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Working Paper 05-2016

Candidate valence in a spatial model with entry

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May, 2016

Abstract

This paper studies electoral competition between two purely office-motivated and heterogeneous (in terms of valence) established candidates when entry of a lower-valence third candidate is anticipated. In this model, when the valence asymmetries among candidates are not very large, there always exists an essentially unique pure strategy equilibrium and it is such that: a) the high valence established candidate offers a more moderate platform than the low valence established candidate, b) the entrant locates between the two established candidates and nearer to the high valence established candidate and, surprisingly, c) both established candidates receive equal vote-shares. We also show that the platforms that the two established candidates choose in this equilibrium constitute a local equilibrium in the extension of the game in which the third candidate is expected to enter the race with any non-degenerate probability.

Keywords: electoral competition; entry; candidate valence.

JEL classification: D7, H1.

1 Introduction

In the standard Downsian model with two vote-share maximizing candidates of equal valence the unique equilibrium prediction is that both candidates will converge to the ideal policy of the median voter. This is so because the game is dominated by *centripetal*[1] forces: each candidate knows that she can always win more votes by approaching her opponent. But, when we introduce a valence asymmetry[2] between the two candidates, the dynamics of the Downsian model alter dramatically; the described dominance of centripetal forces collapses for one of the two candidates. The high valence candidate still has incentives to *move close* to the low valence candidate, but now the low valence candidate has incentives to *move away* from the high valence candidate in order to secure a non-negligible vote-share. In this framework Aragonès and Palfrey (2002) show that precisely because of these asymmetric incentives (that is, because centripetal forces are dominant for only one of the two candidates) *no pure strategy equilibria exist.* They moreover prove generic existence of a mixed equilibrium and they identify a mixed equilibrium for the case of a discrete policy space, minimal valence asymmetries and a uniform distribution of the voters ideal policies. Hummel (2010) and Aragonès and Xefteris (2012) extend these results to non-minimal valence asymmetries and unimodal distributions of the voters ideal policies while Groseclose (2001) extends the model by considering that candidates care about the implemented policies.[3]

One of the most influential modifications of the Downsian model in the direction of mitigating the extent of centripetal forces in a two-candidate competition, while preserving the assumption of pure vote-share maximization, was provided by Palfrey (1984). He considered that a third candidate enters the electoral race after the two established candidates simultaneously selected

[1] The term *centripetal* force denotes here the force which makes one candidate want to move in the direction of the other candidate and not necessarily towards some notion of a center of the policy space.

[2] Introduction of a valence asymmetry between the two candidates in such a model of electoral competition seems very intuitive as common observation dictates that voters decide which candidate to support not only on the basis of the electoral platforms but also on the basis of non-policy characteristics such as charisma, corruption allegations, personal appeal and others.

[3] Further results on electoral competition between heterogeneous candidates (or parties) may be found in Stokes (1963), Adams (1999), Ansolabehere and Snyder (2000), Erikson and Palfrey (2000), Dix and Santore (2002), Laussel and Le Breton (2002), Aragonès and Palfrey (2005), Herrera et al. (2006), Schofield (2007), Degan (2007), Kartik and McAfee (2007), Carillo and Castanheira (2008), Meirowitz (2008), Zakharov (2009), Ashworth and Bueno de Mesquita (2009), Krasa and Polborn (2012), Pastine and Pastine (2012), Ansolabehere et al. (2012), Xefteris (2012), Bernheim and Kartik (2014) and Aragonès and Xefteris (2016).

their platforms in order to maximize their vote-shares. In this model, the standard centripetal forces continue to exist but they are complemented by centrifugal ones too. Each established candidate knows that by approaching her established competitor too much, she increases the likelihood that the entrant will locate such that she will end up squeezed between her two opponents and, thus, that she will receive a very small vote-share. In the unique equilibrium of the game the two established candidates locate at different positions (equidistantly away from the center of the policy space).[4]

For all these reasons, this setup presents itself as an ideal environment for the study of the effect of valence asymmetries on policy platform determination of the two established candidates. Candidates care only about their vote-shares, so unlike Groseclose (2001) the equilibrium behavior that the model results in does not depend on many parameters (models which assume policy motivation need a whole extra structure and assumptions regarding candidates' policy preferences) and, most importantly, the simultaneous existence of both centripetal and centrifugal forces for both candidates (as opposed to the Downsian model without entry) makes existence of pure strategy equilibria possible.[5]

Introduction of valence asymmetries in Palfrey's (1984) entry model is exactly what we do in this paper. We consider that all three candidates may differ in valence and that, naturally, the two established candidates have higher valence than the entrant. We moreover consider that these valence asymmetries are small (but not degenerate) as these are the valence asymmetries that are considered more important in the literature and to keep the analytical part of the paper

[4]Results for the case in which candidates are office-motivated and the third-candidate entry decision is endogenous can be found in Greenberg and Shepsle (1987), Rubinchik and Weber (2007), Callander and Wilson (2007) and Shapoval et al. (2015). The last study relates to this paper also because one of the two established candidates is considered to have a valence advantage. Unlike the present paper, though, Shapoval et al. (2015): a) do not consider the case in which both established candidates enjoy a valence advantage over the entrant and b) they show existence of an equilibrium conditional on the valence advantage being sufficiently large. Moreover, Osborne (1993) provides results for the case in which entry of all candidates (and not just of a third entrant) is endogenous. Loertscher and Muehlheusser (2011) also study a model such that entry of all candidates is endogenous but in contrast to Osborne (1993), who assumes that all players decide whether to enter or not simultaneously, they consider that candidates' decisions are taken in a sequential manner. For the standard Downsian model without valence asymmetries or endogenous entry one is referred to Eaton and Lipsey (1975), Shaked (1982), Osborne and Pitchik (1986) and Collins and Sherstyuk (2000).

[5]Despite the fact that mixed strategies are commonly used in certain branches of the literature (one is referred, for example, to Baye et al. 1996 who characterize mixed equilibria of all-pay auctions), their use in electoral competition models is still not universally accepted.

as coherent as possible. We find that indeed *pure strategy equilibria exist in this model.* One established candidate locates to the left of the median voter and the other established candidate locates to the right of the median voter just like the no-valence asymmetries case. But when we have valence asymmetries between the established candidates we find that: a) the high valence established candidate offers a more moderate platform than the low valence established candidate and that the entrant locates between them and to the side of the center of the policy space in which the high valence candidate locates[6] and that b) the vote-shares of both established candidates are equal; the entry of a third candidate in the race has an egalitarian effect on the equilibrium payoffs of the two established candidates.

So as far as the two established candidates are concerned, like Aragonès and Palfrey (2002), Hummel (2010) and Aragonès and Xefteris (2012) we find that the high valence candidate is more moderate than the low valence established candidate but unlike them: a) we show this by the means of a pure strategy equilibrium and b) we show that the vote-shares of the two established candidates are equal in equilibrium (all these papers find that the high valence candidate enjoys higher expected payoffs). To sum up, the effect of considering a potential entrant is threefold. First, it allows the model to admit a pure strategy equilibrium without further structure and assumptions about candidate's preferences which distort the fundamental elements of the standard Downsian model, second, it confirms that, out of the two main candidates, the high valence one is more moderate than the low valence one and, third, it counterintuitively contradicts the finding of all previous papers that a valence advantage over one's main opponent translates into a larger vote-share. In other words, the valence dimension seems to be more important in determining the platforms of the two main competitors than in determining who will be the winner of the elections (in our model the two established candidates always tie in the first place).

Finally, we argue that the pair of locations that the two main candidates occupy in the unique equilibrium of our game is: a) an equilibrium pair of locations in the extension of the model in which the probability with which a third candidate actually enters in the race is sufficiently large

[6] Three-candidate electoral competition models with valence asymmetries which consider simultaneous platform decisions may also result in equilibria such that the highest and the lowest valence candidates locate to the left (right) of the median voter while the intermediate valence candidate locates to the right (left) of the median voter (see for example Evrenk and Kha, 2011 and Xefteris, 2014).

but strictly lower than one and b) a *local* equilibrium in the extension of the model in which an entrant appears with any non-degenerate probability.[7] These extra findings suggest that our results are robust even if third candidate entry is not certain.

2 The model

The policy space is the linear segment $[0,1]$ and the policy preferences of a voter i with ideal policy x_i are given by $u(x_i, x) = -|x - x_i|$ for $x \in [0,1]$. We consider a continuum of voters with ideal policies distributed on $[0,1]$ according to a continuous and twice-differentiable distribution function $F : [0,1] \to [0,1]$ with the following properties: a) $F'(x) = F'(1-x)$ for every $x \in [0,1]$ (the density of F is symmetric about the center of the policy space), b) $F''(x) > 0$ for $x < \frac{1}{2}$ (the density of F is unimodal) and c) $F'(\frac{1}{2}) - 2F'(0) \leq 0$ (the concentration of the voters in all compact subsets of the policy space of equal measure is not severely asymmetric).[8]

There are three candidates, indexed by A, B and C, which are purely office-motivated; each candidate $J \in \{A, B, C\}$ proposes a policy platform $y_J \in [0,1]$ in order to maximize her expected vote-share. We consider that candidates may differ in the, so called, valence dimension and we therefore assume that $v_A \geq v_B \geq v_C = 0$. The exact values of v_A, v_B and v_C are known to all three candidates and to all voters. Therefore, the valuation of a voter i with ideal policy x_i for a candidate J with valence v_J who proposes a policy y_J is given by $U(x_i, y_J, v_J) = u(x_i, y_J) + v_J$. We consider that voters vote sincerely for the candidate they value most and that they evenly split their vote among the candidates they value most in case they are more than one.

The game has essentially four stages. In the first stage of the game the two established candidates, A and B, select their policy platforms y_A and y_B simultaneously. In the second stage the entrant C observes y_A and y_B and selects y_C. In the third stage each voter observes y_A, y_B and y_C and votes for the candidate she values most. In the last stage of the game the payoffs (vote-shares)

[7] There is a recent literature which considers that local Nash equilibrium is a reasonable solution concept for electoral competition games. See for example Schofield (2007) and Krasa and Polborn (2012).

[8] Brusco and Roy (2011) employ this assumption too when they analyze the citizen-candidate model with aggregate uncertainty. We partially relax this assumption after the presentation of the equilibrium existence result.

of the three candidates are realized.

This game though cannot be directly solved in its essential form by the use of standard equilibrium notions (just like the case of no valence asymmetries of Palfrey, 1984). In this extensive form game of full information one would naturally try to identify subgame perfect equilibria (SPE). What we notice, though, is that not every subgame possesses a Nash equilibrium. This is due to the fact that the best response of candidate C to candidates A and B playing y_A and y_B, $\hat{y}_C(y_A, y_B)$, is not always well-defined. For example, $y_A = 0$, $y_B = 1$ and the values of v_A, v_B and v_C are sufficiently homogenous $\hat{y}_C(y_A, y_B)$ is well-defined and single-valued but when $y_A = y_B$ (or when they are sufficiently near) the entrant has no well-defined set of best responses. Hence, not all subgames posses a Nash equilibrium and, therefore, a SPE cannot exist in this setup.

To deal with this issue we will introduce the notion of a *Quasi-SPE.* To this end, and exactly as Palfrey (1984), we first need to employ a limit equilibrium analysis. That is, we first define a limit equilibrium as any pair of equilibrium strategies for A and B such that the equilibrium strategy for each candidate is a best response to any other strategy for an infinite sequence of the perturbed games, with the perturbation converging to zero. To properly construct the perturbed game Γ_ε we need some extra definitions.

For a fixed triplet (v_A, v_B, v_C) such that $v_A \geq v_B \geq v_C = 0$ define

$$W_A(y_A, y_B,\, y_C) = \{z \in [0, 1] | U(z, y_A, v_A) \geq \max\{U(z, y_B, v_B), U(z, y_C, v_C)\}$$

$$W_B(y_A, y_B,\, y_C) = \{z \in [0, 1] | U(z, y_B, v_B) \geq \max\{U(z, y_A, v_A), U(z, y_C, v_C)\}$$

$$W_C(y_A, y_B,\, y_C) = \{z \in [0, 1] | U(z, y_C, v_C) \geq \max\{U(z, y_A, v_A), U(z, y_B, v_B)\}$$

and

$$\dot{W}_A(y_A, y_B,\, y_C) = [W_A(\cdot) \cap W_B(\cdot)] \cup [W_A(\cdot) \cap W_C(\cdot)]$$

$$\dot{W}_B(y_A, y_B,\, y_C) = [W_B(\cdot) \cap W_A(\cdot)] \cup [W_B(\cdot) \cap W_C(\cdot)]$$

$$\dot{W}_C(y_A, y_B,\, y_C) = [W_C(\cdot) \cap W_A(\cdot)] \cup [W_C(\cdot) \cap W_B(\cdot)]$$

$$\ddot{W}(y_A, y_B,\, y_C) = [W_A(\cdot) \cap W_B(\cdot) \cap W_C(\cdot)].$$

Using the above we further define

$$V_A(y_A, y_B,\, y_C) = \int_{W_A(\cdot)\backslash \dot{W}_A(\cdot)} dF(x) + \tfrac{1}{2} \int_{\dot{W}_A(\cdot)\backslash \ddot{W}} dF(x) + \tfrac{1}{3} \int_{\ddot{W}} dF(x)$$

$$V_B(y_A, y_B,\, y_C) = \int_{W_B(\cdot)\backslash \dot{W}_B(\cdot)} dF(x) + \tfrac{1}{2} \int_{\dot{W}_B(\cdot)\backslash \ddot{W}} dF(x) + \tfrac{1}{3} \int_{\ddot{W}} dF(x)$$

$$V_C(y_A, y_B,\, y_C) = \int_{W_C(\cdot)\backslash \dot{W}_C(\cdot)} dF(x) + \tfrac{1}{2} \int_{\dot{W}_C(\cdot)\backslash \ddot{W}} dF(x) + \tfrac{1}{3} \int_{\ddot{W}} dF(x)$$

and for any $\varepsilon > 0$ and any pair $(y_A, y_B) \in [0,1]^2$

$$\dot{B}_\varepsilon(y_A, y_B) = \{y_C \in [0,1] | V_C(y_A, y_B, y_C) > V_C(y_A, y_B, x),\ \forall x \in [0,1]\backslash\{y_C\}\}$$

$$\ddot{B}_\varepsilon(y_A, y_B) = \{y_C \in [0,1] | V_C(y_A, y_B, y_C) > V_C(y_A, y_B, x) - \varepsilon,\ \forall x \in [0,1]\}$$

$$E_\varepsilon(y_A, y_B) = \begin{cases} \dot{B}_\varepsilon(y_A, y_B) & \text{, for } \dot{B}_\varepsilon(y_A, y_B) \neq \oslash \\ \ddot{B}_\varepsilon(y_A, y_B) & \text{, for } \dot{B}_\varepsilon(y_A, y_B) = \oslash \end{cases}.$$

That is, we consider that if the entrant has a unique and well-defined best response she locates there, while in every other case she mixes uniformly among her ε-best responses. Therefore, our two-player perturbed game, Γ_ε, is defined by

$$\pi_A^\varepsilon(y_A, y_B) = \begin{cases} V_A(y_A, y_B,\, t \in E_\varepsilon(y_A, y_B)) & \text{, for } \dot{B}_\varepsilon(y_A, y_B) \neq \oslash \\ \int_{E_\varepsilon(y_A, y_B)} \frac{1}{\int_{E_\varepsilon(y_A, y_B)} dt} V_A(y_A, y_B,\, t) dt & \text{, for } \dot{B}_\varepsilon(y_A, y_B) = \oslash \end{cases}$$

$$\pi_B^\varepsilon(y_A, y_B) = \begin{cases} V_B(y_A, y_B,\, t \in E_\varepsilon(y_A, y_B)) & \text{, for } \dot{B}_\varepsilon(y_A, y_B) \neq \oslash \\ \int_{E_\varepsilon(y_A, y_B)} \frac{1}{\int_{E_\varepsilon(y_A, y_B)} dt} V_B(y_A, y_B,\, t) dt & \text{, for } \dot{B}_\varepsilon(y_A, y_B) = \oslash \end{cases}.$$

Exactly as Palfrey (1984) we call $(\hat{y}_A, \hat{y}_B)$ a limit equilibrium if: a) for every $y_A \neq \hat{y}_A$, there is a number, $\varepsilon(y_A)$, such that for all $\varepsilon \in (0, \varepsilon(y_A))$, $\pi_A^\varepsilon(y_A, \hat{y}_B) < \pi_A^\varepsilon(\hat{y}_A, \hat{y}_B)$ and b) for every $y_B \neq \hat{y}_B$, there is a number, $\varepsilon(y_B)$, such that for all $\varepsilon \in (0, \varepsilon(y_B))$, $\pi_B^\varepsilon(\hat{y}_A, y_B) < \pi_B^\varepsilon(\hat{y}_A, \hat{y}_B)$.

The careful reader should have noticed that the perturbed game that we described is slightly different to the one constructed by Palfrey (1984). The difference lies in the fact that we consider that the entrant chooses her best response in case she has a well-defined and unique best response and that she mixes (uniformly) among her ε-best responses whenever she has no unique and/or well-defined best response while Palfrey (1984) considers that the entrant always mixes (uniformly) among her ε-best responses independently of whether she has a unique well-defined best response or not.

We can now define a *Quasi-SPE* of the original game.

Definition 1 *A Quasi-SPE, $(\hat{y}_A, \hat{y}_B, \hat{y}_C)$, of the original game is such that, $(\hat{y}_A, \hat{y}_B)$, is a limit equilibrium of the perturbed game with $E_\varepsilon(\hat{y}_A, \hat{y}_B) = \hat{y}_C$.*[9]

The reason why we modify the solution strategy of Palfrey (1984) is because the original solution cannot be directly applied in this framework. When the two established candidates differ in valence, a perturbed game, as originally defined by Palfrey (1984), does not admit any limit equilibrium when mixing over ε-best responses is assumed to be uniform. But since the use of a uniform mixture is just an auxiliary device, the notion of the limit equilibrium introduced by Palfrey (1984) essentially requires that there is a distribution over ε-best responses for the third candidate that allows to pin down the strategies of the other two candidates. Hence, playing with certainty the best response, if a unique one exists, and mixing uniformly over ε-best responses in all other cases is just an alternative auxiliary device which helps us pin down the strategies of the two main candidates and in no way is our solution strategy conceptually distant from the original one. In fact one can show that: a) when both solution strategies result in an equilibrium (no valence asymmetries case) then their predictions regarding the locations of the two established candidates coincide and b) when there exist valence asymmetries between the two established candidates the two approaches converge to the same solution strategy once we consider that $\varepsilon \to 0$. That is, if we consider the original perturbed game of Palfrey (1984) (let its payoff functions be $\tilde{\pi}^{\varepsilon}_A(y_A, y_B)$

[9] Whenever a set H contains a unique element, η, we slightly abuse notation and instead of writing $H = \{\eta\}$ we write $H = \eta$.

and $\tilde{\pi}_A^\varepsilon(y_A, y_B)$) and define a new two-player game with payoffs $\mathring{\pi}_A(y_A, y_B) = \lim_{\varepsilon\to 0} \tilde{\pi}_A^\varepsilon(y_A, y_B)$ and $\mathring{\pi}_B(y_A, y_B) = \lim_{\varepsilon\to 0} \tilde{\pi}_B^\varepsilon(y_A, y_B)$ then for $b > 0$ sufficiently small this new two-player game will have a Nash equilibrium which will coincide with the *Quasi-SPE* of the game that we analyze in this paper.

One can surely come up with advantages of one approach over the other but all of them are doomed to be minor since they cannot relate to the most important feature of the resulting equilibria - the equilibrium locations of the two established candidates. This is so because, as we already noted, when both solution strategies work then their equilibrium predictions regarding the locations of the two established candidates coincide.

3 Formal results

We directly proceed to the main result of the paper.

Theorem 1 *There exists $b > 0$ such that whenever $b \geq v_A \geq v_B \geq v_C = 0$, $(\hat{y}_A, \hat{y}_B, \hat{y}_C)$ is a Quasi-SPE where: a) $\hat{y}_A < \hat{y}_C \leq \frac{1}{2} < \hat{y}_B$, b) $|\frac{1}{2} - \hat{y}_A| \leq |\frac{1}{2} - \hat{y}_B|$, c) $F(\hat{y}_A - v_A) = 1 - 2F(\frac{1}{4} + \frac{1}{2}v_B + \frac{1}{2}\hat{y}_A)$, d) $\hat{y}_B = 1 - \hat{y}_A + v_A - v_B$ and e) $\hat{y}_C = \frac{1}{2} - v_A + v_B$.*

Proof. According to the definition of a *Quasi-SPE* we have to prove that there exists $b > 0$ such that whenever $b \geq v_A \geq v_B \geq v_C = 0$, $(\hat{y}_A, \hat{y}_B)$ is a limit equilibrium where (i) $F(\hat{y}_A - v_A) = 1 - 2F(\frac{1}{4} + \frac{1}{2}v_B + \frac{1}{2}\hat{y}_A)$, (ii) $\hat{y}_B = 1 - \hat{y}_A + v_A - v_B$ and (iii) $\hat{y}_C = (\frac{1}{2} - v_A + v_B) = E_\varepsilon(\hat{y}_A, \hat{y}_B)$ for any $\varepsilon > 0$.

First of all we notice that, since F is continuous and strictly increasing and F' is symmetric about the center of the policy space and strictly unimodal, there exists a non-degenerate $b > 0$ such that for $b \geq v_A \geq v_B \geq v_C = 0$ condition (i) admits a unique solution $\hat{y}_A$ in $[0, 1]$ such that $\hat{y}_A \in (0, \frac{1}{2} - v_A)$. Subsequently, when condition (i) admits a unique solution, condition (ii) admits as well exactly one solution $\hat{y}_B$ in $[0, 1]$ such that $\hat{y}_B \in (\frac{1}{2} + v_B, 1)$.

We begin our proof by showing that when the strategy pair $(\hat{y}_A, \hat{y}_B)$ is characterized by conditions (i) and (ii), it is the case that the entrant has a unique and well-defined best response given by condition (iii). That is, we start by demonstrating that $E_\varepsilon(\hat{y}_A, \hat{y}_B) = \frac{1}{2} - v_A + v_B$ for any $\varepsilon > 0$.

To this end, we first notice that condition (ii) is equivalent to $\hat{y}_A - v_A = 1 - (\hat{y}_B + v_B)$ which, along with F' being symmetric about the center of the policy space, implies that $F(\hat{y}_A - v_A) = 1 - F(\hat{y}_B + v_B)$.

Second, conditions (i) and (ii) indicate that for $b > 0$ small enough (but not degenerate) we should have $\hat{y}_A + v_A < (\frac{1}{2} - v_A + v_B) \leq \frac{1}{2} < \hat{y}_B - v_B$ and $\hat{y}_A \geq 1 - \hat{y}_B$.

We observe that for any given $\delta > 0$ we have

$$\begin{aligned} V_C(\hat{y}_A, \hat{y}_B, \hat{y}_A - v_A - \delta) &< F(\hat{y}_A - v_A) \\ \lim_{\delta \to 0} V_C(\hat{y}_A, \hat{y}_B, \hat{y}_A - v_A - \delta) &= F(\hat{y}_A - v_A) \end{aligned}$$

and

$$\begin{aligned} \lim_{\delta \to 0} V_C(\hat{y}_A, \hat{y}_B, \hat{y}_A - v_A - \delta) &= 2V_C(\hat{y}_A, \hat{y}_B, \hat{y}_A - v_A) \\ V_C(\hat{y}_A, \hat{y}_B, \hat{y}_A - v_A + \delta) &= 0 \text{ for } \delta \in (0, 2v_A) \\ \lim_{\delta \to 0} V_C(\hat{y}_A, \hat{y}_B, \hat{y}_A + v_A + \delta) &= 2V_C(\hat{y}_A, \hat{y}_B, \hat{y}_A + v_A). \end{aligned}$$

We moreover have that

$$\begin{aligned} V_C(\hat{y}_A, \hat{y}_B, \hat{y}_B + v_B + \delta) &< 1 - F(\hat{y}_B + v_B) = F(\hat{y}_A - v_A) \\ \lim_{\delta \to 0} V_C(\hat{y}_A, \hat{y}_B, \hat{y}_B + v_B + \delta) &= 1 - F(\hat{y}_B + v_B) = F(\hat{y}_A - v_A) \end{aligned}$$

and that

$$\begin{aligned}\lim_{\delta\to 0} V_C(\hat{y}_A, \hat{y}_B, \hat{y}_B + v_B + \delta) &= 2V_C(\hat{y}_A, \hat{y}_B, \hat{y}_B + v_B)\\ V_C(\hat{y}_A, \hat{y}_B, \hat{y}_B + v_B - \delta) &= 0 \text{ for } \delta \in (0, 2v_B)\\ \lim_{\delta\to 0} V_C(\hat{y}_A, \hat{y}_B, \hat{y}_B - v_B - \delta) &= 2V_C(\hat{y}_A, \hat{y}_B, \hat{y}_B - v_B).\end{aligned}$$

We further observe that $\hat{y}_C = \frac{1}{2} - v_A + v_B = \arg\max V_C(\hat{y}_A, \hat{y}_B,\, y_C)$ conditional on $y_C \in (\hat{y}_A + v_A, \hat{y}_B - v_B)$. This is due to the fact that for $y_C \in (\hat{y}_A + v_A, \hat{y}_B - v_B)$, it is the case that $V_C(\hat{y}_A, \hat{y}_B,\, y_C) = F(\frac{\hat{y}_B - v_B + y_C}{2}) - F(\frac{\hat{y}_A + v_A + y_C}{2})$. Since F' is symmetric about $\frac{1}{2}$ and $\frac{\hat{y}_B - v_B + y_C}{2} - \frac{\hat{y}_A + v_A + y_C}{2} = \frac{1}{2}\hat{y}_B - \frac{1}{2}\hat{y}_A - \frac{1}{2}v_A - \frac{1}{2}v_B$ is independent of y_C, it follows that $V_C(\hat{y}_A, \hat{y}_B,\, y_C)$ is maximized when $F'(\frac{\hat{y}_B - v_B + y_C}{2}) = F'(\frac{\hat{y}_A + v_A + y_C}{2})$, that is, when $\frac{\hat{y}_B - v_B + y_C}{2} = 1 - \frac{\hat{y}_A + v_A + y_C}{2}$. We moreover have that $\hat{y}_A - v_A = 1 - (\hat{y}_B + v_B)$ and thus that $\hat{y}_C = \frac{1}{2} - v_A + v_B = \arg\max V_C(\hat{y}_A, \hat{y}_B, y_C)$ conditional on $y_C \in (\hat{y}_A + v_A, \hat{y}_B - v_B)$.

The above imply that

$$V_A(\hat{y}_A, \hat{y}_B, \hat{y}_C) = V_B(\hat{y}_A, \hat{y}_B, \hat{y}_C) = F(\frac{\frac{1}{2} - v_A + v_B + \hat{y}_A + v_A}{2}) = F(\frac{1}{4} + \frac{1}{2}v_B + \frac{1}{2}\hat{y}_A)$$

and, hence, that

$$V_C(\hat{y}_A, \hat{y}_B, \hat{y}_C) = 1 - 2F(\frac{1}{4} + \frac{1}{2}v_B + \frac{1}{2}\hat{y}_A).$$

Therefore, $V_C(\hat{y}_A, \hat{y}_B,\, \hat{y}_C) > V_C(\hat{y}_A, \hat{y}_B, y_C)$ for any $y_C \neq \hat{y}_C$ and hence $E_\varepsilon(\hat{y}_A, \hat{y}_B) = \hat{y}_C = \frac{1}{2} - v_A + v_B$ for any $\varepsilon > 0$.

Before we proceed to the next steps of our proof we note the above finding suggests that $\pi^\varepsilon_A(\hat{y}_A, \hat{y}_B) = \pi^\varepsilon_B(\hat{y}_A, \hat{y}_B) = F(\frac{1}{4} + \frac{1}{2}v_B + \frac{1}{2}\hat{y}_A)$. This observation will be useful for the construction of the arguments that follow.

We now have to show that for each $y_A \neq \hat{y}_A$ there exists $\varepsilon(y_A) > 0$, such that for all $\varepsilon \in (0, \varepsilon(y_A))$, it is the case that $\pi^\varepsilon_A(y_A, \hat{y}_B) < \pi^\varepsilon_A(\hat{y}_A, \hat{y}_B)$ and that for every $y_B \neq \hat{y}_B$, there exists $\varepsilon(y_B) > 0$, such that for all $\varepsilon \in (0, \varepsilon(y_B))$, it is the case that $\pi^\varepsilon_B(\hat{y}_A, y_B) < \pi^\varepsilon_B(\hat{y}_A, \hat{y}_B)$ conditional on $b > 0$ being sufficiently small. We consider six cases of deviations for candidate A and then we briefly discuss how similar steps rule out existence of profitable deviations for candidate B as well.

Case 1 $y_A < \hat{y}_A$

It is true that for any $\varepsilon > 0$ we have that

$$E_\varepsilon(y_A, \hat{y}_B) = \frac{1}{2}(2 - v_A + v_B - y_A + \hat{y}_B)$$

or

$$E_\varepsilon(y_A, \hat{y}_B) = (\hat{y}_B - v_B - \delta(\varepsilon), \hat{y}_B - v_B)$$

and that

$$\frac{1}{2}(2 - v_A + v_B - y_A + \hat{y}_B) > \frac{1}{2} - v_A + v_B$$

where $\frac{\partial \delta(\varepsilon)}{\partial \varepsilon} \geq 0$ and $\lim_{\varepsilon \to 0} \delta(\varepsilon) = 0$.

This is due to the following fact: since $\lim_{\delta \to 0} V_C(\hat{y}_A, \hat{y}_B,\ \hat{y}_A - v_A - \delta) = V_C(\hat{y}_A, \hat{y}_B,\ \frac{1}{2} - v_A + v_B)$ and $\dot{y}_C(y_A, \hat{y}_B) = \frac{1}{2}(2 - v_A + v_B - y_A + \hat{y}_B) = \arg\max V_C(y_A, \hat{y}_B,\ y_C)$ is the unique solution of $F'(\frac{\hat{y}_B - v_B + y_C}{2}) = F'(\frac{y_A + v_A + y_C}{2})$ it directly follows that $\dot{y}_C(y_A, \hat{y}_B)$ is strictly decreasing in y_A and, hence, $V_C(y_A, \hat{y}_B, \dot{y}_C(y_A, \hat{y}_B))$ is strictly decreasing in y_A too. Therefore, and since $\lim_{\delta \to 0} V_C(y_A, \hat{y}_B,$ $\hat{y}_B + v_B + \delta) = \lim_{\delta \to 0} V_C(\hat{y}_A, \hat{y}_B,\ \hat{y}_B + v_B + \delta)$ when $y_A < \hat{y}_A$, we have that: a) if $\frac{1}{2}(2 - v_A + v_B - y_A + \hat{y}_B) < \hat{y}_B - v_B$ then $E_\varepsilon(y_A, \hat{y}_B) = \arg\max V_C(y_A, \hat{y}_B,\ y_C) = \frac{1}{2}(2 - v_A + v_B - y_A + \hat{y}_B)$ and b) if $\frac{1}{2}(2 - v_A + v_B - y_A + \hat{y}_B) \geq \hat{y}_B - v_B$ then $E_\varepsilon(y_A, \hat{y}_B) = (\hat{y}_B - v_B - \delta(\varepsilon), \hat{y}_B - v_B)$.

Now since $\frac{\frac{1}{2}(2 - v_A + v_B - y_A + \hat{y}_B) + y_A + v_A}{2} < \frac{\frac{1}{2} - v_A + v_B + \hat{y}_A + v_A}{2}$ and since $\pi^\varepsilon_A(\hat{y}_A, \hat{y}_B) = F(\frac{1}{4} + \frac{1}{2}v_B + \frac{1}{2}\hat{y}_A)$ for any $\varepsilon > 0$ it directly follows that we should have $\pi^\varepsilon_A(y_A, \hat{y}_B) = F(\frac{\frac{1}{2}(2 - v_A + v_B - y_A + \hat{y}_B) + y_A + v_A}{2}) <$ $F(\frac{1}{4} + \frac{1}{2}v_B + \frac{1}{2}\hat{y}_A)$ when $\frac{1}{2}(2 - v_A + v_B - y_A + \hat{y}_B) < \hat{y}_B - v_B$ and $\pi^\varepsilon_A(y_A, \hat{y}_B) \leq F(\frac{\hat{y}_B - v_B + y_A + v_A}{2}) <$ $F(\frac{1}{4} + \frac{1}{2}v_B + \frac{1}{2}\hat{y}_A)$ when $\frac{1}{2}(2 - v_A + v_B - y_A + \hat{y}_B) \geq \hat{y}_B - v_B$. In other words, there exists $\varepsilon(y_A)$, such that for all $\varepsilon \in (0, \varepsilon(y_A))$, it is the case that $\pi^\varepsilon_A(y_A, \hat{y}_B) < \pi^\varepsilon_A(\hat{y}_A, \hat{y}_B)$ for all $y_A < \hat{y}_A$.

Case 2 $y_A \in (\hat{y}_A, \hat{y}_B - v_A + v_B)$

In this case it is true that there exists $\varepsilon(y_A)$, such that for all $\varepsilon \in (0, \varepsilon(y_A))$ we have that

$$E_\varepsilon(y_A, \hat{y}_B) = (y_A - v_A - \delta(\varepsilon), y_A - v_A)$$

where $\frac{\partial \delta(\varepsilon)}{\partial \varepsilon} \geq 0$ and $\lim_{\varepsilon \to 0} \delta(\varepsilon) = 0$.

This is due to the fact that for $y_A \in (\hat{y}_A, \hat{y}_B - v_A + v_B)$ we have: a) $\lim_{\delta \to 0} V_C(y_A, \hat{y}_B, y_A - v_A - \delta) > F(\hat{y}_A - v_A)$, b) $\lim_{\delta \to 0} V_C(y_A, \hat{y}_B, \hat{y}_B + v_B + \delta) = F(\hat{y}_A - v_A)$ and c) $V_C(y_A, \hat{y}_B, y_C) < F(\hat{y}_A - v_A)$ for any $y_C \in (y_A + v_A, y_B - v_B)$.

Therefore, $\pi_A^\varepsilon(y_A, \hat{y}_B) \leq F(\frac{y_A + v_A + \hat{y}_B - v_B}{2}) - F(\frac{y_A - v_A + y_A - v_A - \delta(\varepsilon)}{2})$. This implies that $\lim_{\varepsilon \to 0} \pi_A^\varepsilon(y_A, \hat{y}_B) = F(\frac{y_A + v_A + \hat{y}_B - v_B}{2}) - F(y_A - v_A)$. We notice that due to unimodality of the density of F and since $F'(\frac{1}{2}) - 2F'(0) \leq 0$ it must hold that $\frac{\partial \lim_{\varepsilon \to 0} \pi_A^\varepsilon(y_A, \hat{y}_B)}{\partial y_A} \leq 0$ for any $y_A \in (\hat{y}_A, \hat{y}_B - v_A + v_B)$. So $\lim_{\varepsilon \to 0} \pi_A^\varepsilon(y_A, \hat{y}_B) \leq \lim_{y_A \to \hat{y}_A^+}(\lim_{\varepsilon \to 0} \pi_A^\varepsilon(y_A, \hat{y}_B)) = F(\frac{\hat{y}_A + v_A + \hat{y}_B - v_B}{2}) - F(\hat{y}_A - v_A)$. We moreover know that for $b = 0$, $F(\frac{\hat{y}_A + v_A + \hat{y}_B - v_B}{2}) - F(\hat{y}_A - v_A) + \zeta = F(\frac{1}{4} + \frac{1}{2}v_B + \frac{1}{2}\hat{y}_A)$ where $\zeta > 0$ is a non-degenerate positive number[10]. That is, there exists $b > 0$ and $\varepsilon(y_A)$, such that for all $\varepsilon \in (0, \varepsilon(y_A))$, it is the case that $\pi_A^\varepsilon(y_A, \hat{y}_B) < \pi_A^\varepsilon(\hat{y}_A, \hat{y}_B)$ for all $y_A \in (\hat{y}_A, \hat{y}_B - v_A + v_B)$.

Case 3 $y_A \in (\hat{y}_B - v_A + v_B, \hat{y}_B + v_A - v_B)$

In this case we can verify that there exists $\varepsilon(y_A)$, such that for all $\varepsilon \in (0, \varepsilon(y_A))$ we have that

$$E_\varepsilon(y_A, \hat{y}_B) = (y_A - v_A - \delta(\varepsilon), y_A - v_A)$$

where $\frac{\partial \delta(\varepsilon)}{\partial \varepsilon} \geq 0$ and $\lim_{\varepsilon \to 0} \delta(\varepsilon) = 0$.

This is due to the fact that for $b = 0$, $\hat{y}_B - \zeta = \frac{1}{2}$ where $\zeta > 0$ is a non-degenerate positive number. That is, there exists $b > 0$ such that $\hat{y}_B - v_A + v_B > \frac{1}{2}$ and hence all the ε-best responses of the entrant are to the left of $y_A - v_A$ for $\varepsilon > 0$ small enough. For $b \to 0$ we have that $\lim_{\varepsilon \to 0} \pi_A^\varepsilon(y_A, \hat{y}_B) = F(\hat{y}_A - v_A) = F(\frac{1}{4} + \frac{1}{2}v_B + \frac{1}{2}\hat{y}_A) - \zeta$ where $\zeta > 0$ is a non-degenerate positive

[10] In specific Palfrey (1984) shows that (for $b = 0$) $F(\hat{y}_A - v_A) \geq \frac{1}{4}$ and that $F(\frac{1}{4} + \frac{1}{2}v_B + \frac{1}{2}\hat{y}_A) \geq \frac{1}{3}$. Hence, $F(\frac{\hat{y}_A + v_A + \hat{y}_B - v_B}{2}) - F(\hat{y}_A - v_A) = \frac{1}{2} - F(\hat{y}_A - v_A) \leq \frac{1}{4}$.

number. Therefore, there exists $b > 0$ and $\varepsilon(y_A)$, such that for all $\varepsilon \in (0, \varepsilon(y_A))$, it is the case that $\pi^\varepsilon_A(y_A, \hat{y}_B) < \pi^\varepsilon_A(\hat{y}_A, \hat{y}_B)$ for all $y_A \in (\hat{y}_B - v_A + v_B, \hat{y}_B + v_A - v_B)$.

Case 4 $y_A > \hat{y}_B + v_A - v_B$

In this case there exists $\varepsilon(y_A)$, such that for all $\varepsilon \in (0, \varepsilon(y_A))$ we have that

$$E_\varepsilon(y_A, \hat{y}_B) = (\hat{y}_B - v_B - \delta(\varepsilon), \hat{y}_B - v_B)$$

where $\frac{\partial \delta(\varepsilon)}{\partial \varepsilon} \geq 0$ and $\lim_{\varepsilon \to 0} \delta(\varepsilon) = 0$.

Again this is due to the fact that for $b = 0$, $\hat{y}_B - \zeta = \frac{1}{2}$ where $\zeta > 0$ is a non-degenerate positive number. That is, there exists $b > 0$ such that $\hat{y}_B - v_B > \frac{1}{2}$ and hence all the ε-best responses of the entrant are to the left of $\hat{y}_B - v_B$ for $\varepsilon > 0$ small enough. For $b \to 0$ we have that $\lim_{\varepsilon \to 0} \pi^\varepsilon_A(y_A, \hat{y}_B) = 1 - F(\frac{y_A - v_A + \hat{y}_B + v_B}{2}) < F(\hat{y}_A - v_A) = F(\frac{1}{4} + \frac{1}{2}v_B + \frac{1}{2}\hat{y}_A) - \zeta$ where $\zeta > 0$ is a non-degenerate positive number. Therefore, there exists $b > 0$ and $\varepsilon(y_A)$, such that for all $\varepsilon \in (0, \varepsilon(y_A))$, it is the case that $\pi^\varepsilon_A(y_A, \hat{y}_B) < \pi^\varepsilon_A(\hat{y}_A, \hat{y}_B)$ for all $y_A > \hat{y}_B + v_A - v_B$.

Case 5 $y_A = \hat{y}_B - v_A + v_B$

In this case there exists $\varepsilon(y_A)$, such that for all $\varepsilon \in (0, \varepsilon(y_A))$ we have that

$$E_\varepsilon(y_A, \hat{y}_B) = (y_A - v_A - \delta(\varepsilon), y_A - v_A)$$

where $\frac{\partial \delta(\varepsilon)}{\partial \varepsilon} \geq 0$ and $\lim_{\varepsilon \to 0} \delta(\varepsilon) = 0$.

This is due to the fact that for $b = 0$, $\hat{y}_B - \zeta = \frac{1}{2}$ where $\zeta > 0$ is a non-degenerate positive number. That is, there exists $b > 0$ such that $\hat{y}_B - v_A + v_B > \frac{1}{2}$ and hence all the ε-best responses of the entrant are to the left of $y_A - v_A$ for $\varepsilon > 0$ small enough. For $b \to 0$ we have that $\lim_{\varepsilon \to 0} \pi^\varepsilon_A(y_A, \hat{y}_B) = \frac{1}{2}F(\hat{y}_A - v_A) < F(\hat{y}_A - v_A) = F(\frac{1}{4} + \frac{1}{2}v_B + \frac{1}{2}\hat{y}_A) - \zeta$ where $\zeta > 0$ is a non-degenerate positive number. Therefore, there exists $b > 0$ and $\varepsilon(y_A)$, such that for all $\varepsilon \in (0, \varepsilon(y_A))$, it is the case that $\pi^\varepsilon_A(y_A, \hat{y}_B) < \pi^\varepsilon_A(\hat{y}_A, \hat{y}_B)$ for $y_A = \hat{y}_B - v_A + v_B$.

Case 6 $y_A = \hat{y}_B + v_A - v_B$

In this case there exists $\varepsilon(y_A)$, such that for all $\varepsilon \in (0, \varepsilon(y_A))$ we have that

$$E_\varepsilon(y_A, \hat{y}_B) = (y_A - v_A - \delta(\varepsilon), y_A - v_A)$$

where $\frac{\partial \delta(\varepsilon)}{\partial \varepsilon} \geq 0$ and $\lim_{\varepsilon \to 0} \delta(\varepsilon) = 0$.

This is due to the fact that for $b = 0$, $\hat{y}_B - \zeta = \frac{1}{2}$ where $\zeta > 0$ is a non-degenerate positive number. That is, there exists $b > 0$ such that $\hat{y}_B - v_A + v_B > \frac{1}{2}$ and hence all the ε-best responses of the entrant are to the left of $y_A - v_A$ for $\varepsilon > 0$ small enough. For $b \to 0$ we have that $\lim_{\varepsilon \to 0} \pi^\varepsilon_A(y_A, \hat{y}_B) = F(\hat{y}_A - v_A) = F(\frac{1}{4} + \frac{1}{2}v_B + \frac{1}{2}\hat{y}_A) - \zeta$ where $\zeta > 0$ is a non-degenerate positive number. Therefore, there exists $b > 0$ and $\varepsilon(y_A)$, such that for all $\varepsilon \in (0, \varepsilon(y_A))$, it is the case that $\pi^\varepsilon_A(y_A, \hat{y}_B) < \pi^\varepsilon_A(\hat{y}_A, \hat{y}_B)$ for $y_A = \hat{y}_B + v_A - v_B$.

We have, thus, proved that for each $y_A \neq \hat{y}_A$ there exists $\varepsilon(y_A)$, such that for all $\varepsilon \in (0, \varepsilon(y_A))$, it is the case that $\pi^\varepsilon_A(y_A, \hat{y}_B) < \pi^\varepsilon_A(\hat{y}_A, \hat{y}_B)$. We can construct six similar cases for player B bearing in mind that the arguments of cases one, two and four are equivalent while the arguments for case three become trivial. When $y_B \in (\hat{y}_A + v_A - v_B, \hat{y}_A - v_A + v_B)$ then $\pi^\varepsilon_B(\hat{y}_A, y_B) = 0$ for any $\varepsilon > 0$. The arguments for the degenerate cases five and six follow from the previous four main cases as for player A. ■

As we have noted in the proof $\pi^\varepsilon_A(\hat{y}_A, \hat{y}_B) = \pi^\varepsilon_B(\hat{y}_A, \hat{y}_B)$; the vote-shares of both established candidates in the described *Quasi-SPE* are identical. Moreover, we generically (that is, for the case in which the established candidates have asymmetric valences[11]) have $|\frac{1}{2} - \hat{y}_A| < |\frac{1}{2} - \hat{y}_B|$ and, hence, the main result of our paper: the high valence established candidate offers a more moderate platform than the low valence established candidate while both receive the same vote-shares.

[11] When there are no valence asymmetries between the two established candidates, that is, when $v_A = v_B \geq 0$ we naturally have that $|\frac{1}{2} - \hat{y}_A| = |\frac{1}{2} - \hat{y}_B|$. Kim (2005) provides an equivalent result by studying this particular case (two established candidates of equal valence face an entrant of lower valence) when the distribution of voters is uniform.

Since we now have the existence proof of a certain *Quasi-SPE* we can revisit the unique extra constraint (compared to Palfrey, 1984) that we imposed on the distribution of voter's ideal policies. We have assumed that $F'(\frac{1}{2})-2F'(0)\leq 0$. That is, that the concentration of voters in all compact subsets of the policy space of equal measure is sufficiently symmetric. This was only used in Case 2 of the proof presented above. We observe that we actually needed only a part of this condition. Our results would still be valid if we had only assumed that $F'(\frac{1}{2})-2F'(\psi)\leq 0$ for $\psi\in(\hat{y}_A-v_A,\hat{y}_B+v_B)$. By relaxing our assumption in this manner it becomes evident that the described equilibrium would hold for a wide class of popular distribution functions. Beta distribution functions, for example, with symmetric densities about the center of the policy space can support the described *Quasi-SPE* for a certain range of their shape parameters under this less stringent condition ($F'(\frac{1}{2})-2F'(\psi)\leq 0$ for $\psi\in(\hat{y}_A-v_A,\hat{y}_B+v_B)$).

Unfortunately, conditions on the nature of the distribution of ideal policies cannot be avoided when we seek to identify equilibrium strategies of an electoral competition game among candidates of asymmetric valence. For example, Aragonès and Palfrey (2002) specifically consider a uniform distribution of voters' ideal policies and Aragonès and Xefteris (2012) specifically assume a certain class of unimodal distributions in order to fully characterize an equilibrium.

Next we discuss equilibrium uniqueness.

Theorem 2 *Whenever the Quasi-SPE of Theorem 1 exists then: a)* $(1-\hat{y}_A,1-\hat{y}_B,1-\hat{y}_C)$ *is also a Quasi-SPE and b) no third Quasi-SPE exists.*

Proof. The first part of this theorem follows trivially from the fact that the density of F is symmetric about the center of the policy space. Now assume that there exists a *Quasi-SPE,* $(\mathring{y}_A,\mathring{y}_B,\mathring{y}_C)$, on top of these two mirror ones. Without loss of generality consider that in this equilibrium $\mathring{y}_A<\mathring{y}_B$ (the case $\mathring{y}_A=\mathring{y}_B$ is trivially ruled out as player B has clear incentives to deviate) and that $(\mathring{y}_A,\mathring{y}_B)\neq(0,1)$ (this case is trivially ruled out as A, for example, has clear incentives to deviate from 0 to ε for $\varepsilon>0$ sufficiently small - see case 2 of the proof of proposition 1 for arguments which back up this claim). Notice, that it should be the case that $\mathring{y}_A<\mathring{y}_C<\mathring{y}_B$

because for any pair $(y_A, y_B) \neq (0,1)$ such that $y_A < y_B$ player C has no unique well-defined best response in $[0, y_A] \cup [y_B, 1]$.[12] Moreover, notice that $\hat{y}_A$ and $\hat{y}_B$ are the unique values that solve

$$\lim_{\delta \to 0} V_C(y_A, y_B, y_A - v_A - \delta) = \lim_{\delta \to 0} V_C(y_A, y_B, y_B + v_B + \delta) = V_C(y_A, y_B, \dot{y}_C(y_A, y_B))$$

where $\dot{y}_C(y_A, y_B) = \arg\max V_C(y_A, y_B,\, y_C)$ conditional on $y_C \in (y_A, y_B)$.

That is, if such third *Quasi-SPE*, $(\mathring{y}_A, \mathring{y}_B, \mathring{y}_C)$, exists it should be such that $V_C(\mathring{y}_A, \mathring{y}_B,\, \mathring{y}_C) > \min\{\lim_{\delta\to 0} V_C(\mathring{y}_A, \mathring{y}_B, \mathring{y}_A - v_A - \delta), \lim_{\delta\to 0} V_C(\mathring{y}_A, \mathring{y}_B, \mathring{y}_B + v_B + \delta)\}$. Again without loss of generality assume that $V_C(\mathring{y}_A, \mathring{y}_B,\, \mathring{y}_C) > \lim_{\delta\to 0} V_C(\mathring{y}_A, \mathring{y}_B, \mathring{y}_A - v_A - \delta)$. This implies that there exists $\zeta > 0$ such that $V_A(\mathring{y}_A + \zeta, \mathring{y}_B,\, \dot{y}_C(\mathring{y}_A + \zeta, \mathring{y}_B)) > V_A(\mathring{y}_A, \mathring{y}_B,\, \mathring{y}_C)$ (this follows directly from the arguments employed in Case 1 of the proof of Theorem 1) where $\dot{y}_C(\mathring{y}_A + \zeta, \mathring{y}_B) = \arg\max V_C(\mathring{y}_A + \zeta, \mathring{y}_B,\, y_C)$ and $\dot{y}_C(\mathring{y}_A + \zeta, \mathring{y}_B) \in (\mathring{y}_A + \zeta, \mathring{y}_B)$. That is, $(\mathring{y}_A, \mathring{y}_B, \mathring{y}_C)$ cannot be a *Quasi-SPE* of the game. ■

We conclude our formal analysis by presenting some comparative statics of the essentially unique equilibrium characterized above. The equilibrium condition $F(\hat{y}_A - v_A) = 1 - 2F(\frac{1}{4} + \frac{1}{2}v_B + \frac{1}{2}\hat{y}_A)$ suggests that

$$\frac{\partial \hat{y}_A}{\partial v_A} = \frac{F'(\hat{y}_A - v_A)}{F'(\hat{y}_A - v_A) + F'(\frac{1}{4} + \frac{1}{2}v_B + \frac{1}{2}\hat{y}_A)} > 0$$

and that

$$\frac{\partial \hat{y}_A}{\partial v_B} = \frac{-F'(\frac{1}{4} + \frac{1}{2}v_B + \frac{1}{2}\hat{y}_A)}{F'(\hat{y}_A - v_A) + F'(\frac{1}{4} + \frac{1}{2}v_B + \frac{1}{2}\hat{y}_A)} < 0.$$

We notice that $\frac{\partial \hat{y}_A}{\partial v_A} < 1$ and that $\frac{\partial \hat{y}_A}{\partial v_B} > -1$ and, hence, the equilibrium condition $\hat{y}_B = 1 - \hat{y}_A + v_A - v_B$ suggests that

$$\frac{\partial \hat{y}_B}{\partial v_A} = -\frac{\partial \hat{y}_A}{\partial v_A} + 1 > 0$$

[12] Just for the completeness of the argument let us note that we consider that the (non-conventional) object $[x, x]$ is the singleton $\{x\}$. This clarification is necessary as a pair $(y_A, y_B) \neq (0,1)$ with $y_A < y_B$ might be such that $y_A = 0$ and $y_B < 1$ or such that $y_A > 0$ and $y_B = 1$. That is, $[0, y_A]$ might be the singleton $\{0\}$ or $[y_B, 1]$ might be the singleton $\{1\}$ but never both ($\{0,1\} \subset [0, y_A] \cup [y_B, 1]$).

and that

$$\frac{\partial \hat{y}_B}{\partial v_B} = -\frac{\partial \hat{y}_A}{\partial v_B} - 1 < 0.$$

Moreover, by $\hat{y}_C = \frac{1}{2} - v_A + v_B$ it trivially follows that $\frac{\partial \hat{y}_C}{\partial v_A} < 0$ and that $\frac{\partial \hat{y}_C}{\partial v_B} > 0$. That is, an increase in the valence level of an established candidate leads: a) this established candidate to offer a more moderate platform, b) the other established candidate to offer a more extremist platform and c) the entrant to locate closer to the established candidate whose valence has increased.

Finally, we consider an equal joint increase of the valences of the established candidates. To this end, we define $v = v_B = v_A - \theta$ where $\theta \geq 0$ is fixed and sufficiently small and we write the equilibrium condition as $F(\hat{y}_A - v - \theta) = 1 - 2F(\frac{1}{4} + \frac{1}{2}v + \frac{1}{2}\hat{y}_A)$. This condition yields

$$\frac{\partial \hat{y}_A}{\partial v} = \frac{F'(\hat{y}_A - v - \theta) - F'(\frac{1}{4} + \frac{1}{2}v + \frac{1}{2}\hat{y}_A)}{F'(\hat{y}_A - v - \theta) + F'(\frac{1}{4} + \frac{1}{2}v + \frac{1}{2}\hat{y}_A)} < 0$$

$$\frac{\partial \hat{y}_B}{\partial v} = -\frac{\partial \hat{y}_A}{\partial v} > 0$$

and

$$\frac{\partial \hat{y}_C}{\partial v} = 0.$$

Hence, the degree of policy differentiation between the two established candidates does not just depend on the valence difference between these two candidates but also on the valence level of the entrant.[13] That is, the higher the valence asymmetry between the established candidates and the entrant, the larger the degree of policy differentiation between the two established candidates.

[13]Recall that the assumption that $v_C = 0$ is without loss of generality. If we considered $v_C > 0$ instead, then all the conditions of the equilibrium should be re-written by substituting v_A with $v_A - v_C$ and v_B with $v_B - v_C$. That is, an increase in the difference between the valence level of an established candidate and the valence level of the entrant (when the valence difference between the established candidates is fixed) can be seen either as an equal increase in the valence levels of the established candidates or as a decrease in the valence level of the entrant.

4 An example of an "almost" uniform distribution.

Consider now that $F \rightarrow U[0,1]$. In this case we should have that

$$F(\hat{y}_A - v_A) \rightarrow \hat{y}_A - v_A$$

$$1 - 2F(\tfrac{1}{4} + \tfrac{1}{2}v_B + \tfrac{1}{2}\hat{y}_A) \rightarrow 1 - 2(\tfrac{1}{4} + \tfrac{1}{2}v_B + \tfrac{1}{2}\hat{y}_A)$$

and, therefore, the *Quasi-SPE* $(\hat{y}_A, \hat{y}_B, \hat{y}_C)$ should be such that

$$\hat{y}_A \rightarrow \tfrac{1}{2}v_A - \tfrac{1}{2}v_B + \tfrac{1}{4}$$

$$\hat{y}_B \rightarrow \tfrac{1}{2}v_A - \tfrac{1}{2}v_B + \tfrac{3}{4}$$

and

$$\hat{y}_C = \tfrac{1}{2} - v_A + v_B.$$

We moreover find that in this case the admissible values for $b > 0$ are relatively large. In specific our equilibrium exists for any $\frac{1}{10} > v_A \geq v_B \geq v_C = 0$.

It is evident that, as in the general case, the degree of extremism of the (dis)advantaged established candidate is (decreasing) increasing in the size of the advantage $(v_A - v_B)$. Moreover, since we always have that $\hat{y}_C = \frac{1}{2} - v_A + v_B$ the entrant comes closer to the advantaged candidate when the valence asymmetry between the two established candidates increases (see Figure 1).

[Insert Figure 1 about here]

Note that, since

$$\pi^{\varepsilon}_A(\hat{y}_A, \hat{y}_B) = \pi^{\varepsilon}_B(\hat{y}_A, \hat{y}_B) \rightarrow \tfrac{1}{4}v_A + \tfrac{1}{4}v_B + \tfrac{3}{8},$$

it follows that the identical payoff of the two established candidates is strictly increasing in the difference between their cumulative valence and the valence of the entrant ($v_A + v_B - v_C$).

Finally, we notice that when $v_A = v_B = v > 0$ we have that $\hat{y}_A \to \frac{1}{4}$ and $\hat{y}_B \to \frac{3}{4}$ (the equilibrium play of both established candidates is independent of the exact value of their common valence $v > 0$). As we argued in the end of the previous section this is not true for strictly unimodal distributions of the voters' ideal policies. When $v_A = v_B = v > 0$ the equilibrium condition $F(\hat{y}_A - v) = 1 - 2F(\frac{1}{4} + \frac{1}{2}v + \frac{1}{2}\hat{y}_A)$ along with the strict unimodality of F' imply that $\frac{\partial \hat{y}_A}{\partial v} < 0$. Hence, the importance of the valence asymmetry between the established candidates and the entrant becomes less important when the density of F becomes "flatter" (that is, when it approaches $U[0,1]$).

5 Probabilistic entry

Here we briefly and informally discuss what would happen if we assumed that the third candidate entered the race after the two established candidates selected their platforms with probability $p < 1$ and did not participate in the elections with probability $1 - p$. Considering the location pair $(\hat{y}_A, \hat{y}_B)$ we first explore deviation incentives of candidate A. From case 1 of the proof of proposition 1 it is evident that if the third candidate enters, candidate A prefers to be located at $\hat{y}_A$ compared to any location to the left of $\hat{y}_A$. It is moreover straightforward that candidate A prefers to be located at $\hat{y}_A$ compared to any location to the left of $\hat{y}_A$ even if no third candidate enters the race. Hence, independently of the exact value of p, candidate A prefers to be located at $\hat{y}_A$ compared to any location to the left of $\hat{y}_A$. Things are distinctly more complicated as far as deviations to locations to the right of $\hat{y}_A$ are concerned. As we saw in cases 2, 3, 4, 5 and 6 of the proof of proposition 1, which all correspond to deviations to the right of $\hat{y}_A$, if a third candidate enters and A is located to the right of $\hat{y}_A$, not only the payoff of candidate A is smaller compared to the payoff she would get if she were located at $\hat{y}_A$ but most importantly there exists a non-degenerate $\zeta > 0$ such that $\pi_A^{\varepsilon}(\hat{y}_A, \hat{y}_B) - \pi_A^{\varepsilon}(y, \hat{y}_B) > \zeta$ for every $y > \hat{y}_A$. This is very important as it suggests that if p is sufficiently large candidate A has no incentives to deviate to a location the right of $\hat{y}_A$ even if that location gives her the maximum possible payoff in case no third candidate enters the race. For example one can easily validate that any value of p strictly larger than $\frac{1}{1+\zeta}$ is sufficient to make $\hat{y}_A$ the unique best response of candidate A to B playing $\hat{y}_B$. The arguments why candidate B does not wish to deviate away from $\hat{y}_B$ when A is expected to locate at $\hat{y}_A$ and p is sufficiently large are equivalent.

We moreover notice that the location pair $(\hat{y}_A, \hat{y}_B)$ is a local equilibrium of the above extension of the game for any $p > 0$. This is because: a) if candidate A marginally deviates to the left of $\hat{y}_A$ she is worse off both when a third candidate enters and when no third candidate enters and b) if candidate A marginally deviates to the right of $\hat{y}_A$ she is marginally better off when a third candidate does not enter but *significantly worse off* when a third candidate enters. That is, for every value of p there exists $\gamma > 0$ such that $\hat{y}_A = \arg\max_{y \in [\hat{y}_A - \gamma, \hat{y}_A + \gamma]} \pi_A^{\varepsilon}(y, \hat{y}_B)$. Equivalent arguments apply for candidate B and hence $(\hat{y}_A, \hat{y}_B)$ is a local equilibrium for any $p > 0$.

6 Concluding remarks

We have demonstrated existence of an essentially unique pure strategy equilibrium in the entry model considering that candidates may be characterized by asymmetric valences. Our analysis is the first to produce a pure strategy equilibrium play of the two heterogeneous (in terms of valence) main candidates when candidates are purely office-motivated. Earlier studies on Downsian competition between candidates of unequal valence (Aragonès and Palfrey 2002; Hummel 2010; Aragonès and Xefteris 2012) suggest that: a) the high valence candidate chooses a more moderate platform than the low valence candidate and that b) the vote-share of the high valence candidate (in expected terms) is strictly larger than the vote-share of the low valence candidate. The entry model that we just analyzed suggests that the first prediction of these models is more robust than the second one. We have proved that in the unique equilibrium of the present game, indeed, the high valence established candidate chooses a more moderate platform than the low valence established candidate and that they both receive equal vote-shares.

As far as the size of valence asymmetries is concerned, the entry model is the unique one that produces complete results for a small but non-degenerate difference in the valence levels of the two established candidates and for a sufficiently dispersed electorate. The most important implication of this characterization of a pure strategy equilibrium is that stable equilibrium outcomes can occur even if candidates differ in valence. Models which did not account for entry resulted in equilibria in which both players mix when valence differences are small but non-degenerate.

As far as robustness of the results is concerned we deem important to note that further studies should care to investigate possibility of strategic choice of entry and alternative payoff functions. Since this is the first work on the effect of candidate entry when the two established candidates are asymmetric in terms of valence, it should be the case that it focused in generalizing the original entry model of Palfrey (1984) and not its subsequent variants. Regarding voters' utility functions, one should note that considering a multiplicative valence model and/or non-linear loss functions should not upset the main messages of the present study. Such generalizations would most probably affect only certain aspects of our equilibrium and not its fundamental characteristics. For example, we conjecture that a model with strictly concave loss functions would mitigate the symmetry of the equilibrium payoffs of the two established candidates without eliminating the egalitarian effect that the addition of an entrant has on the equilibrium payoffs of the two main candidates. We finally conjecture that a model with a multiplicative valence component would generate new incentives for the entrant to approach the advantaged established candidate and new types of limitations which would constrain the entrant away from both established candidates without changing any of the qualitative implications of the present equilibrium.

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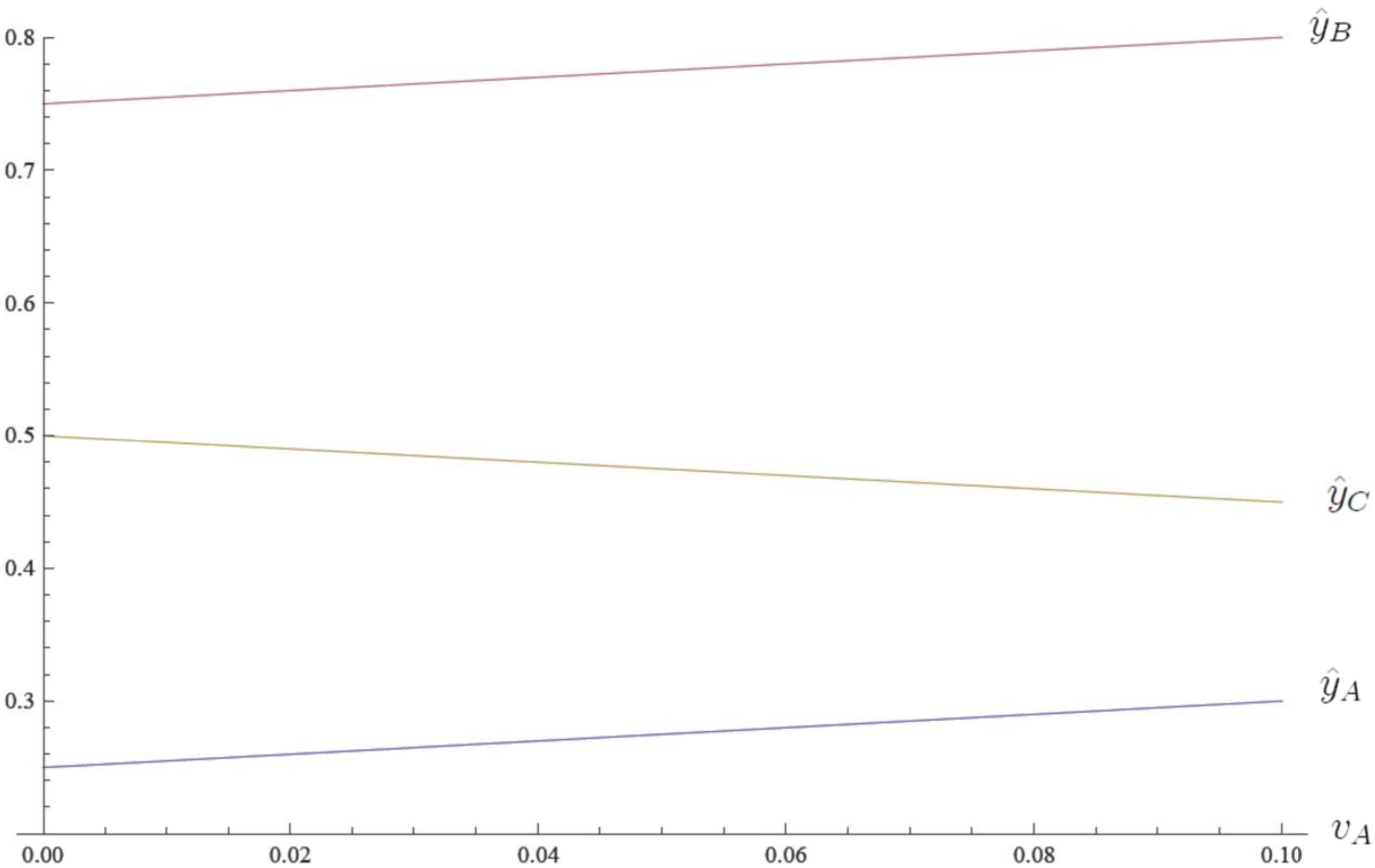

Figure 1. Equilibrium locations as a function of v_A when $v_B = v_C = 0$.