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Christos Mavridis and Nikolas Tsakas

Department of Economics, University of Cyprus, P.O. Box 20537, 1678 Nicosia, Cyprus
Tel.: +357-22893700, Fax: +357-22895028, Web site: http://www.ucy.ac.cy/econ/en

Social Capital, Communication Channels and Opinion Formation*

Christos Mavridis [†1] and Nikolas Tsakas [‡2]

[1]Department of Economics, Lancaster University

[2]Department of Economics, University of Cyprus

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Abstract

We study how different forms of social capital lead to different distributions of multidimensional opinions by affecting the channels through which individuals communicate. We develop a model to compare and contrast the evolution of opinions between societies whose members communicate through bonding associations and societies where communication is through bridging associations. Both processes converge towards opinion distributions where there are groups within which there is consensus in all issues. Bridging processes converge to distributions that have, on average, fewer opinion groups and lower fractionalisation. We provide additional results that highlight the distinct characteristics of the two processes.

JEL Classification: D71, D83, P16, Z1

Keywords: Social Capital, Opinion Formation, Bounded Confidence, Bonding versus Bridging Associations.

1 Introduction

People's opinions are not fixed. Once an individual has formed an opinion about an issue, she may revisit it in the future, once she gets more information about it. Arguably, one of the most important avenues through which new information flows to an individual is by meeting and discussing with other individuals. Of course, one might not necessarily take

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†Department of Economics, Lancaster University, Bailrigg, Lancaster, LA1 4YX, United Kingdom; E-mail: c.mavridis@lancaster.ac.uk

‡Department of Economics, University of Cyprus, P.O. Box 20537, 1678, Nicosia, Cyprus; E-mail: tsakas.nikolaos@ucy.ac.cy

another's opinion into account when updating her own, especially if this opinion is very different than hers. Oftentimes though, when two individuals who hold different opinions discuss, they see each others' points; this way their opinions on the subject they discuss become more similar.

Individuals have plenty of opportunities to meet and interact with others. They interact with their families, friends, colleagues, but they also meet and interact with others through their participation in various clubs and associations. When meeting with her family and friends, an individual expects to talk with people who have opinions and preferences quite close to her own. On the other hand, when she meets other individuals at the gym they may have opinions quite different compared to hers. Obviously, when talking to people she met at the gym, their discussion does not have to be about working out; they may talk about politics or any other issue. However most of the interactions involve individuals who are "similar" in some way. In the case of the gym for example, the individuals will be similar in their interest to work out; whereas in the case of the family individuals will probably be similar to each other in more than one ways. Therefore, there seems to be an apparent connection between the process of opinion formation and the channels through which individuals communicate in order to form their opinions.

In this paper, we present a theoretical framework that incorporates the existence of communication channels with different features into a standard model of multidimensional opinion formation with bounded confidence (Deffuant et al., 2000; Lorenz, 2005). In particular, we consider as the underlying principle of these differences to be the fact that individuals' opportunities to meet and discuss depend to a great extent on the *social capital* of the community they live in (Putnam, 1995), in a sense that will become clear in the following paragraphs.

According to Putnam (1995), social capital refers to *"features of social organization such as networks, norms, and social trust that facilitate coordination and cooperation for mutual benefit"*.[1] In this sense the stock of social capital is greater where individuals have many opportunities to meet, interact and cooperate within a community. This would seem to imply that social capital is associated with good social outcomes. For instance, Knack and Keefer (1997) and Tabellini (2010) find that social capital is associated with stronger economic performance and Guiso et al. (2004) show that social capital brings more trust in a community which improves its level of financial development. Goldin and Katz (1999) show that social capital is also correlated with the spread of secondary schooling. On the other hand, social capital can have negative effects on society. For example, Satyanath et al. (2017) define social capital to be simply a dense network of civic associations and show that the rise of the Nazi Party in Weimar Germany was faster

[1] For other definitions of social capital and for a research survey on the topic see Durlauf and Fafchamps (2005). For an empirical decomposition of the concept see Bjørnskov (2006).

in cities with higher social capital.[2]

In an attempt to understand more deeply the distinct forms of social capital, in his famous work, Putnam (2000) makes the distinction between *"bonding" and "bridging" social capital*. The first type "bonds" similar individuals with each other and the second one "bridges" the gap between different individuals. The point that is made is that only bridging associations are unambiguously "good". There are also two interpretations of "bridging," the internal and the external one. Internal bridging brings together the members of a given association, whereas external bridging brings together members of different associations (see Geys and Murdoch, 2008, for a detailed discussion).[3]

While the importance of social capital on societies and individuals has been extensively studied, it still remains largely unanswered how the different types of social capital give rise to different opinions and levels of fractionalisation within societies. Here, we adopt the view of social capital as a collection of civic associations, meaning that *we distinguish between societies in which citizens interact predominantly through either bonding and bridging associations*. With respect to bridging associations we adopt the internal bridging interpretation. Our aim is to *identify the effect that interactions through bonding or bridging associations respectively have on the distribution of opinions within a society.*

Opinion formation through repeated communication has been the subject of extensive research. A large branch of the literature has focused on conditions that allow individuals to reach consensus (or learn some "correct" action) via communication, considering both Bayesian and non–Bayesian updating processes (see Bala and Goyal, 1998; Gale and Kariv, 2003; Banerjee and Fudenberg, 2004; Acemoglu and Ozdaglar, 2011; DeMarzo et al., 2003; Golub and Jackson, 2010; Mueller-Frank, 2013, 2015). However, by construction, these papers are not adequate to explain persisting disagreement, which is another issue that has puzzled researchers.

For instance, Axelrod (1997) posed the question why even though people tend to become more alike when they interact, the differences across them do not eventually disappear. The author uses a simple model where "similarity leads to interaction, and interaction leads to still more similarity" and where local convergence can lead to global polarisation. Even though individuals are becoming more similar with every interaction, the fact that similarity itself leads to the interaction, makes individuals who are quite different very unlikely to interact, thus making consensus unattainable. This observation pertains the key idea of homophily as a possible explanation for disagreement, which will

[2]Social capital also seems to affect citizens' preferences for redistribution. For instance, Yamamura (2012) and Bergh and Bjørnskov (2014) show that social capital has positive effects on income equality.

[3]In a later paper, Geys and Murdoch (2010) provide an interesting discussion of how the bridging and bonding nature of networks can be measured. Following the discussion on social capital and redistribution, using data from Russia, Borisova et al. (2015) show that it is in fact the bridging social capital that has positive effects on redistribution.

be implicitly considered here as well. Other explanations contain anchoring to initial opinions (Friedkin and Johnsen, 1990), biased assimilation (Dandekar et al., 2013) and opinion fluctuations (Acemoglu et al., 2013).

The theoretical framework that is closer to our analysis is based on the model of average–based updating, introduced by DeGroot (1974), with the addition of "bounded confidence". More specifically, the seminal papers by Deffuant et al. (2000), Krause (2000) and Hegselmann and Krause (2002) introduce the concept of "bounded confidence", which suggests that citizens whose opinions are too far away from each other do not take each other's opinions into account. This introduces implicitly a notion of homophily, not necessarily on interactions per se, but on interactions that might be successful in making an individual revise her opinion. Bounded confidence makes it possible for more than one opinions to survive in the long run and for this reason it has attracted a lot of academic interest, as in many circumstances it can be seen as a more realistic outcome. In the following paragraph we present a list of related papers, which is far from exhaustive.[4]

It is apparent that the tighter the bounds of confidence in a society, the larger the number of opinions surviving in the long run is expected to be (see Ben-Naim et al., 2003; Blondel et al., 2007; Lorenz, 2007, for a thorough discussion). Moreover, Lorenz (2005) provides a proof that formalises the shape of opinion distributions in the long run, which will be helpful for our analysis. Furthermore, Fortunato et al. (2005) and Lorenz (2003, 2008) extend the original models to multidimensional opinions and find significant similarities with the one–dimension models.[5] Finally, Kurahashi-Nakamura et al. (2016) allow individuals to occassionally interact with others who hold distant opinions, which has a similar flavor to the communication through bridging associations in our case.

Overall, the literature has focused almost exclusively on the the evolution of the opinion formation processes, without emphasizing on the underlying principles that might generate one process or another, with different forms of social capital being an example of it. Therefore, in that sense, this paper can be seen as *a revisit to the problems of opinion formation, fractionalisation and consensus–seeking from a social capital perspective.*

In order to do that, we formulate two dynamic processes of opinion formation with bounded confidence, based on Deffuant et al. (2000), that capture the features of communication through bonding and bridging associations respectively. In each of the two processes, citizens hold opinions over two issues, which they revise after meeting and discussing with other citizens of the society. The probability of discussing and subsequently agreeing with a fellow citizen depends on the exact process in the following way: A so-

[4]For a survey on opinion dynamics and bounded confidence see Lorenz (2007) and for some empirical evidence see Lorenz (2017). Moreover, a stream of the literature on average–based updating looks at the shape of persisting disagreement without considering bounded confidence (see DeMarzo et al., 2003; Louis et al., 2017).

[5]Almost all results in these papers are obtained through simulations.

ciety in which citizens interact predominantly through bonding associations is modeled by a process in which two citizens may discuss and agree only as long as their current opinions are sufficiently close *in both issues*. On the contrary, a society in which citizens interact predominantly through bridging associations is modeled by a process in which two citizens may discuss and agree as long as their current opinions are sufficiently close *in at least one of the two issues* (and irrespectively of how far they might be in the other one). For each of these processes we look at the long run distributions of opinions and the conditions that lead opinions to stabilise.

Our aim is twofold: First, we want to understand the shapes that stable opinion distributions are expected to have and the features of each process that lead towards them (or prevent the society from reaching them). Second, we want to examine the differences in opinion distributions that may result from the evolution of the two processes. Of particular interest for our analysis is whether the two processes lead to different levels of fractionalisation.

We find that both processes lead societies to an "island"-type distribution of opinions, where groups of citizens reach consensus between them in both issues. This is a recurrent result in this type of processes (see Deffuant et al., 2000; Lorenz, 2005) and in particular the analysis of Lorenz (2005) has been very important for establishing our main Theorem. Despite the similarities in the shape of the final distributions, we present additional results that show that the two processes have distinct characteristics that determine how these groups of citizens who eventually reach consensus are formed. Furthermore, and in line with the intuition of the theoretical results, we show through simulations that interactions through bridging associations lead on average to fewer opinions that survive in the long run and therefore to lower opinion fractionalisation. Finally, we briefly discuss the speed at which the two processes evolve and we find that the process associated with bonding associations stabilises faster than the one associated with bridging associations.

The rest of the paper is structured as follows: Section provides some empirical motivation for the model. Section 3 describes the model, Sections 4 and 5 discuss the theoretical and numerical results respectively. Section 6 concludes. All proofs are relegated to Appendix A and some additional material can be found in the Online Appendix.

2 Empirical Motivation

Using the last wave of the European Values Study,[6] we can compare the attitudes of citizens of various European countries, given their participation or not in different kinds of associations.

[6]EVS (2011): European Values Study 2008: Integrated Dataset (EVS 2008). GESIS Data Archive, Cologne. ZA4800 Data file Version 3.0.0, doi:10.4232/1.11004

The dataset contains a very rich set of variables spanning the whole spectrum of social life. The data has been collected around the whole of Europe and the place of residence of the participants is recorded at the regional level. This allows us to construct aggregated data points for each region, without restricting the size of our data too much, as it would be the case if location was measured at the national level.

For each country, we observe whether a participant has indicated to be an active member of a number of different associations, as well as whether a participant has declared spontaneously to be a member of no association at all.[7] We aggregate these observations at the regional level and obtain the share of the population that participates in each type of association at each region. The results for each association are presented separately and are not grouped ex–ante to bridging and bonding associations. Nevertheless, despite the fact that there is no commonly accepted way of characterizing particular associations, some types of organisations are often associated with bridging social capital, such as sports, arts, religious and youth associations, whereas others are more often associated with bonding social capital, such as political parties and other politically involved associations, as well as activist organisations. For further discussion of classification of associations see for instance Satyanath et al. (2017); Geys and Murdoch (2008).

In addition to participation in associations we observe the self–declared position of the participants on the left–right political spectrum, which is measured on a scale from 1 (extreme left) to 10 (extreme right). Using this measure we are able to construct a fractionalisation index for the given region. We use the standard version of the Ethnolinguistic Fractionalisation Index $FI = 1 - \sum_{i=1}^{m} s_i^2$.

The aim of this empirical exercise is to check whether the level of participation at a given type of association has any effect on the observed level of fractionalisation in the given region. Note that, the fractionalisation index can be affected by the size of the population used for its calculation, as larger populations are expected to be more fractionalised than smaller ones just because of the fact that their calculation is based on a higher number of opinions. For this reason, the size of the sample for each region is taken into account in our regressions and indeed seems to have a positive and significant effect. In addition to this, we drop regions with fewer than twenty observations, as the results in such cases are affected very much by individual observations, thus being not representative of the entire population.[8]

[7] We focus on the variable that indicates whether or not someone has provided voluntary work for the given organisation, rather than just having participated, as this provides a stronger indication on the extent of involvement in the organisation.

[8] The choice of twenty observations as a limit is obviously ad–hoc, as there is not standard way of making this choice. The idea is that on the one hand a small number of observations induces a lot of noise in the FI, on the other hand setting a high threshold will lead us to drop too many observations, thus altering the nature of the total sample. We have also thought about pooling together different regions with few observations based on their kilometric distance, but this approach would not be less

We regress the calculated fractionalisation indices on the participation level for each of the associations, including as regressors the sample population, the average age, education and wage levels and the share of each gender. All data are aggregated at the regional level. Table 1 presents the coefficients for each of the association in the relevant regression, together with their significance level. The complete table of regression results can be found in the Online Appendix.

Religious	0.122**	Women	0.233*
	(0.0446)		(0.101)
Sports	0.132*	Environment	0.273***
	(0.0518)		(0.0808)
Arts	0.186**	Health	0.295***
	(0.0580)		(0.0722)
Professional	0.194	Political Party	0.300***
	(0.101)		(0.0826)
Trade Union	0.206*	Community	0.325***
	(0.0889)		(0.0870)
Welfare	0.213***	Human Rights	0.397***
	(0.0567)		(0.0895)
Youth	0.221**	Peace	0.404***
	(0.0756)		(0.116)
		None	-0.0749**
			(0.0235)
Observations	386		

Sample population ≥ 20
Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 1: OLS results of the effect of participation in various associations on fractionalisation.

Overall, we find the share of participation in almost any type of association to have a significantly positive effect in the fractionalisation of a society and analogously participating in no association has a significantly negative effect in it.[9] Having said that, we observe that associations of more "bridging nature", such as, for example, arts and sports associations, are found to have the lowest impact among all others and on the contrary associations of more "bonding nature", such as peace and human rights associations, are found to have the largest impact among all others.

ad–hoc than the one actually employed.

[9]In a different set of regressions we found the same result to be true if one looks at partisanship levels instead of FI. By partisanship level we mean the average absolute distance from the average opinion of 5.5. This result could be connected with the findings of Satyanath et al. (2017) where the authors find participation in associations to be linked with increased entry level in the Nazi party before WWII.

The result does not impose any causal effect, but indicates a different relation between different associations and the observed fractionalisation in a society. Our subsequent theoretical analysis provides a framework that could potentially explain the observed differences based on the process of opinion formation that is induced by the participation in different types of associations. Note that our analysis intends to explain the observed differences across associations and not why participation per se is associated with higher fractionalisation.

3 Model

We examine a dynamic model of multidimensional opinion formation in a population of n citizens, denoted by $N = \{1, \ldots, n\}$ with typical elements i, j. Citizens are considered to hold opinions over two issues x and y which can be represented as points on the Euclidean space $[0, 1] \times [0, 1]$. Citizens may meet via two different processes that are described later and upon communication they may adjust their opinions.

Initially, each citizen holds a pair of opinions (x_i^0, y_i^0). Initial opinions are summarised by the vectors $\mathbf{x}(0)$ and $\mathbf{y}(0)$ respectively. Starting from period $t = 1$ onwards, two citizens i, j are randomly selected to communicate. They discuss one of the two issues and if their opinion profiles are sufficiently "close" to each other then they might (but not necessarily) agree and come closer in the issue they discussed. Formally, if two citizens meet, discuss issue x on which they hold opinions x_i^t, x_j^t (with $x_i^t < x_j^t$) and agree, then their opinions become

$$x_i^{t+1} = x_i^t + \mu(x_j^t - x_i^t) \quad \text{and} \quad x_j^{t+1} = x_j^t - \mu(x_j^t - x_i^t)$$

respectively. The parameter $\mu \in (0, 1/2]$ denotes the extent to which opinions come closer to one another upon agreement. On the other hand, if their opinion profiles are very "far" from each other then the two citizens never agree, therefore no adjustment is made if they meet.[10] This is a model of *bounded confidence*, where citizens are willing to exchange opinions only with those that there is some common ground for discussion. The probabilities of discussing each of the issues, as well as the probability of agreeing upon discussing depend on the communication process and are described below.

The two processes that we consider determine the actual meaning of being "close" or not and they reflect the underlying effect of participation in certain associations to the probabilities of meeting and agreeing with citizens given their opinions. We differentiate between two main types of associations, namely *bonding associations* versus *bridging associations* and analyze the respective processes in which citizens interact via one of

[10]Alternatively, one could think that two citizens with very distant opinions do not meet at all. The qualitative results of this alternative mechanism are identical and the only feature that is affected is the speed of convergence and some increased consistency of extreme opinions.

these two types of associations.

Bonding process: Bonding associations are associations whose members tend to be quite similar to each other in all respects. Therefore, a citizen participating in a bonding association expects to meet and discuss with citizens that are not too far away from her own beliefs. We formalise this by constructing a square neighborhood around a citizen that contains the available citizens in the bonding association. This means that citizens i, j with opinion profiles (x_i^t, y_i^t) and (x_j^t, y_j^t) respectively, may only agree as long as:

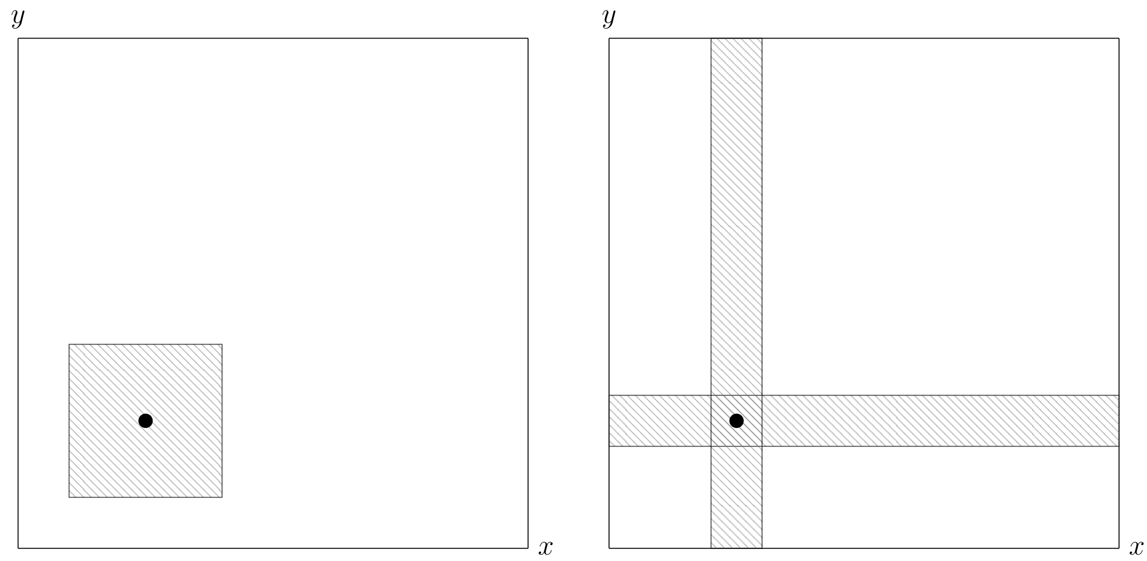
$$\textbf{\textit{Confidence bounds in a bonding process:}} \quad |x_i^t - x_j^t| \leq d \quad \textbf{and} \quad |y_i^t - y_j^t| \leq d$$

where the parameter $d \in (0, 1]$ determines the confidence bounds. Obviously, the higher the value of d the larger the set of citizens who may successfully exchange opinions with each other. A schematic representation of the neighborhood that might lead to opinion adjustment for a citizen i in the bonding case can be seen at the left sub-figure of Figure 1. A citizen i located at the dot faces a positive probability of adjusting her opinion only if citizen j belongs to the shaded area.

Provided that they may agree, the two citizens choose randomly to discuss one of the two issues. We denote by p_x and $p_y = 1 - p_x$ the probabilities with which they discuss each issue. We consider these probabilities to be common for all pairs of agents. Moreover, the probability of agreement is assumed to decay quadratically in the distance of opinions in the issue discussed, i.e. citizens i, j, with opinion profiles (x_i^t, y_i^t) and (x_j^t, y_j^t) respectively, who discuss issue x, agree with probability $1 - (x_i^t - x_j^t)^2$ if $|x_i^t - x_j^t| \leq d$ **and** $|y_i^t - y_j^t| \leq d$ and 0 otherwise.

Bridging process: Bridging associations are associations whose members share a common interest, therefore they tend to agree ex–ante only on the particular issue of interest to the association. Therefore, a citizen participating in a bridging association expects to meet and discuss with citizens with whom she shares a very close opinion in the issue related to the association, but whose opinions might be very different on the other issue. For example, if x would represent sports associations in general, then if the citizen likes football, then she will participate in a football association where she expects to meet other football enthusiasts. However, liking football is not necessarily a predictor for a citizen's stance on issue y. Football lovers come from all walks of life; therefore if issue y represented "politics" then we can reasonably think that football fans attending football associations span the entire political spectrum, from left to right.[11] As such, when interacting with other citizens in the football association, a citizen may end up discussing with someone with vastly different political opinions than her. This is exactly the reason

[11]There might be cases where these two opinions might be ex–ante correlated, but this would not add something to our model.



(a) Bonding process **(b)** Bridging Process

Figure 1: Neighboring areas given the type of process.

why these types of associations are called "bridging".

To formalize this we construct a "cross"–shaped neighborhood around a citizen which will contain the available citizens in the two types of bridging associations. This means that citizens i, j with opinion profiles (x_i^t, y_i^t) and (x_j^t, y_j^t) may agree as long as:

Confidence bounds in a bridging process: $|x_i^t - x_j^t| \leq k$ **or** $|y_i^t - y_j^t| \leq k$

In general, it makes sense to assume that $k < d$, since otherwise the neighborhood of the first process would be included in that of the second one.[12] For this dynamic, the neighborhood of possibly fruitful communication can be seen at the right sub-figure of Figure 1. Hence, in this case an adjustment is possible if the two discussants are sufficiently close in one of the two issues, irrespectively of their disagreement on the other issue.

Defining the probabilities of meeting is slightly trickier for the case of bridging associations, since one needs to define both the probabilities of attending an association and those of discussing the issue related to this association or not. We consider that the two selected citizens always choose to attend the same association and denote by p_x and $p_y = 1 - p_x$ the probabilities with which they both attend association x and y respectively. Subsequently, we denote by q_{xx} (respectively q_{yy}) the conditional probability that they discuss issue x (resp. y) given that they attend association x (resp. y) and $q_{yx} = 1 - q_{xx}$

[12]Later in the paper, we will impose the normalisation $k = \frac{1}{2} - \sqrt{\frac{1}{4} - d^2}$ which ensures that the neighborhoods of communication cover the same area in each case. Observing at Figure 1 the area in which a citizen may find others to agree is $4d^2$ in the bonding case and $2 \cdot 2k - 4k^2$ in the bridging case. Equating the two quantities yields the normalised value of the parameter k.

(resp. $q_{xy} = 1 - q_{yy}$) the remaining probability that they discuss y (resp. x) given that they have attended x (resp. y). It is natural to assume $q_{xx} > 1 - q_{yx}$, although this does not affect our calculations. The probabilities of agreement have again a quadratic decay in the distance of opinions in the discussed issue, i.e. citizens i, j, with opinion profiles (x_i^t, y_i^t) and (x_j^t, y_j^t) respectively, discuss issue x, then the probability that they agree is $1 - (x_i^t - x_j^t)^2$ if $|x_i^t - x_j^t| \leq k$ or $|y_i^t - y_j^t| \leq k$ and 0 otherwise.

Definition 1. The *neighborhood* of a citizen i at period t consists of all the citizens from the population with whom i faces a positive probability of agreeing upon meeting. Formally:

- Bonding: $\mathcal{N}_i^t = \{j \in N \setminus \{i\} : |x_i^t - x_j^t| \leq d \text{ and } |y_i^t - y_j^t| \leq d\}$
- Bridging: $\mathcal{N}_i^t = \{j \in N \setminus \{i\} : |x_i^t - x_j^t| \leq k \text{ or } |y_i^t - y_j^t| \leq k\}$

The nature of both processes imposes an undirected relationship between citizens, i.e. $j \in \mathcal{N}_i^t \Leftrightarrow i \in \mathcal{N}_j^t$. With some abuse of notation, we state an additional definition:

Definition 2. The *neighboring area of a point* (x, y) consists of all points $(\hat{x}, \hat{y})$ such that two citizens with opinion profiles (x, y) and $(\hat{x}, \hat{y})$ face a positive probability of agreeing upon meeting. Formally:

- Bonding: $\mathcal{N}_{(x,y)} = \{(\hat{x}, \hat{y}) \in [0,1]^2 : |x - \hat{x}| \leq d \text{ and } |y - \hat{y}| \leq d\}$
- Bridging: $\mathcal{N}_{(x,y)} = \{(\hat{x}, \hat{y}) \in [0,1]^2 : |x - \hat{x}| \leq k \text{ or } |y - \hat{y}| \leq k\}$

In a completely analogous manner,we can define the neighboring area of a set of multiple opinion profiles as follows:

Definition 3. The *neighboring area of set of points* S consists of all points $(\hat{x}, \hat{y})$ such that a citizen with some opinion profile $(x, y) \in S$ and another citizen with opinion profile $(\hat{x}, \hat{y})$ face a positive probability of agreeing upon meeting. Formally:

- Bonding: $\mathcal{N}_S = \{(\hat{x}, \hat{y}) \in [0,1]^2 : |x - \hat{x}| \leq d \text{ and } |y - \hat{y}| \leq d \text{ for some } (x, y) \in S\}$
- Bridging: $\mathcal{N}_S = \{(\hat{x}, \hat{y}) \in [0,1]^2 : |x - \hat{x}| \leq k \text{ or } |y - \hat{y}| \leq k \text{ for some } (x, y) \in S\}$

It should be noted that Definitions 1 and 2 refer to different sets, but share some similarities. For instance, a citizen j belongs to the neighborhood of citizen i at a given period if and only if her opinion profile (x_j^t, y_j^t) belongs to the neighboring area $\mathcal{N}_{\left(x_i^t, y_i^t\right)}$ of i's current opinion profile, i.e. $j \in \mathcal{N}_i^t \Leftrightarrow \left(x_j^t, y_j^t\right) \in \mathcal{N}_{\left(x_i^t, y_i^t\right)}$. Moreover, while the neighboring areas of points are fixed, the neighborhoods of agents may change across periods, unless the process reaches a phase of *neighborhood stabilisation.*

Definition 4. A process reaches *neighborhood stabilisation* at period t if for all $i \in N$ and $t' > t$ holds that $\mathcal{N}_i^t = \mathcal{N}_i^{t'}$ with probability 1.

4 Theoretical Results

4.1 Preliminary Results

Before stating the main results, it is helpful to report some initial observations regarding the moments of the distributions of opinions in each issue. In particular, both processes preserve the average value of the initial opinions, since each agreement leads to updates of equal magnitude and opposite direction for the two citizens that are involved. In addition to this, the variance of opinions in each issue is weakly decreasing over time, because upon an agreement the opinions come closer to the fixed average, thus the variance decreases. The same holds for all moments of the opinions' distributions. Formally, let $m_x^k(t)$ be the k-th moment of the vector of opinions $\mathbf{x}(t)$ at time t in either of the processes and $\sigma_x^2(t) = m_x^2(t) - [m_x(t)]^2$ be the respective variance. The average of the initial opinion vector is denoted by $m_x(0)$. The definitions and results are totally analogous for issue y.

Proposition 1. *The following statements hold for both a bonding and a bridging process and for any vector of initial opinions* $\mathbf{x}(0)$*:*

1. $m_x(t) = m_x(0)$ *for all* $t \geq 1$.

2. $m_x^k(t)$ *is weakly decreasing in* t *for all* $k \geq 2$, $t \geq 1$.

3. $\sigma_x^2(t)$ *is weakly decreasing in* t *for all* $t \geq 1$.

4.2 Stabilisation and Distribution of Opinions in the Long–Run

The next results are suggestive of the relationships between opinion profiles in the long–run. They provide the intuition behind both Theorem 1 and the results of the simulations that follow. The first result suggests that neighborhood stabilisation implies some sort of transitivity in neighborhoods, in the sense that neighborhoods of neighboring citizens should be common. The reason why this happens is that the existence of a citizen l who belongs to the neighborhood of j but not to the neighborhood of i will lead with positive probability either j out of i's neighborhood or l inside i's neighborhood. Formally,

Proposition 2. *For both a bonding and a bridging process, neighborhood stabilisation at period* t *implies that for all* $t' > t$ *and* $i, j \in N$ *such that* $j \in \mathcal{N}_i^{t'}$ *it must hold that* $\mathcal{N}_i^{t'} \cup \{i\} = \mathcal{N}_j^{t'} \cup \{j\}$.

The fact that, under neighborhood stabilisation, neighboring citizens will have common neighborhoods is suggestive of the emergence of an "island"–type structure, where citizens will be concentrated within groups that will allow successful communication only between members of the same group. However, the formation of common neighborhoods is not sufficient to ensure neighborhood stabilisation by itself. The reason is that successful communication between two citizens who belong to some group might lead one or both of them inside the neighboring area of some other group. This possibility should be ruled out for neighborhood stabilisation to hold. In what follows, we present sufficient conditions that, together with common neighborhoods, guarantee neighborhood stabilisation. As we will see, the existence of common neighborhoods has stronger implications in favor of neighborhood stabilisation in the bonding process compared to the bridging process.

Regarding this aspect, the crucial difference between bonding and bridging processes has to do with the geometry of the neighboring areas of citizens' locations. In order to make clear this distinction, we need some additional notation. Consider a group of citizens $\mathcal{I} \subseteq N$, and let $S^t_{\mathcal{I}}$ be the set that contains the opinion profiles of all citizens who belong to $\mathcal{I}$, i.e. $S^t_{\mathcal{I}} = \{(x, y) \in \mathbb{R}^2 : (x, y) = (x^t_i, y^t_i) \text{ for some } i \in \mathcal{I}\}$. Given this, let $MBR^t_{\mathcal{I}}$ denote the *minimum bounding rectangle* of $S^t_{\mathcal{I}}$, i.e. formally

$$MBR^t_{\mathcal{I}} = \{(x, y) \in \mathbb{R}^2 : \min_{i\in\mathcal{I}} x^t_i \leq x \leq \max_{i\in\mathcal{I}} x^t_i \text{ and } \min_{i\in\mathcal{I}} y^t_i \leq y \leq \max_{i\in\mathcal{I}} y^t_i\}$$

For Propositions 3 and 4 we use the neighboring area of a minimum bounding rectangle (MBR), which is essentially a bigger rectangle that surrounds the said MBR.

The following lemma helps us build an intuition regarding the geometry of common neighborhoods, which will then help in establishing the result of Proposition 3.

Lemma 1. *Consider a bonding process and a group of citizens $\mathcal{I} \subseteq N$ such that at some period t it holds that $\mathcal{N}^t_i = \mathcal{I} \setminus \{i\}$ for all $i \in \mathcal{I}$. Then:*

1. *$\mathcal{N}^t_i = \mathcal{I} \setminus \{i\}$ for all $i \in \mathcal{I}$ is equivalent to saying that there exists a square area Q of size $d \times d$ such that $(x^t_i, y^t_i) \in Q$ for all $i \in \mathcal{I}$,*

2. *$MBR^t_{\mathcal{I}} \subseteq Q \subseteq \mathcal{N}_{(x^t_i, y^t_i)}$ for all $i \in \mathcal{I}$.*

3. *If citizens $j, l \in \mathcal{I}$ discuss and agree at period t then their revised opinion profiles $\left(x^{t+1}_j, y^{t+1}_j\right)$ and $\left(x^{t+1}_l, y^{t+1}_l\right)$ will still belong to $MBR^t_{\mathcal{I}}$, hence also to Q. Therefore, at period $t + 1$ it holds again that $\mathcal{N}^{t+1}_i = \mathcal{I} \setminus \{i\}$ for all $i \in \mathcal{I}$.*

Part 1 of Lemma 1 follows immediately from the fact that all citizens in the group are neighbors, which means that the maximum distance between any two of them, and along each of the two issues, should be at most equal to d. Part 2 follows from the definitions of $MBR^t_{\mathcal{I}}$ and of the neighboring area. Part 3 is a consequence of the previous two, as the new opinions of the two citizens in either issue will never be either larger than the initial maximum or lower than the initial minimum within the group.

Hence, in this case the geometry of citizens' locations guarantees that the members of such a group will remain in it as long as they interact successfully only between themselves. This is not enough to guarantee neighborhood stabilisation, as we still have to ensure that no other citizen will enter the group's neighboring area.

Proposition 3. *In a bonding process, a population reaches neighborhood stabilisation at period t if the following two conditions hold at that period:*

- *there exist disjoint groups of citizens $\mathcal{I}_1 \bigcup \cdots \bigcup \mathcal{I}_p = N$ such that $\mathcal{N}_i^{t'} = \mathcal{I}_g \setminus \{i\}$ for all $g \in \{1, \ldots, p\}$ and for all $i \in \mathcal{I}_g$.*
- *$MBR^t_{\mathcal{I}_{g'}} \bigcap \mathcal{N}_{MBR^t_{\mathcal{I}_g}} = \emptyset$ for all $g, g' \in \{1, \ldots, p\}$ with $g \neq g'$.*

The first condition is necessary, as a consequence of Lemma 1. The second condition ensures that no citizen can potentially enter the neighboring area of a group of citizens other than the one she currently belongs to. A graphical example in which this might occur is presented in Figure 3. The provided condition is still not necessary, despite being quite strict, as there are opinion profiles within the MBR of a group which can never be reached by any citizen, for any sequence of encounters. Therefore, in some cases, although the MBR of a group intersects with the neighboring area of another group, no citizen may reach an opinion profile within this intersection. Notice that, the condition is stricter than considering the intersection between one group's MBR and the union of neighboring areas of another group's members. The reason is that successful interactions between members of a group may change their neighboring areas, without necessarily shrinking them. This means that in subsequent periods the two sets may intersect. On the contrary, both $MBR^t_{\mathcal{I}_g}$ and $\mathcal{N}_{MBR^t_{\mathcal{I}_g}}$ shrink over time, i.e. $MBR^{t+1}_{\mathcal{I}_g} \subseteq MBR^t_{\mathcal{I}_g}$ and $\mathcal{N}_{MBR^{t+1}_{\mathcal{I}_g}} \subseteq \mathcal{N}_{MBR^t_{\mathcal{I}_g}}$, which is sufficient to guarantee neighborhood stabilisation. A visual representation of a population that satisfies the conditions of Proposition 3 is presented in Figure 2.

For the bridging process, achieving neighborhood stabilisation is tougher. The reason is that it is possible for a citizen to leave a group of neighboring citizens, even if she interacts only with other members of the group. This is a consequence of Remark 1 (that follows), which states that the MBR of a group of citizens need not necessarily be fully included in the neighboring area of each agent of the group. An example where Remark 1 holds is presented in Figure 4.

Remark 1. Consider a bridging process and a group of citizens $\mathcal{I} \subseteq N$ such that, at some period t, $\mathcal{N}_i^t = \mathcal{I} \setminus \{i\}$ for all $i \in \mathcal{I}$. Then, it can be true that $MBR^t_{\mathcal{I}} \nsubseteq \mathcal{N}_{(x_i^t, y_i^t)}$ for some $i \in \mathcal{I}$.

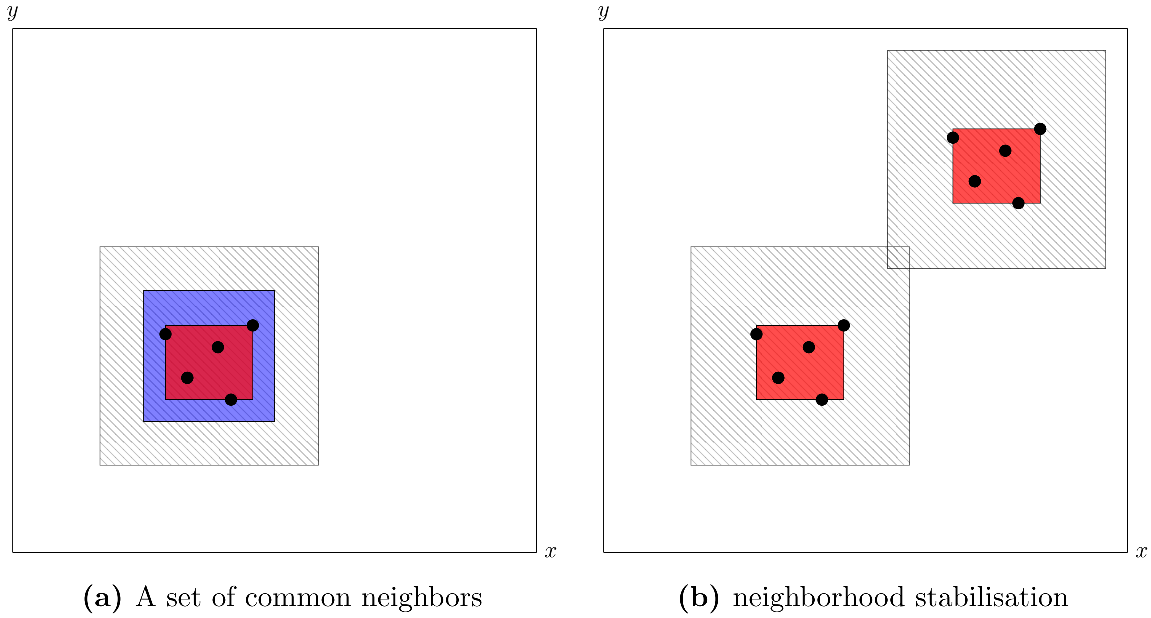


Figure 2: The red rectangles are the groups' MBRs, the blue square is Q and the grey hatched areas are the neighboring areas of each MBR. The neighboring areas may intersect, but they cannot inteset with another group's MBR.

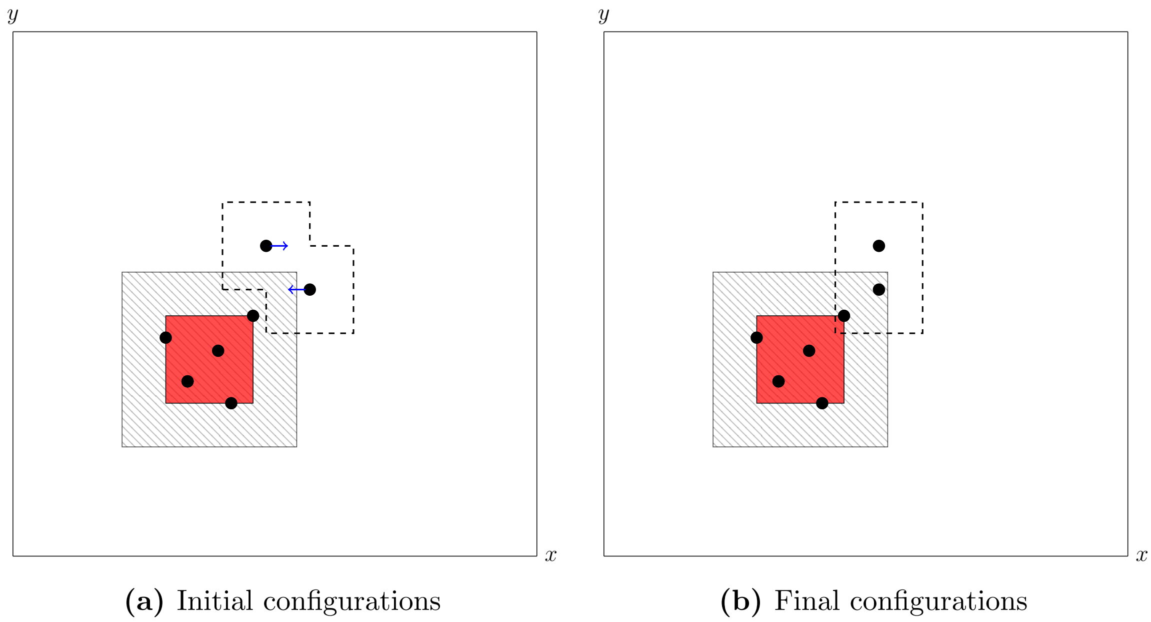


Figure 3: The red rectangle is the left group's MBR and the shaded area is the MBR's neighboring area. The dashed areas are the unions of the neighboring areas of the two remaining citizens. After these two citizens discuss succesfully, the new union of their neighboring areas intersect with the MBR of the left group.

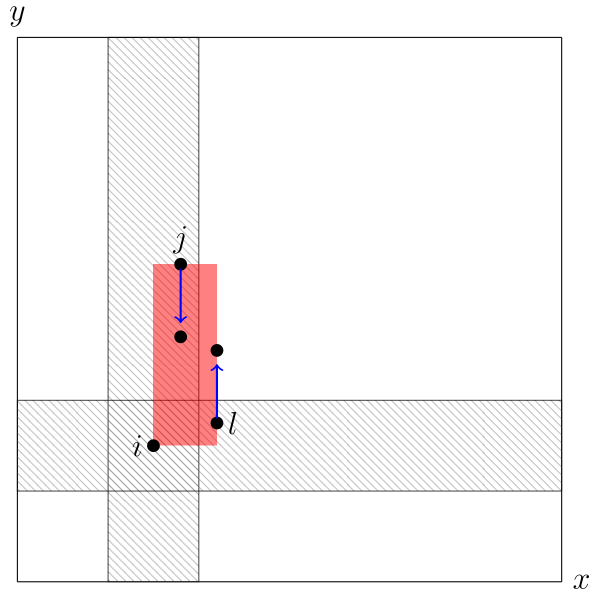


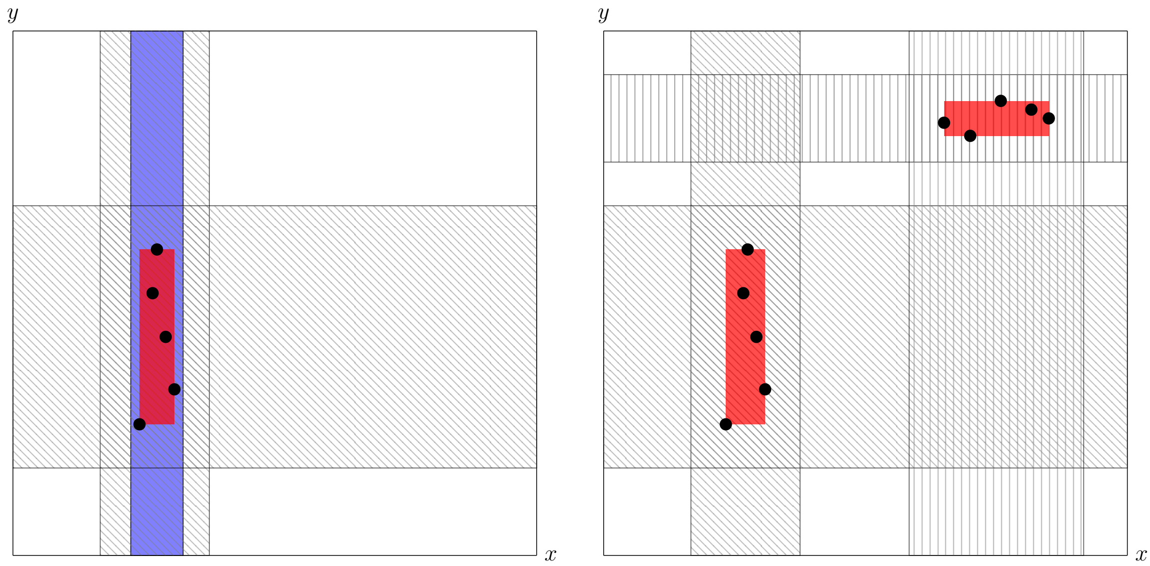
Figure 4: The red rectangle is the group's MBR and the hatched grey area is the neighboring area of citizen i. The blue arrows show the revised opinion profiles of the other two citizens after successfully discussing on issue y. Before the encounter, both citizens were in the neighboring area of i, whereas after the encounter one of them is not anymore.

This result arises as a consequence of the shape of neighboring areas in the bridging process and in particular the fact that the neighboring areas are not convex. Given this, we need some alternative conditions to ensure that citizens will not leave their current neighborhoods, which is a necessary condition for neighborhood stabilisation. Lemma 2 helps in establishing these conditions.

Lemma 2. *Consider a bridging process and a group of citizens $\mathcal{I} \subseteq N$, such that there exists a rectangular area R of size $1 \times k$ or $k \times 1$ such that $(x_i^t, y_i^t) \in R$ for all $i \in \mathcal{I}$ at some period t, then:*

1. $\mathcal{N}_i^t = \mathcal{I} \setminus \{i\}$ *for all* $i \in \mathcal{I}$.
2. $MBR_{\mathcal{I}}^t \subseteq R \subseteq \mathcal{N}_{(x_i^t, y_i^t)}$ *for all* $i \in \mathcal{I}$.
3. *If citizens $j, l \in \mathcal{I}$ discuss and agree at period t then their revised opinion profiles $\left(x_j^{t+1}, y_j^{t+1}\right)$ and $\left(x_l^{t+1}, y_l^{t+1}\right)$ will still belong to $MBR_{\mathcal{I}}^t$, hence also to R. Therefore, at period $t+1$ again holds that $\mathcal{N}_i^{t+1} = \mathcal{I} \setminus \{i\}$ for all $i \in \mathcal{I}$.*

A visual representation of the result is shown in Figure 5(a). Lemma 2 guarantees that citizens remain in the neighborhoods they currently belong to. Neighborhood stabilisation requires also that no other citizens are added to the already formed neighborhoods. The latter condition is equivalent to the last one in the Lemma 1.



(a) A set of common neighbors **(b)** Neighborhood stabilisation

Figure 5: The red rectangle is the set's MBR, the blue rectangle is R_g and the grey hatched area is the union of neighboring areas of each set. The neighboring areas may intersect, but they cannot inteset with other sets' MBR. Moreover, ecah MBR should be included in a rectangle similar to R_g.

Proposition 4. *In a bridging process, a population reaches neighborhood stabilisation at period t if the following two conditions hold at that period:*

- *there exist disjoint groups of citizens $\mathcal{I}_1 \bigcup \cdots \bigcup \mathcal{I}_p = N$ such that $\mathcal{N}_i^t = \mathcal{I}_g \setminus \{i\}$ for all $g \in \{1, \ldots, p\}$ and for all $i \in \mathcal{I}_g$,*
- *for each group $\mathcal{I}_g$, there is a rectangular area R_g of size $k \times 1$ or $1 \times k$ such that $(x_i^t, y_i^t) \in R_g$ for all $i \in \mathcal{I}_g$ and*
- *$MBR^t_{\mathcal{I}_{g'}} \bigcap \mathcal{N}_{MBR^t_{\mathcal{I}_g}} = \emptyset$ for all $g, g' \in \{1, \ldots, p\}$ with $g \neq g'$.*

The intuition behind Proposition 4 is totally analogous to that of Proposition 3. The first two conditions ensure that citizens discuss successfully within disjoint groups, inside which they remain as long as they never face a positive probability of agreeing with any citizen outside of the group. The third condition guarantees that no citizen may reach an opinion profile that will allow her to discuss successfully with a member of a group other than her own. A visual representation of the result for the bridging process is shown in Figure 5(b).

By now we have built some intuition regarding the evolution of the two processes and when we should expect them to stabilise. In the final part of this section, we present the main result on the convergence of the processes. First, we establish that for any finitely large population both processes converge to an "island"–like structure. That is, we observe the emergence of groups of citizens whose opinions converge to consensus in both issues. However, the opinions across groups may differ. This result can be seen as an adaptation of the result of Lorenz (2005) in the current framework. The result provides a clear and intuitive sense of how the long run distributions of opinions in such processes will look like, as well as that the societies indeed move towards these long run outcomes. Moreover, it shows that the geometry of the distribution of opinions does not depend on the actual dynamic, as long as it satisfies certain mild conditions.

In order to state the result formally, we need a formal definition that describes adequately the convergence of opinions in the long–run. One should be very careful with this definition, because exact agreement is never reached for two citizens when $\mu < 1/2$. The following definition helps put things into perspective. In practical terms, it says that a group of citizens reaches consensus almost surely if and only if all its members's opinions in both issues converge over time. Formally:

Definition 5. A group of citizens $\mathcal{I} \subset N$ *reaches consensus almost surely*, if for vectors of initial opinions $\mathbf{x}(0)$ and $\mathbf{y}(0)$ it holds that

$$\left|x_i^t - x_j^t\right| + \left|y_i^t - y_j^t\right| \to 0 \text{ a.s. as } t \to \infty \text{ for all } i, j \in \mathcal{I}$$

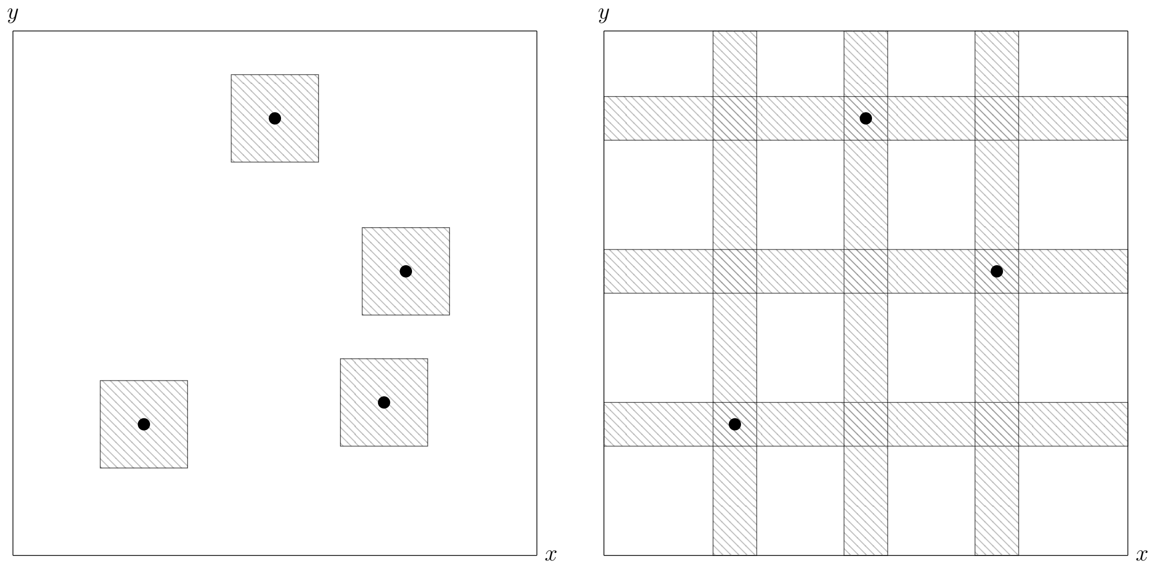
Given this definition, we can state the following result.

Theorem 1. *Let (x_i^t, y_i^t) denote the opinion profile of citizen i at period t in a society where opinions are updated according to either the bonding or the bridging process. Then, there exist pairwise disjoint groups of citizens $\mathcal{I}_1 \bigcup \cdots \bigcup \mathcal{I}_p = N$ such that each group of citizens, $g \in \{1, \ldots, p\}$, reaches consensus almost surely for any vectors of initial opinions $\mathbf{x}(0)$ and $\mathbf{y}(0)$.*

A graphical representation of the result is presented in Figure 6. It becomes apparent that this is a special case of neighborhood stabilisation.

4.3 Speed of Convergence

Apart from the shape of opinion distributions in the long–run, we are also interested in the relative speed at which the two processes converge. In order to do so, we focus on the population density, $f(x, y)$ on each opinion profile (x, y) at each point in time. For small values of d (resp. k) and a sufficiently large population, the density can be approximated by a continuous function. We can then calculate the law that governs the variation of this function at each opinion profile and each elementary time step. This allows us to compare the speed of convergence of the two processes and also verifies the previous finding regarding the "island"–type distribution of opinions in the long–run.



(a) Opinion distribution in a bonding process **(b)** Opinion distribution in a bridging process

Figure 6: The dots represent the consensus opinions of different sets of citizens and the hatched grey areas represent the respective neighboring areas for each set of citizens that has reached cosensus.

Given that this is not the primary focus of this paper, we have relegated the complete mathematical analysis to the Online Appendix and we present here only the laws that govern the variations of the two distribution functions.

In order to understand the evolution of opinions in each case we need to find for each opinion profile (x, y) the expected variation in the population density at this profile at each elementary time step. We do so by considering the density variation at a given opinion profile (x, y) as the sum of two contributions, one positive and one negative.[13]. More specifically, we calculate the probability that a citizen located at a different profile moves towards (x, y) (positive contribution) minus the probability that a citizen located in (x, y) moves towards another profile (negative contribution).

For the *bonding process*, the law that governs the rate of change of the density function over time is the following:

$$I^o(x,y) = \frac{\partial f_o(x,y,t)}{\partial t} \approx \frac{8}{3}(\mu-1)\mu d^4 \left[p_x \frac{\partial^2 (f^2)}{\partial x^2} + (1-p_x)\frac{\partial^2 (f^2)}{\partial y^2} \right]$$

The result provides a multidimensional generalisation of Deffuant et al. (2000) and immediately verifies the prior observation that local higher opinion densities are amplified, leading to "islands" of opinions held by non–trivial parts of the population, surrounded by intermediate opinions held by no one.

[13]This technique has been used extensively in the literature on bounded confidence (see for instance Neau, 2000; Deffuant et al., 2000; Gómez-Serrano and Le Boudec, 2012).

For the *bridging process* we follow the same methodology, constructing the two contributions to the variation. Now, p_x describes the probability that a citizen goes to association related to x and q_{xx} describes the probability that issue x is discussed during participation at association x. We assume that $q_{xx} = q_{yy}$. Overall, the law that governs the rate of change of the density function over time is the following:

$$I^b(x,y) = \frac{\partial f_b(x,y,t)}{\partial t} \approx \frac{16}{15}(1-q_{xx})(\mu-1)\mu k \left[p_x \frac{\partial (f^2)}{\partial y^2} + (1-p_x)\frac{\partial (f^2)}{\partial x^2} \right]$$

One can see that the dynamic is governed by encounters in which the discussed issue is different than the one related to the association the two citizens participated in. This is intuitive since for low values of k even an agreement in the issue of common interest has a very low impact on the distribution, because the opinions are already very similar. On the contrary an agreement in the other issue (even if it is less likely) leads to larger opinion changes with a larger effect on the distribution.

The confidence bound k is considered to be tighter than d, so that the results are not driven by the fact that citizens can interact successfully with a larger number of other citizens. More specifically, we consider $k = \frac{1}{2} - \sqrt{\frac{1}{4} - d^2}$, which ensures that in both processes the neighboring areas of citizens are of equal size. Given this relationship, k is of similar size as $5d^4$, which is a helpful approximation for the subsequent comparison.

In particular, we observe that the dynamic of the bonding process evolves faster than the one of the bridging process as long as the citizens discuss much more often the issue related to the association they participated in, i.e. $q_{xx} \gg 1/2$. This can be seen from the factors that multiply each of the dynamics, which are $\frac{8}{3}(\mu-1)\mu d^4$ and $\frac{16}{15}(1-q_{xx})(\mu-1)\mu k$, with the first one being significantly larger for sufficiently high values of q_{xx}. This means that the former process converges faster than the latter one. This result is also quite intuitive for two reasons. One is that encounters among citizens with very different opinions are expected to happen less often ($1-q_{xx}$ small) and also they are expected to be unsuccessful much more often, since the probability of agreement depended quadratically on the distance between two citizens opinions.

5 Simulations

We have performed an extensive set of simulations to verify the validity of our results and get some quantitative properties of the two processes. Our main goal is to compare the number of disjoint groups of citizens who reach consensus between themselves ("islands") that arise in each process. The analysis is repeated for different values of the confidence bounds' parameters.[14]

To do this we generated 100 two-dimensional opinion profiles (citizens). The opinion profiles are initially drawn uniformly at random and at each period two citizens are randomly chosen to interact. New interactions are drawn until the process stabilises. The analysis is performed independently for each of the two processes. In Table 2 we present descriptive statistics of the number of opinion islands that arise in the long–run for each the two processes. For each process we ran 1000 simulations for different values of the d (k resp.) parameter, keeping the rest of the parameters equal.[15] We used the normalisation $k = \frac{1}{2} - \sqrt{\frac{1}{4} - d^2}$ for the bridging case to make the results directly comparable to each other. For large confidence values both communication channels will trivially lead to consensus, therefore we focus our attention to relatively lower values.

Result 1. Almost always the bridging process results in fewer opinion islands compared to the bonding process.

In fact, even in absolute terms, the bridging process reaches complete consensus (among all citizens) in the vast majority of simulations.

Bonding				Bridging			
d	Mean	Median	Sd	k	Mean	Median	Sd
0.1	27.56	28	2.52	0.01	17.50	17	2.72
0.2	7.40	7	1.25	0.04	1.14	1	0.38
0.3	3.66	4	0.96	0.1	1.01	1	0.13
0.4	2.12	2	0.84	0.2	1	1	0

The values of the other parameters used are as follows: $\mu = 1/3$, $p_x = 1/2$, $q_{yy} = q_{xx} = 0.9$.

Table 2: Simulation results - Number of Islands

A more detailed view of the data is presented in Figure 7, where we plot the empirical cumulative distribution functions of the number of islands that arise in each of the two processes. There we can see that, for each process, if we look at two confidence bounds, the distribution that corresponds to larger confidence is always above the one that corresponds to lower confidence, i.e. it first–order stochastically dominates it. Therefore it is not only the mean or median number of islands that decrease with larger confidence, but the whole distribution of islands shifts to the left.

More importantly, the same holds true if we compare similar confidence bounds (k, d that satisfy $k = \frac{1}{2} - \sqrt{\frac{1}{4} - d^2}$) across the two processes, see Figure 8. In fact, the distribution of the bridging process always first–order stochastically dominates the distribution

[14] For economy of space we do not include the results of the robustness checks we have performed in which we tried slight modifications of the selection process and the functions that determine agreement and meeting probabilities, since they did not lead to any significant change of the results.

[15] See the notes of Table 2 for a complete list of the values of the rest of the parameters.

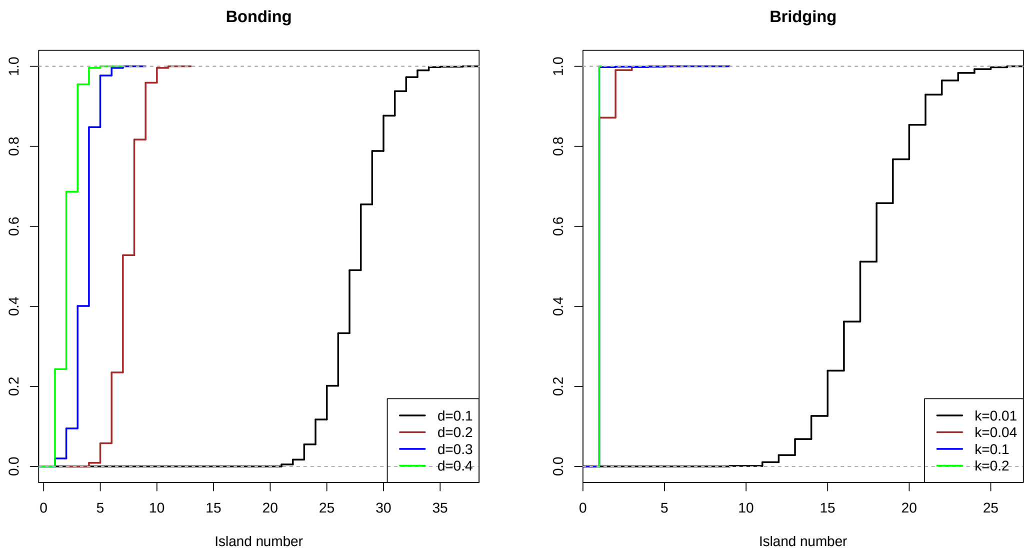


Figure 7: Empirical cumulative distribution functions of the number of islands in the two processes. The horizontal axis contains the number of islands, whereas the vertical axis contains the proportion of simulations in which the number of islands was less or equal than the respective number on the horizontal axis.

of the bonding process. Note that this holds even for the lowest confidence bounds where for the bridging process k is equal to just 0.01.

Table 3 shows the descriptive statistics for the levels of fractionalisation in equilibrium in each case, using the Ethnolinguistic Fractionalisation Index (ELF) and the Greenberg Index (GI).[16] Confirming the results of the previous table, the bridging process produces lower fractionalisation compared to the bonding one.

While we have not included the results on that, we have also observed that the bridging process needed a significantly higher number of rounds to stabilise for each confidence bound compared to the bonding case.

Bonding							Bridging						
d	μ_{ELF}	$\widetilde{ELF}$	σ_{ELF}	μ_{GI}	$\widetilde{GI}$	σ_{GI}	k	μ_{ELF}	$\widetilde{ELF}$	σ_{ELF}	μ_{GI}	$\widetilde{GI}$	σ_{GI}
0.1	0.94	0.94	0.01	0.51	0.51	0.02	0.01	0.85	0.86	0.05	0.20	0.20	0.03
0.2	0.78	0.78	0.04	0.43	0.43	0.03	0.04	0.01	0	0.04	0.00	0	0.01
0.3	0.43	0.51	0.22	0.22	0.26	0.12	0.1	0.00	0	0.00	0.00	0	0.00
0.4	0.03	0.02	0.04	0.02	0.01	0.02	0.2	0	0	0	0	0	0

The values of the other parameters used are as follows: $\mu = 1/3$, $p_x = 1/2$, $q_{yy} = q_{xx} = 0.9$. The values denoted by 0.00 refer to quantities that are positive but lower than 10^{-2}, whereas the values denoted by 0 refer to quantities with value exactly equal to zero.

Table 3: Fractionalisation Indices – μ_x, $\widetilde{x}$ and σ_x correspond to the mean, median and standard deviation of the two fractionalisation indexes respectively.

Overall, the main theoretical predictions and intuitions are indeed verified by the

[16] As a reminder $ELF = 1 - \sum s_i^2$ and $GI = \sum \sum s_i s_j d_{ij}$, where s_i and s_j are shares of distinct opinion islands, and d_{ij} is the euclidean distance between opinion islands i and j.

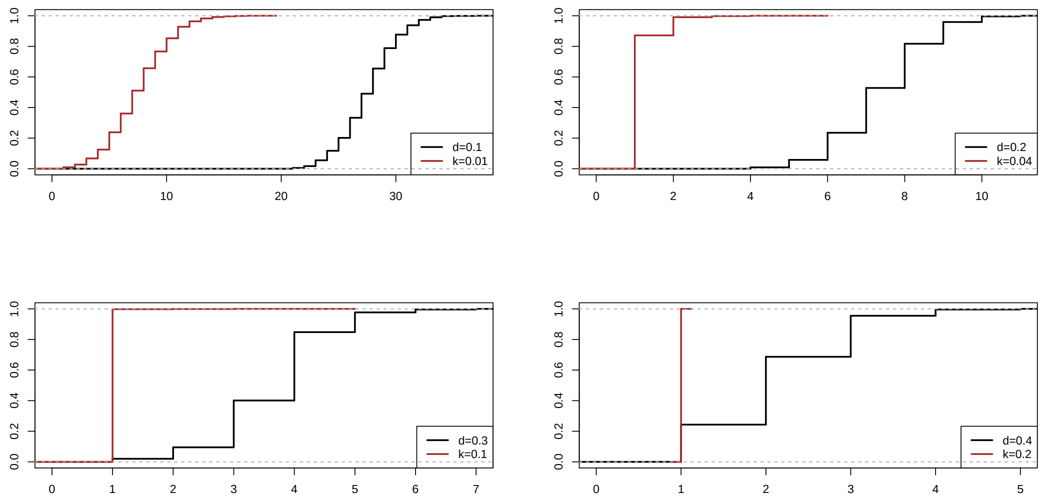


Figure 8: Empirical cumulative distribution functions of the number of islands by confidence parameters d and k. The horizontal axis contains the number of islands, whereas the vertical axis contains the proportion of simulations in which the number of islands was less or equal than the respective number on the horizontal axis. Black for bonding process, brown for bridging.

numerical simulations. Both processes converge towards an "island"–type distribution of opinions, the bridging process yields a lower number of "islands" than the bonding process and it also yields lower levels of fractionalisation. In fact, for most of the chosen parameter values we observe the citizens of bridging processes to reach full consensus.

6 Conclusion

Social capital has been considered as a major factor that affects the establishment of predominant opinions within societies. The aim of this paper has been to identify the effect of different forms of social capital on the distribution of opinions within a society, as a result of the differences that channels of communication they encourage.

It turns out that both of the considered processes converge towards distributions where large masses of citizens are concentrated around common opinions, with intermediate opinions gradually dying out. This points out an absorbing effect of popular opinions that arises from the updating process and certainly needs a more concise analysis. In addition to this, there is very little we can say about the kind of opinions around which large masses are concentrated. It could be the purpose of a different study to understand how different processes lead to more moderate or extreme opinions, as well as the importance of influential citizens to the prevalence of certain opinions.

Regarding social dynamics, the fact that societies where citizens meet through bridging associations seem more prone to agreement is intuitive. It indicates that societies in which communication takes place predominantly through bridging associations allows the interaction between citizens with very distant opinions, providing them the opportunity to come closer, even if this occurs with low probability. On top of that, this indirectly induces smaller distances between unconnected citizens. This last observation can have further implications. If one represents meetings that yield positive probability of agreement as network links, then the result suggests that networks with smaller average distance between unconnected nodes are more likely to reach consensus.

A less intuitive and possibly more intriguing observation is that the effect of social capital on opinions can be explained by a purely abstract model that takes into account only the potentially fruitful conversations within a society. This seems to suggest that models of opinion formation might provide a useful machinery for further research on issues related to social capital and communication structures within a society.

Overall, and to the best of our knowledge, this is the first work that explores avenues connecting the literature on social capital with that on opinion formation and provides a new set of tools that could be useful for the analysis of other important problems broadly related to social capital and its applications.

A Proofs

Proof of Proposition 1. 1. It is enough to consider only the case where issue x is discussed and there is an agreement between the two citizens i, j. In any other case there is no opinion update, therefore the average remains unchanged. Without loss of generality let $x_i(t-1) < x_j(t-1)$, which means that after an agreement $x_i(t) = x_i(t-1) + \mu[x_j(t-1) - x_i(t-1)]$ and $x_j(t) = x_j(t-1) - \mu[x_j(t-1) - x_i(t-1)]$ respectively. For every other citizen it holds that $x_k(t) = x_k(t-1)$, for $k \neq i, j$

$$m(t) = \frac{x_1(t) + x_2(t) + ... + x_i(t) + ... + x_j(t) + ... + x_n(t)}{n} =$$

$$= \frac{x_1(t-1) + x_2(t-1) + ... + x_i(t-1) + \mu[x_j(t-1) - x_i(t-1)] +}{n} + \dots$$

$$\dots \frac{+ x_j(t-1) - \mu[x_j(t-1) - x_i(t-1)] + ... + x_n(t-1)}{n} =$$

$$= \frac{\sum_i x_i(t-1) + \mu[x_j(t-1) - x_i(t-1)] - \mu[x_j(t-1) - x_i(t-1)]}{n} = m(t-1)$$

2. Once again, it is enough if we focus only on the rounds when there is agreement, hence there is some change in the opinion vector. The result follows almost immediately from the following lemma.

Lemma 3. *The function $f(x) = (A+x)^k + (B-x)^k$, with $0 < A < B$, $k \in \mathbb{N}$ and $x \in [0, \frac{B-A}{2}]$ is decreasing in x.*

Proof of Lemma 3. Let k an odd number, then:

$$
\begin{aligned}
&f^{(k-1)}(x) = k!\left[(A+x)+(B-x)\right] > 0 \Rightarrow \\
&f^{(k-2)}(x) = \frac{k!}{2!}\left[(A+x)^2-(B-x)^2\right] \uparrow x,\, f^{(k-2)}\left(\frac{B-A}{2}\right) = 0 \Rightarrow f^{(k-2)} < 0 \Rightarrow \\
&f^{(k-3)}(x) = \frac{k!}{3!}\left[(A+x)^3+(B-x)^3\right] \downarrow x,\, f^{(k-3)}\left(\frac{B-A}{2}\right) > 0 \Rightarrow f^{(k-2)} > 0 \Rightarrow \\
&\dots \\
&f^{(2)}(x) = k(k-1)\left[(A+x)^{k-2}+(B-x)^{k-2}\right] \downarrow x,\, f^{(2)}\left(\frac{B-A}{2}\right) > 0 \Rightarrow f^{(2)} > 0 \Rightarrow \\
&f^{(1)}(x) = k\left[(A+x)^{k-1}-(B-x)^{k-1}\right] \uparrow x,\, f^{(1)}\left(\frac{B-A}{2}\right) = 0 \Rightarrow \mathbf{f^{(1)} < 0}
\end{aligned}
$$

Let k being an even number, then:

$$
\begin{aligned}
&f^{(k-1)}(x) = k!\left[(A+x)+(B-x)\right] < 0 \Rightarrow \\
&f^{(k-2)}(x) = \frac{k!}{2!}\left[(A+x)^2-(B-x)^2\right] \downarrow x,\, f^{(k-2)}\left(\frac{B-A}{2}\right) > 0 \Rightarrow f^{(k-2)} > 0 \Rightarrow \\
&\dots \\
&f^{(2)}(x) = k(k-1)\left[(A+x)^{k-2}+(B-x)^{k-2}\right] \downarrow x,\, f^{(2)}\left(\frac{B-A}{2}\right) > 0 \Rightarrow f^{(2)} > 0 \Rightarrow \\
&f^{(1)}(x) = k\left[(A+x)^{k-1}-(B-x)^{k-1}\right] \uparrow x,\, f^{(1)}\left(\frac{B-A}{2}\right) = 0 \Rightarrow \mathbf{f^{(1)} < 0}
\end{aligned}
$$

□

Having proven Lemma 3, the result follows immediately if one notices that

$$
\begin{aligned}
&m_x^k(t) - m_x^k(t-1) = \frac{1}{n}\left[x_i^k(t) + x_j^k(t) - x_i^k(t-1) - x_j^k(t-1)\right] = \\
&= \frac{1}{n}\{[x_i(t-1) + \mu(x_j(t-1) - x_i(t-1))]^k + [x_j(t-1) - \mu(x_j(t-1) - x_i(t-1))]^k - \\
&- x_i^k(t-1) - x_j^k(t-1)\} = \\
&= \frac{1}{n}\{[x_i(t-1) + \mu\Delta]^k + [x_j(t-1) - \mu\Delta]^k - x_i^k(t-1) - x_j^k(t-1)\} < 0
\end{aligned}
$$

where the inequality follows from Lemma 3.

3. It follows directly from parts (1) and (2) since $\sigma_x^2(t) = m_x^2(t) - [m_x(t)]^2$, which implies $\sigma_x^2(t) - \sigma_x^2(t-1) = m_x^2(t) - m_x^2(t-1) \le 0$. Part (1) implies the equality and part (2) implies the inequality.

□

Proof of Proposition 2. We prove the result by contradiction. The proof is presented for the bonding process, but it is completely analogous for the bridging process.

Consider a period $t' > t$ and let $i, j, l \in N$ such that $j \in \mathcal{N}_i^{t'}$ and $l \in \mathcal{N}_j^{t'}$, but $l \notin \mathcal{N}_i^{t'}$. In a bonding process, this means that $|x_i^{t'} - x_j^{t'}| \leq d$ and $|y_i^{t'} - y_j^{t'}| \leq d$, $|x_l^{t'} - x_j^{t'}| \leq d$ and $|y_l^{t'} - y_j^{t'}| \leq d$, but $|x_i^{t'} - x_l^{t'}| > d$ or/and $|y_i^{t'} - y_l^{t'}| > d$. Let us focus on the case where $|x_i^{t'} - x_l^{t'}| > d$ and without loss of generality assume that $x_i^{t'} < x_j^{t'} < x_l^{t'}$. This means that the projection of opinion profiles on the x axis is as depicted in Figure 9, where $a := |x_i^{t'} - x_j^{t'}| < d$, $b := |x_l^{t'} - x_j^{t'}| < d$ and $a + b > d$. Such a configuration violates

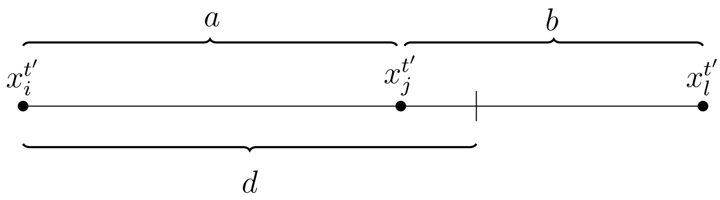


Figure 9: Projection of opinion profiles of agents i, j and l on the x axis.

neighborhood stabilisation because there is positive probability of some neighborhood changing after a finite number of periods. To show that, let us first consider the case where $a + \frac{b}{2} < d$ and consider a finite sequence of consecutive periods in which only citizens j and l interact. For any $\mu \leq 1/2$, $\lim_{t\to\infty} x_j^t = \lim_{t\to\infty} x_l^t = x_j^{t'} + \frac{b}{2}$. Therefore, by definition of the limit, for every $\epsilon > 0$, there exists some $\hat{t}_\epsilon$ such that for all $t > \hat{t}_\epsilon$ it holds that $|x_l^t - x_j^{t'} - \frac{b}{2}| < \epsilon$ or equivalently $|x_l^t - x_i^t| < a + \frac{b}{2} + \epsilon$. Hence, for $\hat{\epsilon} = \frac{d-a-b/2}{2}$, there exists some $\hat{t}_{\hat{\epsilon}}$, such that for all $t > \hat{t}_{\hat{\epsilon}}$, $|x_l^t - x_i^t| < d$. This, in turn, means that there is a sufficiently long sequence of encounters between citizens j and l that can bring l in the neighborhood of i. Analogously, if $a + \frac{b}{2} > d$, there is a sufficiently long sequence of encounters between citizens j and l that can bring j out of the neighborhood of i. If $a + \frac{b}{2} = d$ and $\mu < 1/2$, the above result holds only in the limit, however it is enough to consider that there is first a successful encounter between citizens i and j. Depending on the values of μ and d, this encounter either drives i sufficiently close to l, or j sufficiently far from l, or to a case where $a' + \frac{b'}{2} < d$. If l is far from i in both issues, then in the case in which $|x_l^t - x_i^t| < d$ after a finite number of rounds, the sequence of encounters between j and l is repeated in the issue y, where the analysis is exactly the same. Therefore, overall, if there is $i, j, l \in N$ such that $j \in \mathcal{N}_i^{t'}$, $l \in \mathcal{N}_j^{t'}$, but $l \notin \mathcal{N}_i^{t'}$, then neighborhood stabilisation cannot hold. Hence, neighborhood stabilisation at period t implies that for all $t' > t$ and $i, j, l \in N$ such that $j \in \mathcal{N}_i^{t'}$ and $l \in \mathcal{N}_j^{t'}$, it must also hold that $l \in \mathcal{N}_i^{t'}$. $\square$

Proof of Lemma 1. 1. ($\Rightarrow$) $\mathcal{N}_i^t = \mathcal{I} \setminus \{i\}$ implies that $|x_j^t - x_i^t| \leq d$ and $|y_j^t - y_i^t| \leq d$ for all $j \in \mathcal{I}$. Therefore, $\max_{j\in I} x_j^t - \min_{j\in I} x_j^t \leq d$ and as a result one can find $x_1 < x_2$ such that $x_2 - x_1 = d$ and $x_1 \leq \min_{j\in I} x_j^t \leq \max_{j\in I} x_j^t \leq x_2$. Analogously, for issue y, one can find $y_1 < y_2$ such that $y_2 - y_1 = d$ and $y_1 \leq \min_{j \in I} y_j^t \leq \max_{j \in I} y_j^t \leq y_2$. Hence, if $Q := \{(x, y) \in \mathbb{R}^2 : x_1 \leq x \leq x_2 \text{ and } y_1 \leq y \leq y_2\}$, then $(x_i^t, y_i^t) \in Q$ for all $i \in \mathcal{I}$.

($\Leftarrow$): Let Q as defined above. Then, by the definition of the neighboring area in a bonding process it holds that $Q \subseteq \mathcal{N}_{(x_i^t, y_i^t)}$ for all $i \in \mathcal{I}$. Hence, $(x_j^t, y_j^t) \in \mathcal{N}_{(x_i^t, y_i^t)}$ for all $j \in \mathcal{I}$ and equivalently $j \in \mathcal{N}_i^t$ for all $j \in \mathcal{I} \setminus \{i\}$.

2. It follows immediately by the definition of Q in part (1) that for all $(x, y) \in MBR_{\mathcal{I}}^t$ it also holds that $(x, y) \in Q$. The second relation holds by the definition of a neighboring area in a bonding process. Overall, $MBR_{\mathcal{I}}^t \subseteq Q \subseteq \mathcal{N}_{(x_i^t, y_i^t)}$ for all $i \in \mathcal{I}$.

3. Without loss of generality, assume that the agents $j, l \in \mathcal{I}$ discuss issue x and their opinion in that issue at period t satisfy $x_j^t < x_l^t$. By construction, it holds that $\min_{i \in \mathcal{I}} x_i^t \leq x_j^t < x_l^t \leq \max_{i \in \mathcal{I}} x_i^t$. But, upon agreeing $x_j^{t+1} = x_j^t + \mu(x_l^t - x_j^t)$ and $x_l^{t+1} = x_l^t - \mu(x_l^t - x_j^t)$, therefore given that $\mu \in (0, 1/2]$ we get that $x_j^t \leq x_j^{t+1} \leq x_l^{t+1} \leq x_l^t$. Therefore, both opinion profiles will still belong to $MRB_{\mathcal{I}}^t$, hence also to Q, both of which are still included in the neighboring area of each citizen i's location.

$\square$

Proof of Proposition 3. The following two conditions are jointly equivalent with neighborhood stabilisation: For each citizen $i \in N$ (i) with probability 1, i does not lose any of his current neighbors at a subsequent period (ii) with probability 1, no additional citizen enters i's neighborhood at a subsequent period.

Proposition 2 ensures that the existence of disjoint groups of citizens with common neighborhoods are necessary for conditions (i) and (ii) to be satisfied. Absent of such groups, there would be positive probability of some citizen's neighborhood changing.

Given the existence of these disjoint groups of citizens, Lemma 1 guarantees that a member of such a group will not face a change in his neighborhood (with probability 1), as long as the members of that group may discuss successfully only among themselves.

The second condition guarantees the argument that (with probability 1) the members of each of the disjoint groups may discuss successfully only among themselves. This is because at period t each citizen may only discuss successfully with another member of her own group. But a successful discussion between citizens $i, j \in \mathcal{I}_g$ weakly shrinks both the minimum bounding rectangle of the group and as a result also the neighboring area of the minimum bounding rectangle, i.e. $MBR_{\mathcal{I}_g}^{t+1} \subseteq MBR_{\mathcal{I}_g}^t$, which then implies that $\mathcal{N}_{MBR_{\mathcal{I}_g}^{t+1}} \subseteq \mathcal{N}_{MBR_{\mathcal{I}_g}^t}$. Therefore, if $MBR_{\mathcal{I}_{g'}}^t \bigcap \mathcal{N}_{MBR_{\mathcal{I}_g}^t} = \emptyset$, for all $g' \neq g$, it must also hold that $MBR_{\mathcal{I}_{g'}}^{t+1} \bigcap \mathcal{N}_{MBR_{\mathcal{I}_g}^{t+1}} = \emptyset$, for all $g' \neq g$, with probability 1. This completes the argument, as it guarantees (with probability 1) that in all subsequent periods no additional citizen enters the neighborhood of any $i \in \mathcal{I}_g$. $\square$

Proof of Lemma 2. Without loss of generality, we prove the result considering that the area R is of size $k \times 1$.

1. By definition of the area R, it holds that $|x_i^t - x_j^t| \leq k$ for all $i, j \in \mathcal{I}$. Therefore, for all $i \in \mathcal{I}$ it also holds that $j \in \mathcal{N}_i^t$ for all $j \in \mathcal{I} \setminus \{i\}$, or equivalently for all $\mathcal{N}_i^t = \mathcal{I} \setminus \{i\}$ for all $i \in \mathcal{I}$.

2. $MBR_{\mathcal{I}}^t$ is also a rectangle with sizes $|\max_{i \in \mathcal{I}} x_i^t - \min_{i \in \mathcal{I}} x_i^t| \times |\max_{i \in \mathcal{I}} y_i^t - \min_{i \in \mathcal{I}} y_i^t|$, by definition. Given that all $i \in \mathcal{I}$ are such that $(x_i^t, y_i^t) \in R$, it must hold that $|\max_{i \in \mathcal{I}} x_i^t - \min_{i \in \mathcal{I}} x_i^t| = k - \epsilon$ for some $\epsilon \in [0, k]$ and also the left bound of R, denote it $\underline{x}$, must satisfy $\underline{x} \in [\min_{i \in \mathcal{I}} x_i^t - \epsilon, \min_{i \in \mathcal{I}} x_i^t]$. These last two conditions guarantee that the right bound of R, $\overline{x}$, will have to satisfy $\overline{x} \in [\max_{i \in \mathcal{I}} x_i^t, \min_{i \in \mathcal{I}} x_i^t + \epsilon]$. Therefore, recalling that R extends in the whole length of the vertical axis, it follows that $MBR_{\mathcal{I}}^t \subseteq R$.

   The neighboring area of the citizen $j = \operatorname{argmin}_{i \in \mathcal{I}} x_i^t$ extends for all y at least until $\min_{i \in \mathcal{I}} x_i^t - k \leq \underline{x}$. Analogously, the neighboring area of the citizen $l = \operatorname{argmin}_{i \in \mathcal{I}} x_i^t$ extends for all y at least until $\max_{i \in \mathcal{I}} x_i^t - k \leq \overline{x}$. Hence, it follows immediately that $R \subseteq \mathcal{N}_{(x_i^t, y_i^t)}$ for all $i \in \mathcal{I}$.

3. The proof is identical to that of part 3 in Lemma 1, substituting R for Q.

□

Proof of Proposition 4. The following two conditions are jointly equivalent with neighborhood stabilisation: For each citizen $i \in N$ (i) with probability 1, i does not lose any of his current neighbors at a subsequent period (ii) with probability 1, no additional citizen enters i's neighborhood at a subsequent period. As explained also in the bonding process, the first condition (existence of disjoint groups of neighbors) is necessary for neighborhood stabilisation because of Proposition 2.

By Lemma 2, the first two conditions are sufficient to guarantee that a citizen who is a member of such a group will not face a change in her neighborhood (with probability 1), as long as the members of each group may discuss successfully only among themselves.

The third condition guarantees that (with probability 1) the members of each of the disjoint groups may discuss successfully only among themselves. The reasoning is identical to that in Proposition 3. □

Proof of Theorem 1. In each of the two issues, the system evolves following a Markov chain with a randomly determined transition matrix in each period, i.e. $\mathbf{x}(t+1) = A_x(\mathbf{x}(t), t)\mathbf{x}(t)$ and $\mathbf{y}(t+1) = A_y(\mathbf{y}(t), t)\mathbf{y}(t)$, where $A_x(\mathbf{x}(t), t), A_y(\mathbf{y}(t), t)$ are the realisations of two stochastic matrices that are randomly selected according to the following process:

The two matrices have typical elements $a_{x,ij}$ and $a_{y,ij}$ respectively. At each period t, two citizens i, j are selected at random from the population. Every other citizen keeps both opinions constant, which means that for all $k \neq i, j$ it holds that $a^t_{x,kk} = a^t_{y,kk} = 1$ and $a^t_{x,kl} = a^t_{y,kl} = 1$ for all $l \neq k$. Moreover, citizens i, j may be affected only by the opinion of each other, hence $a^t_{x,ik} = a^t_{y,ik} = a^t_{x,jk} = a^t_{y,jk} = 0$ for all $k \neq i, j$. Their positions and the process determine the probabilities with which they discuss each issue, as well as the probabilities of agreement in the issue discussed. For the issue that is not discussed, for instance x, the two citizens do not revise their opinions, hence $a^t_{x,ij} = a^t_{x,ji} = 0$ and $a^t_{x,ii} = a^t_{x,jj} = 1$. This means that there has been no adjustment in this issue at period t and $A^t_x = I$, where I is the identity matrix. For the issue that is discussed, in this case y, there is a revision only when the two citizens agree. If this is the case then $a^t_{y,ij} = a^t_{y,ji} = \mu$ and $a^t_{y,ii} = a^t_{y,jj} = 1 - \mu$, where μ has been defined to be the adjustment rate after agreement. In case of disagreement there is no revision, therefore $A^t_y = I$. The argument is analogous when issue x is discussed.

Therefore, given the realized values of the transition matrices in each period, we can consider the evolution of the process in each of the issues separately. Stability of the process as a whole is equivalent to stability in each of the issues. Notice also that these values are not affected by the process we consider, since this only affects the probabilities of discussing each issue and subsequently the probability of agreeing on it.

The previous observation is crucial, because it shows that multidimensionality does not affect the result, since each issue can be considered independently. This in turn means that it is enough to show that in each issue the process satisfies the sufficient conditions estblished in Lorenz (2005) for the population to get divided in pairwise disjoint groups within which the citizens reach consensus almost surely. The essence of these conditions is that they ensure that the series of accumulation of transition matrices in each issue converges to a constant matrix over time. For this to be the case the following three conditions must hold in each of the two processes:

(i) Self–confidence: At each period t, each citizen $i \in \underline{n}$ puts positive weight to her own opinion, i.e. it holds that $a^t_{\cdot,ii} > 0$, which is clearly true in our case.

(ii) Mutual confidence: Zero entries in the transition matrix are symmetric. For every two citizens $i, j \in \underline{n}$ it holds that $a^t_{\cdot,ij} > 0 \Leftrightarrow a^t_{\cdot,ji} > 0$. This is again clearly true in both our processes, given that after agreeing both citizens revise their opinions. Notice that this condition does not even impose the two weights to be equal, which is anyway true in our case.

(iii) Positive weights not converging to zero: There is a $\delta > 0$ such that the lowest postitive entry of the transition matrix is greater than δ. This is also true in our case, since the minimum positive value that can arise in the transition matrix is $\min\{\mu, 1 - \mu\}$ which is strictly positive.

These three conditions ensure that in each issue pairwise disjoint groups $\mathcal{J}_{x,1} \bigcup \cdots \bigcup \mathcal{J}_{x,r}$ and $\mathcal{J}_{y,1} \bigcup \cdots \bigcup \mathcal{J}_{y,s}$ of citizens that reach consensus in this issue are formed. To complete the argument, let $\mathcal{I}_{kl} = \mathcal{J}_{x,k} \bigcap \mathcal{J}_{y,l}$. There is a total of $p = r \times s$ of such groups and within each of them citizens reach consensus almost surely (in both issues). $\square$

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