity to depend on workers’ ability, which seems to be a natural assumption, while in the Mortensen and Pissarides model it is randomly drawn. The interaction between worker heterogeneity and an endogenous match accep-
tance/continuation decision generates cyclical changes in matching efficiency that help in amplifying the response of job creation to productivity shocks. Specifically, as the firms’ threshold for hiring or retaining a worker becomes tighter in a recession, job destruction rises and unemployment is increased, but at the same time, matching efficiency falls. The fall in matching efficiency offsets the positive externality from the fall in tightness on vacancy creation and acts to further depress vacancy creation in a recession.

I have stressed that this mechanism provides a solution to the standard model’s failure to generate sufficient unemployment volatility, but incorporating this mechanism into more generalized settings, can also shed light on some other important questions that remain open. One such question is whether search theoretic models of the labor market can explain the wedge between the marginal rate of substitution of consumption for leisure and the marginal product of labor, often called the “labor wedge.” As Rogerson and Shimer (2011) argue, in a model with search frictions the labor wedge is positively correlated with employment, but the opposite holds in the data. This is because search frictions act as an adjustment cost that dampens fluctuations in employment. Specifically, increasing the vacancy to unemployment ratio in response to a positive productivity shock is costly, because doing so reduces the probability for each vacancy-posting firm to match with a worker. As this paper has shown, the negative externality from the rise in the vacancy to unemployment ratio on the matching probability of firms can be mitigated when there is worker heterogeneity in the model. Moreover, as firms become more selective about filling vacancies during bad times and less selective during booms, some workers are constrained from working as much as they would like to in a recession and vice-versa in a boom. These provide potential explanations to the counter-cyclical labor wedge. Further investigation along these lines might give new insights into the cyclical behavior of the labor wedge in the presence of search frictions.
References


