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Optimal Contracts and Investment in General Human Capital under Common Agency

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Abstract

This paper studies contracts and incentives to invest in general human capital under common agency. Both the worker and the employer have too weak investment incentives in equilibrium. The employer's underinvestment results from his failure to internalize the positive impact of his investment on other firms' productivity as well as from the fact that he gives a share of output to the worker in order to induce a higher effort contribution. The worker anticipates that she will not be the full residual claimant of benefits and underinvests in equilibrium, too. A benevolent government will choose a set of subsidies such that the worker's investment relative to the employer is equal to the first-best relative investment intensity. If the number of employers is small, then the worker's investment level is relatively low and the government must give a relatively higher subsidy to the worker in order to stimulate her investment incentives.

Keywords: General Human Capital, Common Agency, Contracts.

JEL Classification: D82, J24, J31.

1. Introduction

In his seminal contribution to the debate over the provision of human capital, Becker (1964) predicts that the employer will make zero investment in his employees' general skills if the labor market is perfectly competitive. The rationale behind this theoretical conclusion (which, however, contrasts with empirical evidence) is that competition between employers to attract the trained worker enables the latter to reap all productivity benefits associated with acquired general skills. The incumbent employer anticipates that he will not be able to recoup the cost of investment and, therefore, is unwilling to make any investment at all. In this framework, the worker faces first-best incentives to invest in her own general human capital since she is the full residual claimant of associated benefits (i.e. she has all the bargaining power vis-à-vis the employer, who receives zero expected profits).

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The more recent economic literature on the provision of general skills (Acemoglu and Pischke, 1998, 1999a, 1999b) departs from the assumption of perfect competition between employers in the labor market. The introduction of frictions in the labor market implies a pattern of wage compression (in the sense that the marginal effect of general training on worker's wage is lower than the marginal effect on productivity). As a result, the employer will now be willing to make some positive but still inefficiently low investment in general human capital. Again, the (however imperfect) competition between employers pushes the worker's outside wage upwards and forces the incumbent employer to pay a higher wage in order to keep his employee from moving to another firm. The employer anticipates that the worker will be able to extract a proportion of the additional surplus and thus has too weak investment incentives relative to the socially optimal outcome[1].

This paper tries to study general training in a unified framework by allowing both the employer and the worker to invest in human capital. We use a common agency setting in which the worker can be employed by multiple firms at the same time and thus must decide how to distribute her effort between them. One of these employers also makes an investment in general human capital after having observed the worker's investment choice. Given these choices, each employer offers a contract to the worker, linking the latter's wage compensation with output realization in the respective firm. After observing the set of contract offers, the worker chooses the (nonverifiable) level of effort contributions and output is realized in each firm. We study the properties of subgame perfect equilibrium in comparison to the first-best outcome.

The equilibrium allocation involves underprovision of effort to all firms. This is a typical result of models incorporating moral hazard and can be attributed to the standard tradeoff between limited liability and efficiency (see e.g. Laffont and Martimort 2002, Ch. 4). Since the agent is constrained by limited liability, the principal cannot impose a severe enough punishment for the case of low output realization. Therefore, the equilibrium contract involves too weak wage incentives and implements a suboptimally low level of effort. Furthermore, it is shown that both the worker and the employer underinvest in general human capital (relative to the first-best). The prediction of employer's underinvestment is in

[1] The problem of underinvestment in the context of asymmetric information has also been studied by Katz and Ziderman (1990) or Chang and Wang (1996), who assume that the incumbent employer has superior information about the worker's training. An empirical investigation of employee training and wage compression in Britain can be found in Almeida-Santos and Mumford (2005). For a survey of the literature, see Leuven (2005).

line with findings in the previous literature but has a different explanation here: First, the investing employer does not internalize the positive external effect of his investment on other firms' productivity. Second, the presence of moral hazard implies that the employer has to give a share of output to the worker in the form of a wage necessary to implement a higher level of effort. Therefore, the net social benefit exceeds the employer's private benefit from investment, implying that the latter has too weak investment incentives in equilibrium.

On the other hand, the worker also does not internalize the full positive impact of her investment on firms' productivity and thus commits herself to a suboptimally low level of investment, too. Since there is no perfect competition between employers in the labor market (i.e. employers receive more than zero expected profits in equilibrium), the worker anticipates that she will not be able to extract all productivity benefits and thus has too weak investment incentives in the first place. In this context, it is shown that the worker's willingness to invest (relative to the employer) in equilibrium is increasing in the number of firms as well as in output elasticity with respect to worker's effort and investment but can be either increasing or decreasing in output elasticity with respect to employer's investment.

A standard question addressed in the related literature concerns the appropriate policy instrument to alleviate inefficiencies associated with human capital investment. In this context, the original game is modified by assuming a government which chooses the set of investment subsidies in order to maximize social welfare. In the new equilibrium outcome, the government chooses a set of subsidies such that the worker's investment relative to the employer equals the first-best relative investment intensity. If the number of firms in the market is small enough, then the worker's investment incentives are relatively weak in the original equilibrium and the government must give a higher subsidy to the worker than to the employer in order to stimulate the former's investment level.

The seminal contribution to the debate over common agency has been made by Bernheim and Whinston (1986). More recent applications and further contributions to common agency theory include Dixit et al (1996), Peters (2001), Attar et al (2007) or Martimort and Stole (2009).

The rest of the paper has the following structure: In Section 2, we introduce the basic model and in Section 3 we calculate the first-best allocation, which is used as a benchmark outcome thereafter. In Section 4, we compute the subgame perfect equilibrium of the associated game and in Section 5 we study the implications of this equilibrium outcome in comparison to the first-best. In Section 6, we study the optimal subsidization policy chosen by a government that seeks to maximize social welfare. Finally, Section 7 provides some concluding remarks and outlines possible directions for future research.

2. The Model

We consider an economy which consists of $n+1$ agents: one worker (A) and n employers ($P_1,\ldots,P_n$). There is one consumption good (x) produced according to the technology specified below. The worker can be employed by all principals and contributes effort to each firm for the production of the consumption good[2]. Both the worker and the first employer (P_1) can invest in general human capital. We assume the following production technology for each firm:

$$x_i = \begin{cases} HI_o^{\beta} I_1^{\delta} & \text{, with probability } p_i = \min\{e_i^{\alpha}, 1\} \\ 0 & \text{, with probability } 1 - p_i \end{cases} \qquad \textbf{(1)}$$

where x_i denotes the output produced in firm $i=1,\ldots,n$ and $e_i \geq 0$ denotes the level of effort contributed to firm i. Furthermore, I_o (I_1) represents the level of investment in general training made by the worker (employer 1), where $I_o, I_1 \geq 0$. We also assume that the total factor productivity is $H>0$ and $\alpha+\beta+\delta<1$. The latter assumption guarantees that the production function is strictly concave. The values of parameters $\alpha,\beta,\delta>0$ represent output elasticities with respect to inputs e_i, I_o and I_1 respectively. Finally, agents' preferences are represented by the following utility functions:

$$U_A = x_A - \gamma\sum_{i=1}^{n} e_i - \theta I_o$$
$$U_{P1} = x_{P1} - \theta I_1$$
$$U_{Pj} = x_{Pj}\ ,\ j = 2,\ldots,n.$$

where x_A denotes A's consumption, x_{Pi} denotes employer i's consumption ($i=1,\ldots,n$), $\gamma>0$ represents the marginal disutility of labor and $\theta>0$ represents the marginal cost of investment[3].

[2] The distribution of worker's nonverifiable effort among different tasks and principals implies a common agency setting.

[3] For simplicity, it is assumed here that both the worker and employer 1 face the same marginal cost of investment in general human capital. The relaxation of this assumption does not qualitatively affect the results presented below.

3. The First-Best Outcome

Since all agents in the economy are risk-neutral, the first-best allocation can be found by the maximization of aggregate surplus (i.e. the sum of agents' expected utilities) subject to technological and resource constraints:

$$\max\ W = EU_A + \sum_{i=1}^{n} EU_{Pi} = E\left(x_A + \sum_{i=1}^{n} x_{Pi} \right) - \gamma \sum_{i=1}^{n} e_i - \theta(I_o + I_1)$$

s.t. x_i given by (1) for $i=1,\dots,n$: Technological Constraints

$$x_A + \sum_{i=1}^{n} x_{Pi} = \sum_{i=1}^{n} x_i \qquad \text{: Resource Constraint}$$

$$x_A, x_{Pi}, x_i, e_i, I_0, I_1 \geq 0 \qquad \text{: Nonnegativity Constraints}$$

The above problem can be written equivalently:

$$\max_{\{e_i, I_o, I_1\}_{i=1}^{n}} W = E\left(\sum_{i=1}^{n} x_i \right) - \gamma \cdot \sum_{i=1}^{n} e_i - \theta(I_o + I_1) = HI_o^{\alpha} I_1^{\alpha} \sum_{i=1}^{n} e_i^{\alpha} - \gamma \sum_{i=1}^{n} e_i - \theta(I_o + I_1)$$

s.t. $I_0, I_1 \geq 0$, $0 \leq e_i \leq 1$

It should be noted that the first-best outcome always involves $e_i \leq 1$, since any increase in e_i beyond one is costly for society but yields no social benefits ($p_i=1$ for all $e_i>1$). The objective function is concave and the set of constraints is convex, implying that the (Kuhn-Tucker) necessary conditions are also sufficient for maximization. We write the Lagrangian and the first-order conditions:

$$L = HI_o^{\beta} I_1^{\delta} \sum_{i=1}^{n} e_i^{\alpha} - \gamma \sum_{i=1}^{n} e_i - \theta(I_o + I_1) + \sum_{i=1}^{n} \lambda_i (1 - e_i)$$

$$\partial L / \partial e_i = a e_i^{\alpha-1} HI_o^{\beta} I_1^{\delta} - \gamma - \lambda_i \leq 0,\ (\partial L / \partial e_i) e_i = 0,\ i=1,\dots,n$$

$$\partial L / \partial I_0 = \beta HI_o^{\beta-1} I_1^{\delta} \sum_{i=1}^{n} e_i^{\alpha} - \theta \leq 0,\ (\partial L / \partial I_0) I_0 = 0$$

$$\partial L / \partial I_1 = \delta HI_o^{\beta} I_1^{\delta-1} \sum_{i=1}^{n} e_i^{\alpha} - \theta \leq 0,\ (\partial L / \partial I_1) I_1 = 0$$

$$\partial L / \partial \lambda_i = 1 - e_i \geq 0,\ (\partial L / \partial \lambda_i) \lambda_i = 0,\ i=1,\dots,n$$

The solution of these conditions yields the first-best outcome summarized below.

<u>Proposition 1</u>. The first-best outcome involves $I_0 = I_0^{FB}, I_1 = I_1^{FB}, e_i^{FB} = e^{FB}$ for *i=1,…,n*, where:

$$\begin{pmatrix} e^{FB} \\ I_0^{FB} \\ I_1^{FB} \end{pmatrix} = \begin{cases} \begin{pmatrix} n^{\frac{\beta+\delta}{1-\alpha-\beta-\delta}} H^{\frac{1}{1-\alpha-\beta-\delta}} \beta^{\frac{\beta}{1-\alpha-\beta-\delta}} \delta^{\frac{\delta}{1-\alpha-\beta-\delta}} (\frac{1}{\theta})^{\frac{\beta+\delta}{1-\beta-\delta}} (\frac{\alpha}{\gamma})^{\frac{1-\beta-\delta}{1-\alpha-\beta-\delta}} \\ n^{\frac{1-\alpha}{1-\alpha-\beta-\delta}} H^{\frac{1}{1-\alpha-\beta-\delta}} \beta^{\frac{1-\alpha-\delta}{1-\alpha-\beta-\delta}} \delta^{\frac{\delta}{1-\alpha-\beta-\delta}} (\frac{1}{\theta})^{\frac{1-\alpha}{1-\beta-\delta}} (\frac{\alpha}{\gamma})^{\frac{\alpha}{1-\alpha-\beta-\delta}} \\ n^{\frac{1-\alpha}{1-\alpha-\beta-\delta}} H^{\frac{1}{1-\alpha-\beta-\delta}} \beta^{\frac{\beta}{1-\alpha-\beta-\delta}} \delta^{\frac{1-\alpha-\beta}{1-\alpha-\beta-\delta}} (\frac{1}{\theta})^{\frac{1-\alpha}{1-\beta-\delta}} (\frac{\alpha}{\gamma})^{\frac{\alpha}{1-\alpha-\beta-\delta}} \end{pmatrix}, \text{ if } \gamma \geq \tilde{\gamma} \\ \begin{pmatrix} 1 \\ n^{\frac{1}{1-\beta-\delta}} H^{\frac{1}{1-\beta-\delta}} \beta^{\frac{1-\delta}{1-\beta-\delta}} \delta^{\frac{\delta}{1-\beta-\delta}} (\frac{1}{\theta})^{\frac{1}{1-\beta-\delta}} \\ n^{\frac{1}{1-\beta-\delta}} H^{\frac{1}{1-\beta-\delta}} \beta^{\frac{\beta}{1-\beta-\delta}} \delta^{\frac{1-\beta}{1-\beta-\delta}} (\frac{1}{\theta})^{\frac{1}{1-\beta-\delta}} \end{pmatrix}, \text{ if } \gamma \leq \tilde{\gamma} \end{cases} \quad \textbf{(2)}$$

where: $\tilde{\gamma} = \alpha n^{\frac{\beta+\delta}{1-\beta-\delta}} H^{\frac{1}{1-\beta-\delta}} \beta^{\frac{\beta}{1-\beta-\delta}} \delta^{\frac{\delta}{1-\beta-\delta}} (\frac{1}{\theta})^{\frac{\beta+\delta}{1-\beta-\delta}}$

In what follows, we assume $\gamma \geq \tilde{\gamma}$ – i.e. we assume that the first-best allocation is given by the interior solution of the above problem (the second branch of (2)).

4. The Second-Best Outcome: Subgame Perfect Equilibrium

In the second-best environment, the worker's effort contribution e_i to each firm is nonverifiable and thus noncontractible. The sequence of moves in the associated game is the following:

- At stage 1, the worker A chooses her investment in general human capital (I_0).
- At stage 2, employer 1 (P_1) also makes an investment (I_1) in general training after having observed I_0.
- At stage 3, all employers simultaneously offer a wage contract to the worker. Each wage contract is contingent on output realized in the respective firm. In particular, the wage structure has the following form:

$$w_i(x_i) = \begin{cases} h_i & \text{, if } x_i = H I_0^{\beta} I_1^{\delta} \\ l_i & \text{, if } x_i = 0 \end{cases} \quad , i=1,\dots,n. \qquad \textbf{(3)}$$

The assumption that the agent is constrained by limited liability implies $h_i, l_i \geq 0$. Furthermore, the constraint $w_i(x_i) \leq x_i$ (i.e. the requirement of nonnegative profitability for each firm) implies $l_i \leq 0$. Therefore, we can immediately set $l_i = 0$ and conclude that the contract offer made by employer *i* is represented by h_i.

- At stage 4, the worker A chooses her effort contribution e_i to each firm *i*. Then, the output level x_i is realized in each firm, wage compensations are paid and the game ends.

We use backward induction to find the subgame perfect equilibrium of this game. At stage 4, the worker chooses her effort contribution to each firm (e_i) so as to maximize her expected utility (given I_0, I_1, h_i):

$$\max_{\{e_i\}_{i=1}^n} EU_A = \sum_{i=1}^{n} h_i e_i^{\alpha} - \gamma \sum_{i=1}^{n} e_i - \theta I_0$$

s.t. $0 \leq e_i \leq 1$, $i=1,\dots,n$.

It should be noted that the worker will never choose $e_i>1$, because any choice of effort level greater than one involves labor disutility without yielding any benefit to her ($p_i=1$ for all $e_i>1$). Therefore, we can focus on the interval $0 \leq e_i \leq 1$.

We write the Lagrangian and the first-order necessary (and sufficient) conditions for maximization:

$$L = \sum_{i=1}^{n} h_i e_i^{\alpha} - \gamma \sum_{i=1}^{n} e_i - \theta I_0 + \sum_{i=1}^{n} \lambda_i (1 - e_i)$$

The FOCs are:

$$\partial L / \partial e_i = a h_i e_i^{\alpha - 1} - \gamma - \lambda_i \leq 0, \ (\partial L / \partial e_i) e_i = 0, \ i=1,\dots,n.$$
$$\partial L / \partial \lambda_i = 1 - e_i \geq 0, \ (\partial L / \partial \lambda_i) \lambda_i = 0, \ i=1,\dots,n.$$

The solution of these conditions yields the following stage-4 outcome:

$$e_i = \begin{cases} \left(\dfrac{\alpha h_i}{\gamma} \right)^{\frac{1}{1-\alpha}} & \text{, if } h_i \leq \gamma / \alpha \\ 1 & \text{, if } h_i \geq \gamma / \alpha \end{cases} \quad , i=1,\dots,n. \qquad \textbf{(4)}$$

At stage 3, employers $P_1,\dots,P_n$ simultaneously offer contracts $h_1,\dots,h_n$ respectively to the worker so as to maximize their expected profits (given I_0, I_1 and anticipating e_i as given in (4)). Consider the case where $h_i \geq \gamma / \alpha$ first. Then, P_i's expected payoff is:

$$EU_{Pi} = (H I_0^{\beta} I_1^{\delta} - h_i) e_i^{\alpha} = H I_0^{\beta} I_1^{\delta} - h_i$$

The maximization of this payoff under the constraint $h_i \geq \gamma/\alpha$ obviously yields the solution $h_i = \gamma/\alpha$. This means that P_i never has an inventive to set $h_i > \gamma/\alpha$ and we can focus on the case where $h_i \leq \gamma/\alpha$ to write P_i's maximization problem as:

$$\left.\begin{aligned} &\max_{\{h_i\}} EU_{Pi} = (HI_0^\beta I_1^\delta - h_i)e_i^\alpha \\ &\text{s.t. } e_i = \left(\frac{\alpha h_i}{\gamma}\right)^{\frac{1}{1-\alpha}} && (IC) \\ &\sum_{i=1}^{n} h_i e_i^\alpha - \gamma \sum_{i=1}^{n} e_i \geq \bar{U} && (PC) \\ &0 \leq h_i \leq \gamma/\alpha \end{aligned}\right\} \quad \textbf{(}\boldsymbol{PP_i}\textbf{)}$$

where the constraint $h_i \geq 0$ is due to worker's limited liability, (IC) is the worker's incentive compatibility constraint and (PC) is the worker's participation constraint[4]. We assume that $\bar{U} = 0$ and use (IC) to write (PC) as:

$(PC) \Leftrightarrow \sum_{i=1}^{n} h_i \left(\frac{\alpha h_i}{\gamma}\right)^{\frac{\alpha}{1-\alpha}} - \gamma \sum_{i=1}^{n} \left(\frac{\alpha h_i}{\gamma}\right)^{\frac{1}{1-\alpha}} \geq 0$ or, equivalently:

$\left[\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right]\left(\frac{1}{\gamma}\right)^{\frac{\alpha}{1-\alpha}} \sum_{i=1}^{n} h_i^{\frac{1}{1-a}} \geq 0$. The last inequality always holds for $\alpha<1$. This means that the participation constraint is non-binding under the normalization $\bar{U} = 0$. It is also convenient to make a change of variables at this point. In particular, we have:

$$e_i = \left(\frac{\alpha h_i}{\gamma}\right)^{\frac{1}{1-\alpha}} \Leftrightarrow h_i = \gamma e_i^{1-a}/\alpha \qquad \textbf{(5)}$$

We use (5) to rewrite each employer's problem (PP_i) and solve it with respect to e_i instead of h_i:

$$\max_{\{e_i\}} EU_{Pi} = HI_0^\beta I_1^\delta e_i^\alpha - \gamma e_i/\alpha$$
$$\text{s.t. } 0 \leq e_i \leq 1$$

We write the Lagrangian and the first-order conditions for maximization:

$$L = HI_0^\beta I_1^\delta e_i^\alpha - \gamma e_i/a + \lambda(1-e_i)$$

$$\partial L/\partial e_i = \alpha HI_o^\beta I_1^\delta e_i^{\alpha-1} - \gamma/\alpha - \lambda_i \leq 0\,,\ (\partial L/\partial e_i)e_i = 0$$
$$\partial L/\partial \lambda_i = 1 - e_i \geq 0\,,\ (\partial L/\partial \lambda_i)\lambda_i = 0$$

The solution of these conditions yields the following stage-3 outcome:

[4] For $i=1$, the objective function of (PP_1) is $EU_{P1} = (HI_0^\beta I_1^\delta - h_1)e_1^\alpha - \theta I_1$ but the additional term does not affect the choice of contract and thus can be omitted at this stage for notational simplicity.

$$e_i = \begin{cases} (\frac{a^2H}{\gamma})^{\frac{1}{1-\alpha}} I_0^{\frac{\beta}{1-\alpha}} I_1^{\frac{\delta}{1-\alpha}} & \text{, if } I_1 \le (\frac{\gamma}{a^2H})^{\frac{1}{\delta}} I_0^{-\frac{\beta}{\delta}} \\ 1 & \text{, if } I_1 \ge (\frac{\gamma}{a^2H})^{\frac{1}{\delta}} I_0^{-\frac{\beta}{\delta}} \end{cases} \quad , i=1,\ldots,n \qquad \textbf{(6)}$$

From (5) and (6) we obtain the set of optimal contract offers (given I_0 and I_1):

$$h_i = \begin{cases} \alpha H I_0^{\beta} I_1^{\delta} & \text{, if } I_1 \le (\frac{\gamma}{a^2H})^{\frac{1}{\delta}} I_0^{-\frac{\beta}{\delta}} \\ \gamma / \alpha & \text{, if } I_1 \ge (\frac{\gamma}{a^2H})^{\frac{1}{\delta}} I_0^{-\frac{\beta}{\delta}} \end{cases} \quad , i=1,\ldots,n \qquad \textbf{(7)}$$

Since effort contributions and contracts are symmetric, we can write $e_i = e$ and $h_i = h$ hereafter. It should be noted that wage incentives (and effort contributions) increase with the level of investment made by the worker (I_0) and the employer (I_1) at previous stages of the interaction. Any increase in human capital investment induces employers to implement higher levels of effort by giving stronger wage incentives to the worker[5].

The set of players' stage-3 payoffs is:

$$\begin{aligned} EU_A &= nhe^{\alpha} - n\gamma e - \theta I_0 = \frac{n\gamma(1-\alpha)}{\alpha} \cdot e - \theta I_0 \\ EU_{P1} &= (HI_0^{\beta} I_1^{\delta} - h)e^{\alpha} - \theta I_1 \\ EU_{Pj} &= (HI_0^{\beta} I_1^{\delta} - h)e^{\alpha} \;\; , j=2,\ldots,n \end{aligned} \qquad \textbf{(8)}$$

At stage 2, P_1 chooses his level of investment I_1 (given I_0 and anticipating h, e as given in (6), (7)) so as to maximize his expected payoff. The appropriate series of calculations (which can be found in the Appendix) yields the following stage-2 outcome:

$$I_1 = \begin{cases} (\frac{\delta H}{\theta})^{\frac{1-\alpha}{1-\alpha-\delta}} (\frac{a^2H}{\gamma})^{\frac{\alpha}{1-\alpha-\delta}} I_0^{\frac{\beta}{1-\alpha-\delta}} & \text{, if } I_0 \le (\frac{\gamma}{a^2H})^{\frac{1-\delta}{\beta}} (\frac{\theta}{\delta H})^{\frac{\delta}{\beta}} \qquad \textbf{(9a)} \\ (\frac{\delta H}{\theta})^{\frac{1}{1-\delta}} I_0^{\frac{\beta}{1-\delta}} & \text{, if } I_0 \ge (\frac{\gamma}{a^2H})^{\frac{1-\delta}{\beta}} (\frac{\theta}{\delta H})^{\frac{\delta}{\beta}} \qquad \textbf{(9b)} \end{cases}$$

It should be noted from (9a) and (9b) that the principal is more willing to invest in general human capital given a higher level of investment by the worker, because any increase in I_0 also increases the marginal productivity of I_1.

The solution (9a) implies:

[5] It can be easily verified that the solution (7) also satisfies the nonnegative profitability constraint: $h_i \le HI_0^{\beta} I_0^{\delta}$.

$e=(\frac{\delta H}{\theta})^{\frac{\delta}{1-\alpha-\delta}}(\frac{a^2H}{\gamma})^{\frac{1-\delta}{1-\alpha-\delta}}I_0^{\frac{\beta}{1-\alpha-\delta}}$, $h=\alpha H I_0^{\beta}I_1^{\delta}$, and

$$EU_A=\frac{n\gamma(1-\alpha)}{\alpha}(\frac{a^2H}{\gamma})^{\frac{1-\delta}{1-\alpha-\delta}}(\frac{\delta H}{\theta})^{\frac{\delta}{1-\alpha-\delta}}I_0^{\frac{\beta}{1-\alpha-\delta}}-\theta I_0 \qquad \textbf{(10)}$$

Similarly, the solution (9b) implies:

$e=1$, $h=\gamma/\alpha$ and $EU_A=\frac{n\gamma(1-\alpha)}{\alpha}-\theta I_0$ **(11)**

We proceed backwards to stage 1, where the worker *A* chooses the level of investment (I_0) to maximize her expected payoff (anticipating I_1, e, h as given in (9), (10) and (11)).

For $I_0\geq(\gamma/a^2H)^{(1-\delta)/\beta}(\theta/\delta H)^{\delta/\beta}$, *A*'s expected payoff is $EU_A=n\gamma(1-\alpha)/\alpha-\theta I_0$ and the solution to the associated maximization problem is $I_0=(\gamma/a^2H)^{(1-\delta)/\beta}(\theta/\delta H)^{\delta/\beta}$. Therefore, we can focus on the interval $I_0\leq(\gamma/a^2H)^{(1-\delta)/\beta}(\theta/\delta H)^{\delta/\beta}$ and write A's maximization problem as follows:

$$\max_{\{I_0\}} EU_A=\frac{n\gamma(1-\alpha)}{\alpha}(\frac{a^2H}{\gamma})^{\frac{1-\delta}{1-\alpha-\delta}}(\frac{\delta H}{\theta})^{\frac{\delta}{1-\alpha-\delta}}I_0^{\frac{\beta}{1-\alpha-\delta}}-\theta I_0$$

s.t. $0\leq I_0\leq(\frac{\gamma}{a^2H})^{\frac{1-\delta}{\beta}}(\frac{\theta}{\delta H})^{\frac{\delta}{\beta}}$

The objective function of the problem is concave with respect to I_0 (for $\alpha+\beta+\delta<1$). We write the Lagrangian and the first-order conditions for maximization:

$$L=\frac{n\gamma(1-\alpha)}{\alpha}(\frac{a^2H}{\gamma})^{\frac{1-\delta}{1-\alpha-\delta}}(\frac{\delta H}{\theta})^{\frac{\delta}{1-\alpha-\delta}}I_0^{\frac{\beta}{1-\alpha-\delta}}-\theta I_0+\lambda\left[(\frac{\gamma}{a^2H})^{\frac{1-\delta}{\beta}}(\frac{\theta}{\delta H})^{\frac{\delta}{\beta}}-I_0\right]$$

$$\partial L/\partial I_0=\frac{n\beta\gamma(1-\alpha)}{\alpha(1-\alpha-\delta)}(\frac{a^2H}{\gamma})^{\frac{1-\delta}{1-\alpha-\delta}}(\frac{\delta H}{\theta})^{\frac{\delta}{1-\alpha-\delta}}I_0^{\frac{\alpha+\beta+\delta-1}{1-\alpha-\delta}}-\theta-\lambda\leq 0,\ (\partial L/\partial I_0)I_0=0$$

$$\partial L/\partial\lambda=(\frac{\gamma}{a^2H})^{\frac{1-\delta}{\beta}}(\frac{\theta}{\delta H})^{\frac{\delta}{\beta}}-I_0\geq 0,\ (\partial L/\partial\lambda)\lambda=0$$

The solution is:

$$I_0=\begin{cases}(\frac{\gamma}{a^2H})^{\frac{1-\delta}{\beta}}(\frac{\theta}{\delta H})^{\frac{\delta}{\beta}} & \text{, if } \gamma\leq\hat{\gamma}\\ \left[\frac{n(1-\alpha)}{1-\alpha-\delta}\right]^{\frac{1-\alpha-\delta}{1-\alpha-\beta-\delta}}\alpha^{\frac{1-\delta}{1-\alpha-\beta-\delta}}H^{\frac{1}{1-\alpha-\beta-\delta}}\beta^{\frac{1-\alpha-\delta}{1-\alpha-\beta-\delta}}\delta^{\frac{\delta}{1-\alpha-\beta-\delta}}(\frac{1}{\theta})^{\frac{1-\alpha}{1-\alpha-\beta-\delta}}(\frac{\alpha}{\gamma})^{\frac{\alpha}{1-\alpha-\beta-\delta}} & \text{, if } \gamma\geq\hat{\gamma}\end{cases}$$

where $\hat{\gamma}=[\frac{n(1-\alpha)}{1-\alpha-\delta}]^{\frac{\beta}{1-\beta-\delta}}\alpha^{\frac{2-\beta-2\delta}{1-\beta-\delta}}H^{\frac{1}{1-\beta-\delta}}\beta^{\frac{\beta}{1-\beta-\delta}}\delta^{\frac{\delta}{1-\beta-\delta}}(\frac{1}{\theta})^{\frac{\beta+\delta}{1-\beta-\delta}}$

We have already assumed that $\gamma \geq \tilde{\gamma} = \alpha \cdot n^{\frac{\beta+\delta}{1-\beta-\delta}} H^{\frac{1}{1-\beta-\delta}} \cdot \beta^{\frac{\beta}{1-\beta-\delta}} \cdot \delta^{\frac{\delta}{1-\beta-\delta}} \cdot (\frac{1}{\theta})^{\frac{\beta+\delta}{1-\beta-\delta}}$ (in order to guarantee an interior first-best solution). Now, we have:

$$\tilde{\gamma} > \hat{\gamma} \Leftrightarrow n^{\delta} > \left(\frac{1-\alpha}{1-\alpha-\delta}\right)^{\beta} \cdot \alpha^{1-\delta}$$

It is easy to verify that the last inequality always holds for $n \geq 2$, $\alpha+\beta+\delta<1$ and $\alpha,\beta,\delta>0$. Therefore, the subgame perfect equilibrium involves:

$$I_0 = \left[\frac{n(1-\alpha)}{1-\alpha-\delta}\right]^{\frac{1-\alpha-\delta}{1-\alpha-\beta-\delta}} \alpha^{\frac{1-\delta}{1-\alpha-\beta-\delta}} H^{\frac{1}{1-\alpha-\beta-\delta}} \beta^{\frac{1-\alpha-\delta}{1-\alpha-\beta-\delta}} \delta^{\frac{\delta}{1-\alpha-\beta-\delta}} (\frac{1}{\theta})^{\frac{1-\alpha}{1-\alpha-\beta-\delta}} (\frac{\alpha}{\gamma})^{\frac{\alpha}{1-\alpha-\beta-\delta}} \quad \textbf{(12)}$$

We substitute (12) into (9a) to get:

$$I_1 = \left[\frac{n(1-\alpha)}{1-\alpha-\delta}\right]^{\frac{\beta}{1-\alpha-\beta-\delta}} \alpha^{\frac{\alpha+\beta}{1-\alpha-\beta-\delta}} H^{\frac{1}{1-\alpha-\beta-\delta}} \beta^{\frac{\beta}{1-\alpha-\beta-\delta}} \delta^{\frac{1-\alpha-\beta}{1-\alpha-\beta-\delta}} (\frac{1}{\theta})^{\frac{1-\alpha}{1-\alpha-\beta-\delta}} (\frac{\alpha}{\gamma})^{\frac{\alpha}{1-\alpha-\beta-\delta}} \quad \textbf{(13)}$$

Finally, we use (12) and (13) to compute the equilibrium effort contributions and contract offers summarized in Proposition 2 below.

Proposition 2. The subgame perfect equilibrium of the game is:

$$\begin{aligned}
I_0^* &= \left[\frac{n(1-\alpha)}{1-\alpha-\delta}\right]^{\frac{1-\alpha-\delta}{1-\alpha-\beta-\delta}} \alpha^{\frac{1-\delta}{1-\alpha-\beta-\delta}} H^{\frac{1}{1-\alpha-\beta-\delta}} \beta^{\frac{1-\alpha-\delta}{1-\alpha-\beta-\delta}} \delta^{\frac{\delta}{1-\alpha-\beta-\delta}} (\frac{1}{\theta})^{\frac{1-\alpha}{1-\alpha-\beta-\delta}} (\frac{\alpha}{\gamma})^{\frac{\alpha}{1-\alpha-\beta-\delta}} \\
I_1^* &= \left[\frac{n(1-\alpha)}{1-\alpha-\delta}\right]^{\frac{\beta}{1-\alpha-\beta-\delta}} \alpha^{\frac{\alpha+\beta}{1-\alpha-\beta-\delta}} H^{\frac{1}{1-\alpha-\beta-\delta}} \beta^{\frac{\beta}{1-\alpha-\beta-\delta}} \delta^{\frac{1-\alpha-\beta}{1-\alpha-\beta-\delta}} (\frac{1}{\theta})^{\frac{1-\alpha}{1-\alpha-\beta-\delta}} (\frac{\alpha}{\gamma})^{\frac{\alpha}{1-\alpha-\beta-\delta}} \quad \textbf{(14)} \\
h^* &= \left[\frac{n(1-\alpha)}{1-\alpha-\delta}\right]^{\frac{\beta(1-\alpha)}{1-\alpha-\beta-\delta}} \alpha^{\frac{(1-\delta)(1-\alpha)}{1-\alpha-\beta-\delta}} H^{\frac{1-\alpha}{1-\alpha-\beta-\delta}} \beta^{\frac{\beta(1-\alpha)}{1-\alpha-\beta-\delta}} \delta^{\frac{\delta(1-\alpha)}{1-\alpha-\beta-\delta}} (\frac{1}{\theta})^{\frac{(\beta+\delta)(1-\alpha)}{1-\alpha-\beta-\delta}} (\frac{\alpha}{\gamma})^{\frac{\alpha(\beta+\delta)}{1-\alpha-\beta-\delta}} \\
e^* &= \left[\frac{n(1-\alpha)}{1-\alpha-\delta}\right]^{\frac{\beta}{1-\alpha-\beta-\delta}} \alpha^{\frac{1-\delta}{1-\alpha-\beta-\delta}} H^{\frac{1}{1-\alpha-\beta-\delta}} \beta^{\frac{\beta}{1-\alpha-\beta-\delta}} \delta^{\frac{\delta}{1-\alpha-\beta-\delta}} (\frac{1}{\theta})^{\frac{\beta+\delta}{1-\alpha-\beta-\delta}} (\frac{\alpha}{\gamma})^{\frac{1-\beta-\delta}{1-\alpha-\beta-\delta}}
\end{aligned}$$

5. Implications

We proceed with a series of comparisons between the first-best and equilibrium levels of investment and effort. First, we have:

$I_0^* < I_0^{FB} \Leftrightarrow n^{\delta} > (1-\alpha)/(1-\alpha-\delta)^{1-\alpha-\delta} \alpha^{1-\delta}$, which is always true for $n \geq 2$, $\alpha+\beta+\delta<1$ and $\alpha,\beta,\delta>0$. This means that the worker underinvests in general training relative to the first-best. Similarly:

$I_1^* < I_1^{FB} \Leftrightarrow n^{1-\alpha-\beta} > (1-\alpha)/(1-\alpha-\delta)^{\beta} \alpha^{\alpha+\beta}$, which is always true for $n \geq 2$, $\alpha+\beta+\delta<1$ and $\alpha,\beta,\delta>0$. This means that the employer also underinvests in general training. Finally:

$e^* < e^{FB} \Leftrightarrow n^{\delta} > (1-\alpha)/(1-\alpha-\delta)^{\beta}\alpha^{1-\delta}$, which is again true for all parameter values. In other words, there is inefficiently low provision of effort in equilibrium relative to the first-best (i.e. the wage incentives h^* offered by employers are too weak to implement the socially optimal level of effort). These results are summarized in Proposition 3.

Proposition 3.

(a) In the subgame perfect equilibrium, both the worker and the employer underinvest in general human capital relative to the first-best. Therefore, the overall investment in the economy is suboptimally low.

(b) In the subgame perfect equilibrium, there is underprovision of effort relative to the first-best. In other words, the contracts offered by employers give too weak incentives to the worker.

The underprovision of effort by the worker is a hardly surprising result which can be attributed to the standard tradeoff between limited liability and efficiency. Since the worker is constrained by limited liability, employers cannot impose a severe enough punishment for the case of low output realization. As a result, wage incentives are too weak and implement inefficiently low levels of effort in equilibrium.

The employer's underinvestment fits well the standard predictions of the related literature but has a different explanation here. First, the employer does not internalize the positive external effect of his investment on other firms' productivity. Second, the presence of moral hazard implies that the employer must give part of the produced output to the worker in the form of an incentive scheme that induces a higher level of effort. Therefore, the employer cannot reap the full positive impact of his investment on his own firm's productivity. As a result, he has too weak investment incentives in equilibrium (i.e. the net social benefit exceeds the employer's net private benefit from investment in general human capital). On the other hand, the worker also has too weak investment incentives because neither she is the full residual claimant of investment benefits. Since there is no perfect competition between employers in the labor market, the produced output in each firm is divided between the employer and the worker. The latter anticipates this distribution of benefits and chooses an inefficiently low level of investment in the first place.

From (14), it is easy to see that:

$$I_0^* > I_1^* \Leftrightarrow n > \frac{1-\alpha-\delta}{\alpha(1-\alpha)} \cdot \frac{\delta}{\beta}$$

As the number of firms increases, the worker's investment is more likely to be higher than the employer's investment in equilibrium (e.g. for $\alpha=\beta=\delta=1/4$ we have $I_0^* > I_1^*$ for $n \geq 3$). More generally, we can define the worker's relative investment intensity as $r \equiv I_0 / I_1$. Then, we can use (2) to find that the first-best relative investment intensity is $r^{FB} = I_0^{FB} / I_1^{FB} = \beta / \delta$. As expected, r^{FB} is increasing in output elasticity with respect to I_0 and decreasing in output elasticity with respect to I_1. On the other hand, the worker's relative investment intensity in equilibrium is:

$$r^* = I_0^* / I_1^* = \frac{n\alpha(1-\alpha)}{1-\alpha-\delta} \cdot \frac{\beta}{\delta}$$

This expression can be used to state the following proposition:

Proposition 4. The worker's relative investment intensity in equilibrium is:

(i) Increasing in the number of firms: $\partial r^* / \partial n > 0$.

(ii) Increasing in output elasticity with respect to worker's investment: $\partial r^* / \partial \beta > 0$.

(iii) Increasing in output elasticity with respect to worker's effort: $\partial r^* / \partial \alpha > 0$.

(iv) Decreasing (increasing) in output elasticity with respect to employer's investment for low (high) values of this elasticity: $\frac{\partial r^*}{\partial \delta} < (>) 0$ for $\delta < (>) \frac{1-\alpha}{2}$.

A stronger positive impact of worker's effort on output (i.e. a higher value of α) makes the worker relatively more willing to invest in human capital. Furthermore, a stronger positive impact of employer's investment on output (i.e. a higher value of δ) also increases the worker's investment incentives relatively more than the employer's investment incentives for high values of δ. In other words, as the employer's investment becomes highly productive the worker becomes more willing to invest relative to the employer.

6. Optimal Subsidization Policy

This section studies the optimal set of subsidies given by a benevolent government to the worker and employer who invest in general training. The original interaction is now enriched by assuming an initial stage where the social planner optimally chooses the set of subsidies in order to maximize social welfare. In particular, we assume that the worker receives a subsidy s_o per unit of investment I_o, while the investing employer receives a subsidy s_1 per unit of I_1. These subsidies are paid by the worker as a lump-sum tax T[6]. Therefore, the government faces the following budget constraint:

[6] Alternatively, it could be assumed that the cost of the subsidization policy is paid by the firms as a lump-sum tax or any other tax scheme that maintains a balanced government budget.

$$s_0 I_0 + s_1 I_1 = T \quad \textbf{(15)}$$

The new set of players' expected payoffs is the following:

$$EU_A = \sum_{i=1}^{n} h_i e_i^{\alpha} - \gamma \cdot \sum_{i=1}^{n} e_i - (\theta - s_0) I_0 - T$$

$$EU_{P1} = (H I_0^{\beta} I_1^{\delta} - h_1) e_1^{\alpha} - (\theta - s_1) I_1$$

$$EU_{Pj} = (H I_0^{\beta} I_1^{\delta} - h_j) e_j^{\alpha} \quad , \; j = 2, \ldots, n.$$

We use backward induction to find the new subgame perfect equilibrium. At the last stage, the worker's problem has the same solution with the one calculated in (4). In turn, the employers offer the set of contracts and implement the set of effort levels given in (7) and (6), respectively. Anticipating these effort contributions, the employer P_1 chooses I_1 to maximize his expected payoff. The solution of the associated program now is:

$$I_1 = \begin{cases} \left(\frac{\delta H}{\theta - s_1}\right)^{\frac{1-\alpha}{1-\alpha-\delta}} \left(\frac{a^2 H}{\gamma}\right)^{\frac{\alpha}{1-\alpha-\delta}} I_0^{\frac{\beta}{1-\alpha-\delta}} & \text{, if } I_0 \le \left(\frac{\gamma}{a^2 H}\right)^{\frac{1-\delta}{\beta}} \left(\frac{\theta - s_1}{\delta H}\right)^{\frac{\delta}{\beta}} \\ \left(\frac{\delta H}{\theta - s_1}\right)^{\frac{1}{1-\delta}} I_0^{\frac{\beta}{1-\delta}} & \text{, if } I_0 \ge \left(\frac{\gamma}{a^2 H}\right)^{\frac{1-\delta}{\beta}} \left(\frac{\theta - s_1}{\delta H}\right)^{\frac{\delta}{\beta}} \end{cases} \quad \textbf{(16)}$$

Given s_0, s_1 and T, the worker chooses I_o to maximize her expected payoff. As before, we only have to consider the second branch of (16) and solve the problem:

$$\max_{\{I_0\}} EU_A = \frac{n\gamma(1-\alpha)}{\alpha} \left(\frac{a^2 H}{\gamma}\right)^{\frac{1-\delta}{1-\alpha-\delta}} \left(\frac{\delta H}{\theta - s_1}\right)^{\frac{\delta}{1-\alpha-\delta}} I_0^{\frac{\beta}{1-\alpha-\delta}} - (\theta - s_0) I_0 - T$$

$$s.t. \quad 0 \le I_0 \le \left(\frac{\gamma}{a^2 H}\right)^{\frac{1-\delta}{\beta}} \left(\frac{\theta - s_1}{\delta H}\right)^{\frac{\delta}{\beta}}$$

The solution of this problem is[7]:

$$I_0 = \left[\frac{n(1-\alpha)}{1-\alpha-\delta}\right]^{\frac{1-\alpha-\delta}{1-\alpha-\beta-\delta}} \alpha^{\frac{1-\delta}{1-\alpha-\beta-\delta}} H^{\frac{1}{1-\alpha-\beta-\delta}} \left(\frac{\beta}{\theta - s_0}\right)^{\frac{1-\alpha-\delta}{1-\alpha-\beta-\delta}} \left(\frac{\delta}{\theta - s_1}\right)^{\frac{\delta}{1-\alpha-\beta-\delta}} \left(\frac{\alpha}{\gamma}\right)^{\frac{\alpha}{1-\alpha-\beta-\delta}} \quad \textbf{(17)}$$

We can use this solution to get I_1, h and e as functions of s_0 and s_1:

$$I_1 = \left[\frac{n(1-\alpha)}{1-\alpha-\delta}\right]^{\frac{\beta}{1-\alpha-\beta-\delta}} \alpha^{\frac{\alpha+\beta}{1-\alpha-\beta-\delta}} H^{\frac{1}{1-\alpha-\beta-\delta}} \left(\frac{\beta}{\theta - s_0}\right)^{\frac{\beta}{1-\alpha-\beta-\delta}} \left(\frac{\delta}{\theta - s_1}\right)^{\frac{1-\alpha-\beta}{1-\alpha-\beta-\delta}} \left(\frac{\alpha}{\gamma}\right)^{\frac{\alpha}{1-\alpha-\beta-\delta}} \quad \textbf{(18)}$$

$$e = \left[\frac{n(1-\alpha)}{1-\alpha-\delta}\right]^{\frac{\beta}{1-\alpha-\beta-\delta}} \alpha^{\frac{1-\delta}{1-\alpha-\beta-\delta}} H^{\frac{1}{1-\alpha-\beta-\delta}} \left(\frac{\beta}{\theta - s_0}\right)^{\frac{\beta}{1-\alpha-\beta-\delta}} \left(\frac{\delta}{\theta - s_1}\right)^{\frac{\delta}{1-\alpha-\beta-\delta}} \left(\frac{\alpha}{\gamma}\right)^{\frac{1-\beta-\delta}{1-\alpha-\beta-\delta}} \quad \textbf{(19)}$$

$$h = \left[\frac{n(1-\alpha)}{1-\alpha-\delta}\right]^{\frac{\beta(1-\alpha)}{1-\alpha-\beta-\delta}} \alpha^{\frac{(1-\delta)(1-\alpha)}{1-\alpha-\beta-\delta}} H^{\frac{1-\alpha}{1-\alpha-\beta-\delta}} \left(\frac{\beta}{\theta - s_0}\right)^{\frac{\beta(1-\alpha)}{1-\alpha-\beta-\delta}} \left(\frac{\delta}{\theta - s_1}\right)^{\frac{\delta(1-\alpha)}{1-\alpha-\beta-\delta}} \left(\frac{\alpha}{\gamma}\right)^{\frac{\alpha(\beta+\delta)}{1-\alpha-\beta-\delta}} \quad \textbf{(20)}$$

Now, we can write all players' expected payoffs as a function of s_o, s_1 and T:

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[7] In what follows, we focus on the interior solution involving $e_i<1$. Indeed, only this solution is relevant under the initial assumption of a high enough disutility of labor ($\gamma \ge \tilde{\gamma}$).

$$EU_A^S = (1-\alpha-\beta-\delta)\left[\frac{n(1-\alpha)}{1-\alpha-\delta}\right]^{\frac{1-\alpha-\delta}{1-\alpha-\beta-\delta}} \alpha^{\frac{1-\delta}{1-\alpha-\beta-\delta}} H^{\frac{1}{1-\alpha-\beta-\delta}} (\frac{\beta}{\theta-s_0})^{\frac{\beta}{1-\alpha-\beta-\delta}} (\frac{\delta}{\theta-s_1})^{\frac{\delta}{1-\alpha-\beta-\delta}} (\frac{\alpha}{\gamma})^{\frac{\alpha}{1-\alpha-\beta-\delta}} - T$$

$$EU_{P1}^S = [(1-\delta)\alpha^{\frac{\alpha+\beta}{1-\alpha-\beta-\delta}} - \alpha^{\frac{1-\delta}{1-\alpha-\beta-\delta}}]\left[\frac{n(1-\alpha)}{1-\alpha-\delta}\right]^{\frac{\beta}{1-\alpha-\beta-\delta}} H^{\frac{1}{1-\alpha-\beta-\delta}} (\frac{\beta}{\theta-s_0})^{\frac{\beta}{1-\alpha-\beta-\delta}} (\frac{\delta}{\theta-s_1})^{\frac{\delta}{1-\alpha-\beta-\delta}} (\frac{\alpha}{\gamma})^{\frac{\alpha}{1-\alpha-\beta-\delta}}$$

$$EU_{Pj}^S = [\alpha^{\frac{\alpha+\beta}{1-\alpha-\beta-\delta}} - \alpha^{\frac{1-\delta}{1-\alpha-\beta-\delta}}]\left[\frac{n(1-\alpha)}{1-\alpha-\delta}\right]^{\frac{\beta}{1-\alpha-\beta-\delta}} H^{\frac{1}{1-\alpha-\beta-\delta}} (\frac{\beta}{\theta-s_0})^{\frac{\beta}{1-\alpha-\beta-\delta}} (\frac{\delta}{\theta-s_1})^{\frac{\delta}{1-\alpha-\beta-\delta}} (\frac{\alpha}{\gamma})^{\frac{\alpha}{1-\alpha-\beta-\delta}}, j=2,\ldots,n.$$

At the first stage, the government chooses the set of subsidies s_0 and s_1 so as to maximize social welfare subject to its budget constraint:

$$\max_{\{s_0,s_1\}} W_s = EU_A + EU_{P1} + \sum_{j=2}^{n} EU_{Pj}$$

$$s.t. \quad s_0 I_0 + s_1 I_1 = T$$

We can use the above expressions to write this problem in the following form:

$$\max_{\{s_0,s_1\}} W_s = B\left(\frac{1}{\theta-s_0}\right)^{\frac{\beta}{1-\alpha-\beta-\delta}} \left(\frac{1}{\theta-s_1}\right)^{\frac{\delta}{1-\alpha-\beta-\delta}} - \Gamma s_0 \left(\frac{1}{\theta-s_0}\right)^{\frac{1-\alpha-\delta}{1-\alpha-\beta-\delta}} \left(\frac{1}{\theta-s_1}\right)^{\frac{\delta}{1-\alpha-\beta-\delta}} -$$

$$-\Delta s_1 \left(\frac{1}{\theta-s_0}\right)^{\frac{\beta}{1-\alpha-\beta-\delta}} \left(\frac{1}{\theta-s_1}\right)^{\frac{1-\alpha-\beta}{1-\alpha-\beta-\delta}}$$, where:

$$B \equiv \left\{(1-\alpha-\beta-\delta)\alpha^{\frac{1-\delta}{1-\alpha-\beta-\delta}}\left[\frac{n(1-\alpha)}{1-\alpha-\delta}\right]^{\frac{1-\alpha-\delta}{1-\alpha-\beta-\delta}} + [(n-\delta)\alpha^{\frac{\alpha+\beta}{1-\alpha-\beta-\delta}} - n\alpha^{\frac{1-\delta}{1-\alpha-\beta-\delta}}]\left[\frac{n(1-\alpha)}{1-\alpha-\delta}\right]^{\frac{\beta}{1-\alpha-\beta-\delta}}\right\}\cdot$$

$$\cdot H^{\frac{1}{1-\alpha-\beta-\delta}} \beta^{\frac{\beta}{1-\alpha-\beta-\delta}} \delta^{\frac{\delta}{1-\alpha-\beta-\delta}} (\frac{\alpha}{\gamma})^{\frac{\alpha}{1-\alpha-\beta-\delta}} ,$$

$$\Gamma \equiv \left[\frac{n(1-\alpha)}{1-\alpha-\delta}\right]^{\frac{1-\alpha-\delta}{1-\alpha-\beta-\delta}} \alpha^{\frac{1-\delta}{1-\alpha-\beta-\delta}} H^{\frac{1}{1-\alpha-\beta-\delta}} \beta^{\frac{1-\alpha-\delta}{1-\alpha-\beta-\delta}} \delta^{\frac{\delta}{1-\alpha-\beta-\delta}} (\frac{\alpha}{\gamma})^{\frac{\alpha}{1-\alpha-\beta-\delta}} ,$$

$$\Delta \equiv \left[\frac{n(1-\alpha)}{1-\alpha-\delta}\right]^{\frac{\beta}{1-\alpha-\beta-\delta}} \alpha^{\frac{\alpha+\beta}{1-\alpha-\beta-\delta}} H^{\frac{1}{1-\alpha-\beta-\delta}} \beta^{\frac{\beta}{1-\alpha-\beta-\delta}} \delta^{\frac{1-\alpha-\beta}{1-\alpha-\beta-\delta}} (\frac{\alpha}{\gamma})^{\frac{\alpha}{1-\alpha-\beta-\delta}}$$

The first-order conditions for maximization are:

$$\frac{\partial W_s}{\partial s_0} = 0 \Leftrightarrow \frac{\beta B}{1-\alpha-\beta-\delta} - \Gamma\left[1 + \frac{1-\alpha-\delta}{1-\alpha-\beta-\delta}\cdot\frac{s_0}{\theta-s_0}\right] - \frac{\beta\Delta}{1-\alpha-\beta-\delta}\cdot\frac{s_1}{\theta-s_1} = 0$$

$$\frac{\partial W_s}{\partial s_1} = 0 \Leftrightarrow \frac{\delta B}{1-\alpha-\beta-\delta} - \Delta\left[1 + \frac{1-\alpha-\beta}{1-\alpha-\beta-\delta}\cdot\frac{s_1}{\theta-s_1}\right] - \frac{\delta\Gamma}{1-\alpha-\beta-\delta}\cdot\frac{s_0}{\theta-s_0} = 0$$

The solution of these conditions yields the set of optimal subsidies[8]:

[8] It can be shown that the second-order conditions for maximization also hold in this case.

$$s_0 = \frac{\beta(B+\Delta)-(1-\alpha-\beta)\Gamma}{\beta(B+\Gamma+\Delta)} \cdot \theta$$

$$s_1 = \frac{\delta(B+\Gamma)-(1-\alpha-\delta)\Delta}{\delta(B+\Gamma+\Delta)} \cdot \theta$$

Finally, we can substitute this solution into (17), (18), (19) and (20) to get the equilibrium outcome summarized in the following proposition:

<u>Proposition 5</u>. The subgame perfect equilibrium of the game with subsidies is:

$$s_0^* = \frac{\beta(B+\Delta)-(1-\alpha-\beta)\Gamma}{\beta(B+\Gamma+\Delta)} \cdot \theta \quad , \quad s_1^* = \frac{\delta(B+\Gamma)-(1-\alpha-\delta)\Delta}{\delta(B+\Gamma+\Delta)} \cdot \theta$$

$$I_0^S = \alpha^{\frac{\alpha}{1-\alpha-\beta-\delta}} \left[\frac{B+\Gamma+\Delta}{(1-\alpha)\Delta}\right]^{\frac{1-\alpha}{1-\alpha-\beta-\delta}} \cdot H^{\frac{1}{1-\alpha-\beta-\delta}} \beta^{\frac{1-\alpha-\delta}{1-\alpha-\beta-\delta}} \delta^{\frac{1-\alpha+\delta}{1-\alpha-\beta-\delta}} (\frac{1}{\theta})^{\frac{1-\alpha}{1-\alpha-\beta-\delta}} (\frac{\alpha}{\gamma})^{\frac{\alpha}{1-\alpha-\beta-\delta}}$$

$$I_1^S = \alpha^{\frac{\alpha}{1-\alpha-\beta-\delta}} \left[\frac{B+\Gamma+\Delta}{(1-\alpha)\Delta}\right]^{\frac{1-\alpha}{1-\alpha-\beta-\delta}} \cdot H^{\frac{1}{1-\alpha-\beta-\delta}} \beta^{\frac{\beta}{1-\alpha-\beta-\delta}} \delta^{\frac{2(1-\alpha)-\beta}{1-\alpha-\beta-\delta}} (\frac{1}{\theta})^{\frac{1-\alpha}{1-\alpha-\beta-\delta}} (\frac{\alpha}{\gamma})^{\frac{\alpha}{1-\alpha-\beta-\delta}}$$

$$e^s = \alpha^{\frac{1-\beta-\delta}{1-\alpha-\beta-\delta}} \left[\frac{B+\Gamma+\Delta}{(1-\alpha)\Delta}\right]^{\frac{\beta+\delta}{1-\alpha-\beta-\delta}} \cdot H^{\frac{1}{1-\alpha-\beta-\delta}} \beta^{\frac{\beta}{1-\alpha-\beta-\delta}} \delta^{\frac{\beta+2\delta}{1-\alpha-\beta-\delta}} (\frac{1}{\theta})^{\frac{\beta+\delta}{1-\alpha-\beta-\delta}} (\frac{\alpha}{\gamma})^{\frac{1-\beta-\delta}{1-\alpha-\beta-\delta}}$$

$$h^s = \alpha^{\frac{(1-\beta-\delta)(1-\alpha)}{1-\alpha-\beta-\delta}} \left[\frac{B+\Gamma+\Delta}{(1-\alpha)\Delta}\right]^{\frac{(\beta+\delta)(1-\alpha)}{1-\alpha-\beta-\delta}} \cdot H^{\frac{1-\alpha}{1-\alpha-\beta-\delta}} \beta^{\frac{\beta(1-\alpha)}{1-\alpha-\beta-\delta}} \delta^{\frac{(\beta+2\delta)(1-\alpha)}{1-\alpha-\beta-\delta}} (\frac{1}{\theta})^{\frac{(\beta+\delta)(1-\alpha)}{1-\alpha-\beta-\delta}} (\frac{\alpha}{\gamma})^{\frac{\alpha(\beta+\delta)}{1-\alpha-\beta-\delta}}$$

We can now study the implications of this equilibrium. First, we compare the new equilibrium levels of investment and effort with the first-best outcome given in (2) to get:

$$I_0^S < I_0^{FB} \Leftrightarrow \frac{n(1-\alpha)\Delta}{\delta(B+\Gamma+\Delta)} > \alpha^{\frac{\alpha}{1-\alpha}}$$

$$I_1^S < I_1^{FB} \Leftrightarrow \frac{n(1-\alpha)\Delta}{\delta(B+\Gamma+\Delta)} > \alpha^{\frac{\alpha}{1-\alpha}}$$

$$e^S < e^{FB} \Leftrightarrow \frac{n(1-\alpha)\Delta}{\delta(B+\Gamma+\Delta)} > \alpha^{\frac{1-\beta-\delta}{\beta+\delta}}$$

It is easy to verify that these inequalities hold for all values of parameters and state the following conclusion:

<u>Lemma 1</u>. The new equilibrium outcome with subsidies also involves underinvestment by the worker and the employer $(I_0^S < I_0^{FB}, I_1^S < I_1^{FB})$ as well as underprovision of effort $(e^S < e^{FB})$ relative to the first-best allocation.

Second, we can compare the worker's with the employer's new level of investment to get the next result.

Lemma 2. **(i)** In the new equilibrium with subsidies, the worker chooses a higher (lower) level of investment than the employer if and only if the elasticity of output with respect to I_0 is higher (lower) than the elasticity of output with respect to I_1:

$$I_0^S > (<) I_1^S \Leftrightarrow \beta > (<) \delta$$

(ii) The worker's relative investment intensity in equilibrium with subsidies is equal to the worker's first-best relative investment intensity:

$$r^S = \frac{I_0^S}{I_1^S} = \frac{\beta}{\delta} = \frac{I_0^{FB}}{I_1^{FB}} = r^{FB}$$

Finally, we can compare the levels of subsidy received by the worker and the employer to derive the last result of this section.

Lemma 3. In the new equilibrium with subsidies, we have:

$$s_0^* > (<) s_1^* \Leftrightarrow r^* = I_0^* / I_1^* < (>) \beta / \delta$$

In other words, the worker receives a higher (lower) of subsidy than the employer if and only if the number of employers is small (large) enough:

$$s_0^* > (<) s_1^* \Leftrightarrow n < (>) \frac{1-\alpha-\delta}{\alpha(1-\alpha)}$$

It is easy to see the intuition behind these findings. As already found in Proposition 4, if there is only a small number of firms in the market then the worker's investment intensity (relative to the employer) is low in the original equilibrium allocation. In this case, the government optimally chooses to give a relatively higher subsidy to the worker in order to strengthen her investment incentives in the new equilibrium.

7. Conclusions

In this paper, we have studied optimal contracts and incentives to invest in general human capital under common agency. In the original subgame perfect equilibrium, there is underprovision of effort due to the standard tradeoff between limited liability and efficiency, which is typically present in models with moral hazard. It has been shown that both the employer and the worker choose an inefficiently low level of investment. The employer's underinvestment results from his failure to internalize the positive impact of his investment on other firms' productivity as well as from the fact that he has to give a share of output to the worker in order to induce a higher level of effort. On the other hand, the worker also has too weak investment incentives in absence of perfect competition in the labor market. The worker anticipates that she will not be the full residual claimant of productivity benefits and chooses a suboptimally low level of investment in the first place. The worker's relative investment intensity is increasing in the number of firms as well as in output elasticity with respect to worker's effort and investment but can be either increasing or decreasing in output elasticity with respect to employer's investment. Furthermore, a benevolent government will choose a set of investment subsidies such that the worker's relative investment intensity in the new equilibrium equals the first-best relative investment intensity. If the number of firms in the marker is small (large) enough, then the worker's investment level is relatively low (high) in the original equilibrium and the government must give a higher (lower) subsidy to the worker than to the employer in order to stimulate more the former's (latter's) investment incentives.

A natural extension of the model developed here might consider the presence of many workers and/or study the case where the number of firms is endogenously determined in the market. In another possible extension, the additional assumption that the worker's ability (as well as her effort) is not observable by employers would give a more complete picture of inefficiencies associated with informational asymmetries. These extensions are left for future research.

Appendix

Case 1. For $I_1 \leq (\frac{\gamma}{a^2 H})^{\frac{1}{\delta}} I_0^{-\frac{\beta}{\delta}}$, P_1's maximization problem at stage 2 becomes:

$$\max_{\{I_1\}} EU_{P1} = (HI_0^{\beta} I_1^{\delta} - h)e^{\alpha} - \theta I_1 = (1-a)H(\frac{a^2 H}{\gamma})^{\frac{\alpha}{1-\alpha}} \cdot I_0^{\frac{\beta}{1-\alpha}} I_1^{\frac{\delta}{1-\alpha}} - \theta I_1$$

s.t. $0 \leq I_1 \leq (\frac{\gamma}{a^2 H})^{\frac{1}{\delta}} I_0^{-\frac{\beta}{\delta}}$

The objective function is concave with respect to I_1 (for $\alpha+\beta+\delta<1$). We write the Lagrangian and the first-order conditions for maximization:

$$L = (1-a)H(\frac{a^2 H}{\gamma})^{\frac{\alpha}{1-\alpha}} \cdot I_0^{\frac{\beta}{1-\alpha}} I_1^{\frac{\delta}{1-\alpha}} - \theta I_1 + \lambda \left[(\frac{\gamma}{a^2 H})^{\frac{1}{\delta}} I_0^{-\frac{\beta}{\delta}} - I_1 \right]$$

$$\partial L / \partial I_1 = \delta H(\frac{a^2 H}{\gamma})^{\frac{\alpha}{1-\alpha}} I_0^{\frac{\beta}{1-\alpha}} I_1^{\frac{\alpha+\delta-1}{1-\alpha}} - \theta - \lambda \leq 0, \ (\partial L / \partial I_1) I_1 = 0$$

$$\partial L / \partial \lambda = (\frac{\gamma}{a^2 H})^{\frac{1}{\delta}} I_0^{-\frac{\beta}{\delta}} - I_1 \geq 0, \ (\partial L / \partial \lambda)\lambda = 0$$

The solution is:

$$I_1 = \begin{cases} (\frac{\delta H}{\theta})^{\frac{1-\alpha}{1-\alpha-\delta}} (\frac{a^2 H}{\gamma})^{\frac{\alpha}{1-\alpha-\delta}} I_0^{\frac{\beta}{1-\alpha-\delta}} & \text{, if } I_0 \le (\frac{\gamma}{a^2 H})^{\frac{1-\delta}{\beta}} (\frac{\theta}{\delta H})^{\frac{\beta}{\delta}} \quad \textbf{(Ia)} \\ (\frac{\gamma}{a^2 H})^{\frac{1}{\delta}} I_0^{-\frac{\beta}{\delta}} & \text{, if } I_0 \ge (\frac{\gamma}{a^2 H})^{\frac{1-\delta}{\beta}} (\frac{\theta}{\delta H})^{\frac{\beta}{\delta}} \quad \textbf{(Ib)} \end{cases}$$

Case 2. For $I_1 \ge (\frac{\gamma}{a^2 H})^{\frac{1}{\delta}} I_0^{-\frac{\beta}{\delta}}$, P_1's maximization problem becomes:

$$\max_{\{I_1\}} EU_{P1} = HI_0^{\beta} I_1^{\delta} - \frac{\gamma}{\alpha} - \theta I_1$$

s.t. $I_1 \ge (\frac{\gamma}{a^2 H})^{\frac{1}{\delta}} I_0^{-\frac{\beta}{\delta}}$

The Lagrangian and the first-order conditions for maximization are:

$$L = HI_0^{\beta} I_1^{\delta} - \frac{\gamma}{\alpha} - \theta I_1 + \lambda \left[I_1 - (\frac{\gamma}{a^2 H})^{\frac{1}{\delta}} I_0^{-\frac{\beta}{\delta}} \right]$$

$$\partial L / \partial I_1 = \delta H I_0^{\beta} I_1^{\delta-1} - \theta + \lambda \le 0 \,,\ (\partial L / \partial I_1) I_1 = 0$$

$$\partial L / \partial \lambda = I_1 - (\frac{\gamma}{a^2 H})^{\frac{1}{\delta}} I_0^{-\frac{\beta}{\delta}} \ge 0 \,,\ (\partial L / \partial \lambda) \lambda = 0$$

The solution is:

$$I_1 = \begin{cases} (\frac{\gamma}{a^2 H})^{\frac{1}{\delta}} I_0^{-\frac{\beta}{\delta}} & \text{, if } I_0 \le (\frac{\gamma}{a^2 H})^{\frac{1-\delta}{\beta}} (\frac{\theta}{\delta H})^{\frac{\delta}{\beta}} \quad \textbf{(IIa)} \\ (\frac{\delta H}{\theta})^{\frac{1}{1-\delta}} I_0^{\frac{\beta}{1-\delta}} & \text{, if } I_0 \ge (\frac{\gamma}{a^2 H})^{\frac{1-\delta}{\beta}} (\frac{\theta}{\delta H})^{\frac{\delta}{\beta}} \quad \textbf{(IIb)} \end{cases}$$

The derivation of the stage-2 outcome requires comparing the values of EU_{P1} at solutions (9a), (9b), (10a) and (10b) within the appropriate intervals of I_0.

For $I_0 \le (\frac{\gamma}{a^2 H})^{\frac{1-\delta}{\beta}} (\frac{\theta}{\delta H})^{\frac{\delta}{\beta}}$, we compare the values of EU_{P1} associated with solutions (9a) and (10a) to find: $EU_{P1}\left(I_1 = (\frac{\delta H}{\theta})^{\frac{1-\alpha}{1-\alpha-\delta}} (\frac{a^2 H}{\gamma})^{\frac{\alpha}{1-\alpha-\delta}} I_0^{\frac{\beta}{1-\alpha-\delta}} \right) \ge EU_{P1}\left(I_1 = (\frac{\gamma}{a^2 H})^{\frac{1}{\delta}} I_0^{-\frac{\beta}{\delta}} \right)$

Similarly, for $I_0 \ge (\frac{\gamma}{a^2 H})^{\frac{1-\delta}{\beta}} (\frac{\theta}{\delta H})^{\frac{\delta}{\beta}}$, we compare the values of EU_{P1} associated with solutions (9b) and (10b) to find:

$$EU_{P1}\left(I_1 = (\frac{\delta H}{\theta})^{\frac{1}{1-\delta}} I_0^{\frac{\beta}{1-\delta}} \right) \ge EU_{P1}\left(I_1 = (\frac{\gamma}{a^2 H})^{\frac{1}{\delta}} I_0^{-\frac{\beta}{\delta}} \right)$$

Therefore, the stage-2 outcome is:

$$I_1 = \begin{cases} (\frac{\delta H}{\theta})^{\frac{1-\alpha}{1-\alpha-\delta}} (\frac{a^2 H}{\gamma})^{\frac{\alpha}{1-\alpha-\delta}} I_0^{\frac{\beta}{1-\alpha-\delta}} & \text{, if } I_0 \le (\frac{\gamma}{a^2 H})^{\frac{1-\delta}{\beta}} (\frac{\theta}{\delta H})^{\frac{\delta}{\beta}} \\ (\frac{\delta H}{\theta})^{\frac{1}{1-\delta}} I_0^{\frac{\beta}{1-\delta}} & \text{, if } I_0 \ge (\frac{\gamma}{a^2 H})^{\frac{1-\delta}{\beta}} (\frac{\theta}{\delta H})^{\frac{\delta}{\beta}} \end{cases}$$

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